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AMB NONLINEAR CONTROL WITH INPUT-OUTPUT EXACT LINEARIZATION CONSIDERING MODAL CHARACTERISTICS

Michihiro Kawanishi
kawa@mech.kobe-u.ac.jp

Hiroshi Kanki
kanki@mech.kobe-u.ac.jp

Department of Mechanical Engineering, Kobe University
Kobe city, Hyogo Prefecture, Japan

Abstract : In this paper, exact linearization technique considering modal characteristics of a rotor is proposed for AMB systems. The 1st and the 2nd modes of the rotor are taken into account of the control system design. In order to utilize modal characteristics of the rotor with exact linearization technique, AMB control systems are treated as MIMO systems. The effectiveness of the proposed method is confirmed by levitation experiments.

1 INTRODUCTION

Nonlinearity of magnetic force is one of difficulties for AMB (Active Magnetic Bearing) control. To overcome the difficulty, push-pull coil configuration has been widely used. From the view point of control methods, almost all of the AMB control systems are however designed as linear due to its simplicity and easiness of the parameter tuning. In order to improve the performance of the AMB control system, it is necessary to develop a nonlinear control method for AMB.

Recently, exact linearization technique[1] has been adopted to AMB control[2][3]. Conventional exact linearization technique for AMB treats the AMB systems as several SISO (Single Input Single Output) subsystems of each axis. Input-State linearization is then carried out for each SISO subsystem. However, due to the lack of the consideration of the rotor dynamics, the AMB control system based on the SISO subsystems of each axis has the limitation at critical situations, e.g. critical speed and spillover. The MIMO (Multi Inputs Multi Outputs) AMB model considering the dynamics of the rotor is essential for constructing the high performance control system based on recent advanced control theory.

In this paper, exact linearization technique considering modal characteristics of the rotor is proposed for AMB systems. In order to utilize modal characteristics of the rotor with exact linearization technique, AMB systems are treated as MIMO systems, i.e. 4 inputs (current) and 4 outputs (gap), and the control system is designed as MIMO system. The 1st and the 2nd mode of the rotor are taken into account of the control system design. The modal characteristics of the rotor enables us to construct the high performance control systems.

In case that AMB systems are treated as MIMO systems which consider the dynamics of the rotor, the Input-State linearization technique cannot be applicable because of the complexity of the models. It can be proven that there exist no AMB nonlinear feedback controllers which attain Input-State linearization. It is however shown that the Input-Output exact linearization technique can be applicable. Based on the results, the nonlinear state feedback controller, i.e. the inputs exchange and the coordinate transformation, is constructed. The MIMO Input-Output exact linearization of AMB systems enables us to consider the dynamics of the rotor on linear controller design procedure. It then make recent advanced linear control methods, e.g. H_∞, Gain Scheduling, applicable in wide range of the AMB systems variables. The effectiveness of the proposed control method is evaluated by experiments.

2 MIMO MODEL OF AMB

MIMO model of AMB is here derived. The schematic of a considered AMB rotor system is shown in Fig.1. Let the mass of the rotor be M, the moment of inertia J, displacement from the equilibrium point x_G and the inclination angle of rotor θ_G. And let the displacements at left and right AMB be x_{lb} and x_{rb}, the displacements at left and right sensor x_{ls} and x_{rs} respectively. The length from the rotor center of gravity to the bearing is l_b and the length from the rotor center of gravity to the sensor is l_s. The rotor is assumed to be rigid.

The following relationships then hold.

$$x_{ls} = x_G - l_s\theta_G, \quad (1)$$

$$x_{rs} = x_G + l_s\theta_G, \quad (2)$$

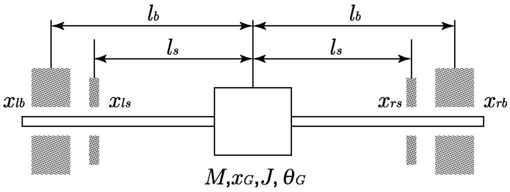

Figure 1: AMB rotor model

$$x_{lb} = x_G - l_b\theta_G, \tag{3}$$
$$x_{rb} = x_G + l_b\theta_G. \tag{4}$$

The inductance of upper and lower left coils L_{l1} and L_{l2} are expressed as

$$L_{l1} = \frac{Q}{X + W - x_{lb}} + L_0, \tag{5}$$
$$L_{l2} = \frac{Q}{X + W + x_{lb}} + L_0, \tag{6}$$

where Q, X and L_0 are constant parameters determined by identification experiments, and W is the gap between the rotor and bearing at the equilibrium point. Similarly, the inductance of upper and lower right coils L_{r1} and L_{r2} are

$$L_{r1} = \frac{Q}{X + W - x_{rb}} + L_0, \tag{7}$$
$$L_{r2} = \frac{Q}{X + W + x_{rb}} + L_0. \tag{8}$$

The magnetic levitation forces of left and right AMB are denoted by f_l and f_r respectively. The dynamical equations are then expressed as

$$M\ddot{x}_G = -Mg + f_l + f_r, \tag{9}$$
$$J\ddot{\theta}_G = (f_r - f_l)l_b. \tag{10}$$

The currents of upper and lower coils of left AMB are i_{l1} and i_{l2}, and the current of coils of right AMB are i_{r1} and i_{r2}. The magnetic levitation forces f_l and f_r are described as follows:

$$f_l = k_l\left(\frac{I_1 + \tilde{i}_{l1}}{X_W - x_{lb}}\right)^2 - k_l\left(\frac{I_2 - \tilde{i}_{l2}}{X_W + x_{lb}}\right)^2, \tag{11}$$
$$f_r = k_r\left(y\frac{I_1 + \tilde{i}_{r1}}{X_W - x_{rb}}\right)^2 - k_r\left(\frac{I_2 - \tilde{i}_{r2}}{X_W + x_{rb}}\right)^2 \tag{12}$$

where k_l and k_r are constant variables which are determined by identification experiments and $X_W := X+W$. The circuit equations are expressed as

$$\dot{\tilde{i}}_{l1} = -\frac{R\tilde{i}_{l1} + \frac{Q\dot{x}_{lb}}{(X_W - x_{lb})^2}(I_1 + \tilde{i}_{l1})}{\frac{Q}{X_W - x_{lb}} + L_0} + \frac{1}{\frac{Q}{X_W - x_{lb}} + L_0}\tilde{e}_l, \tag{13}$$

$$\dot{\tilde{i}}_{l2} = -\frac{R\tilde{i}_{l2} + \frac{Q\dot{x}_{lb}}{(X_W + x_{lb})^2}(I_2 - \tilde{i}_{l2})}{\frac{Q}{X_W + x_{lb}} + L_0} + \frac{1}{\frac{Q}{X_W + x_{lb}} + L_0}\tilde{e}_l, \tag{14}$$

$$\dot{\tilde{i}}_{r1} = -\frac{R\tilde{i}_{r1} + \frac{Q\dot{x}_{rb}}{(X_W - x_{rb})^2}(I_1 + \tilde{i}_{r1})}{\frac{Q}{X_W - x_{rb}} + L_0} + \frac{1}{\frac{Q}{X_W - x_{rb}} + L_0}\tilde{e}_r, \tag{15}$$

$$\dot{\tilde{i}}_{r2} = -\frac{R\tilde{i}_{r2} + \frac{Q\dot{x}_{rb}}{(X_W + x_{rb})^2}(I_2 - \tilde{i}_{r2})}{\frac{Q}{X_W + x_{rb}} + L_0} + \frac{1}{\frac{Q}{X_W + x_{rb}} + L_0}\tilde{e}_r. \tag{16}$$

Now, define the state variable as $\boldsymbol{x} = [x_G \quad \theta_G \quad \dot{x}_G \quad \dot{\theta}_G \quad \dot{\tilde{i}}_{l1} \quad \dot{\tilde{i}}_{l2} \quad \dot{\tilde{i}}_{r1} \quad \dot{\tilde{i}}_{r2}]^T$. The state space equation is then obtained as follows:

$$\dot{x} = \begin{bmatrix} \dot{x}_G \\ \dot{\theta}_G \\ \alpha(x) \\ \beta(x) \\ \gamma_1(x) \\ \gamma_2(x) \\ \gamma_3(x) \\ \gamma_4(x) \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ \frac{1}{\frac{Q}{X_W - x_G + l_b\theta_G} + L_0} & 0 \\ \frac{1}{\frac{Q}{X_W + x_G - l_b\theta_G} + L_0} & 0 \\ 0 & \frac{1}{\frac{Q}{X_W - x_G - l_b\theta_G} + L_0} \\ 0 & \frac{1}{\frac{Q}{X_W + x_G + l_b\theta_G} + L_0} \end{bmatrix} \begin{bmatrix} \tilde{e}_l \\ \tilde{e}_r \end{bmatrix}$$

$$\begin{aligned} \alpha(x) = \quad & -g + \frac{k}{M}\left(\frac{I_1 + \tilde{i}_{l1}}{X_W - x_G + l_b\theta_G}\right)^2 - \frac{k}{M}\left(\frac{I_2 - \tilde{i}_{l2}}{X_W + x_G - l_b\theta_G}\right)^2 \\ & + \frac{k}{M}\left(\frac{I_1 + \tilde{i}_{r1}}{X_W - x_G - l_b\theta_G}\right)^2 - \frac{k}{M}\left(\frac{I_2 - \tilde{i}_{r2}}{X_W + x_G + l_b\theta_G}\right)^2 \end{aligned} \tag{17}$$

$$\begin{aligned} \beta(x) = \quad & \frac{1}{J}\left(k\left(\frac{I_1 + \tilde{i}_{r1}}{X_W - x_G - l_b\theta_G}\right)^2 - k\left(\frac{I_2 - \tilde{i}_{r2}}{X_W + x_G + l_b\theta_G}\right)^2\right. \\ & \left. - k\left(\frac{I_1 + \tilde{i}_{l1}}{X_W - x_G + l_b\theta_G}\right)^2 + k\left(\frac{I_2 - \tilde{i}_{l2}}{X_W + x_G - l_b\theta_G}\right)^2\right) l_b \end{aligned} \tag{18}$$

$$\gamma_1(x) = -\frac{R\tilde{i}_{l1} + \frac{Q(\dot{x}_G - l_b\dot{\theta}_g)}{(X_W - x_G + l_b\theta_G)^2}(I_1 + \tilde{i}_{l1})}{\frac{Q}{X_W - x_G + l_b\theta_G} + L_0} \tag{19}$$

$$\gamma_2(x) = -\frac{R\tilde{i}_{l2} + \frac{Q(\dot{x}_G - l_b\dot{\theta}_G)}{(X_W + x_G - l_b\theta_G)^2}(I_2 - \tilde{i}_{l2})}{\frac{Q}{X_W + x_G - l_b\theta_G} + L_0} \tag{20}$$

$$\gamma_3(x) = -\frac{R\tilde{i}_{r1} + \frac{Q(\dot{x}_G + l_b\dot{\theta}_G)}{(X_W - x_G - l_b\theta_G)^2}(I_1 + \tilde{i}_{r1})}{\frac{Q}{X_W - x_G - l_b\theta_G} + L_0} \tag{21}$$

$$\gamma_4(x) = -\frac{R\tilde{i}_{r2} + \frac{Q(\dot{x}_G + l_b\dot{\theta}_G)}{(X_W + x_G + l_b\theta_G)^2}(I_2 - \tilde{i}_{r2})}{\frac{Q}{X_W + x_G + l_b\theta_G} + L_0} \tag{22}$$

3 MODELS AND LINEARIZABILITY

In this section, we consider the relationship between employed models of AMB system and its linearizability.

3.1 3rd ORDER PUSH-PULL MODEL

Consider an axis of AMB system, i.e. a pair of coils, as shown in Fig.2. For the simplicity of notation, the suffix l and r are dropped. The volts of upper and lower circuits are denoted by e_1 and e_2.

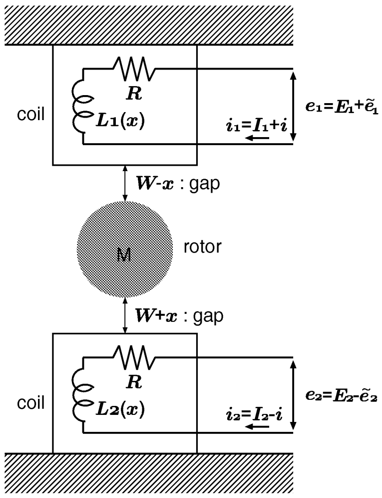

Figure 2: An axis of AMB system (3rd order model)

Assume that the current of the circuits is controlled as follows:

$$i_1 = I_1 + i, i_2 = I_2 - i, \tag{23}$$

where I_1 and I_2 are bias currents. This assumption is actually very strong and somewhat unrealistic due to the separate configuration of the AMB circuits. The assumption however enables us to derive simple 3rd order AMB model which is exactly linearizable.

Let the upper and lower input volts to keep the same current i be denoted by $\tilde{e}_1$ and $\tilde{e}_2$. The volts of the circuits are expressed as

$$e_1 = E_1 + \tilde{e}_1, \quad e_2 = E_2 - \tilde{e}_2, \tag{24}$$

where $E_1 = RI_1$ and $E_2 = RI_2$. The following state space equations then hold.

$$x = \begin{bmatrix} x & \dot{x} & i \end{bmatrix}^T \tag{25}$$

$$\dot{x} = \begin{bmatrix} \dot{x} \\ \ddot{x} \\ \dot{i} \end{bmatrix} = \begin{bmatrix} \dot{x} \\ \alpha(x) \\ \beta(x) \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ \gamma(x) \end{bmatrix} \frac{\tilde{e}_1 + \tilde{e}_2}{2} \tag{26}$$

$$\alpha(x) = \ddot{x} = -g + \frac{Q}{2M}\left(\frac{I_1+i}{X_W - x}\right)^2 - \frac{Q}{2M}\left(\frac{I_2-i}{X_W+x}\right)^2 \tag{27}$$

$$\beta(x) = -\frac{2Ri + Q\dot{x}\left\{\frac{I_1+i}{(X_W-x)^2} + \frac{I_2-i}{(X_W+x)^2}\right\}}{\frac{Q}{X_W-x} + \frac{Q}{X_W+x} + 2L_0} \tag{28}$$

$$\gamma(x) = \frac{2}{\frac{Q}{X_W-x} + \frac{Q}{X_W+x} + 2L_0} \tag{29}$$

The necessary and sufficient conditions for exact linearizability are as follows:

1. $\{ad_f^0 g(x), ad_f^1 g(x), ad_f^2 g(x)\}$ are linearly independent.
2. $^\forall x \;\; [ad_f^0 g(x), ad_f^1 g(x)] \in span\{ad_f^0 g(x), ad_f^1 g(x)\}$

The derived 3rd order model satisfies above conditions, then that is exactly linearizable. Actually, taking the coordinate transformation as

$$y_1 = L_f^0 \phi(\boldsymbol{x}) = x_1, \tag{30}$$

$$y_2 = L_f^1 \phi(\boldsymbol{x}) = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} \dot{x} \\ \alpha(\boldsymbol{x}) \\ \beta(\boldsymbol{x}) \end{bmatrix} = x_2, \tag{31}$$

$$y_3 = L_f^2 \phi(\boldsymbol{x}) = \begin{bmatrix} 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} \dot{x} \\ \alpha(\boldsymbol{x}) \\ \beta(\boldsymbol{x}) \end{bmatrix} = \alpha(\boldsymbol{x}), \tag{32}$$

the state space equation can be transformed to

$$\dot{\boldsymbol{y}} = \begin{bmatrix} y_2 \\ y_3 \\ f_3(\boldsymbol{y}) \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ g_3(\boldsymbol{y}) \end{bmatrix} u, \tag{33}$$

where

$$\begin{aligned} f_3(\boldsymbol{y}) &= L_f^3 \phi = \frac{\partial \alpha}{\partial \boldsymbol{x}^T} f(\boldsymbol{x}) \\ &= \frac{Q}{M}\left[\left\{\frac{(I_1+i)^2}{(X_W-x)^3} + \frac{(I_2-i)^2}{(X_W+x)^3}\right\}\dot{x}\right. \\ &\quad - \frac{2Ri + Q\dot{x}\left\{\frac{I_1+i}{(X_W-x)^2} + \frac{I_2-i}{(X_W+x)^2}\right\}}{\frac{Q}{X_W-x} + \frac{Q}{X_W+x} + 2L_0} \\ &\quad \left. \times \left\{\frac{I_1+i}{(X_W-x)^2} + \frac{I_2-i}{(X_W+x)^2}\right\}\right] \end{aligned} \tag{34}$$

$$\begin{aligned} g_3(\boldsymbol{y}) &= L_g L_f^2 \phi = L_g \alpha(\boldsymbol{x}) = \frac{\partial \alpha}{\partial \boldsymbol{x}^T} g(\boldsymbol{x}) \\ &= \frac{Q}{M}\left\{\frac{I_1+i}{(X_W-x)^2} + \frac{I_2-i}{(X_W+x)^2}\right\} \\ &\quad \cdot \frac{2}{\frac{Q}{X_W-x} + \frac{Q}{X_W+x} + 2L_0}. \end{aligned} \tag{35}$$

Then, it is exactly linearizable with input transformation as shown in eqn.(33).

3.2 4th ORDER PUSH-PULL MODEL

The more natural model is here employed as shown in Fig.3. The control inputs are e for the upper AMB circuit and $-e$ for the lower circuit.

$$e_1 = E_1 + e, \quad e_2 = E_2 - e, \tag{36}$$

where E_1 and E_2 are bias volts for upper and lower AMB circuits. The currents of AMB circuits i_1 and i_2

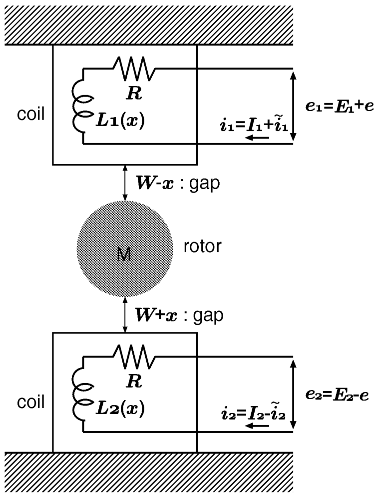

Figure 3: An axis of AMB system (4th order model)

are

$$i_1 = I_1 + \tilde{i}_1, \quad i_2 = I_2 - \tilde{i}_2. \tag{37}$$

where I_1 and I_2 are bias currents caused by bias volts E_1 and E_2. In this case, defining state space variables as $\boldsymbol{x} = \begin{bmatrix} x & \dot{x} & \tilde{i}_1 & \tilde{i}_2 \end{bmatrix}^T$, a state space equation is obtained as follows:

$$\begin{aligned} \dot{x} &= f(x) + g(x)u \\ &= \begin{bmatrix} \dot{x} \\ -g + \frac{Q}{2M}\left(\frac{I_1+\tilde{i}_1}{X_W-x}\right)^2 - \frac{Q}{2M}\left(\frac{I_2-\tilde{i}_2}{X_W+x}\right)^2 \\ -\frac{R\tilde{i}_1 + \frac{Q}{(X_W-x)^2}\dot{x}(I_1+\tilde{i}_1)}{\frac{Q}{X_W-x}+L_0} \\ -\frac{R\tilde{i}_2 + \frac{Q}{(X_W+x)^2}\dot{x}(I_2-\tilde{i}_2)}{\frac{Q}{X_W+x}+L_0} \end{bmatrix} \\ &\quad + \begin{bmatrix} 0 \\ 0 \\ \frac{1}{\frac{Q}{X_W-x}+L_0} \\ \frac{1}{\frac{Q}{X_W+x}+L_0} \end{bmatrix} e. \end{aligned} \tag{38}$$

The necessary and sufficient conditions for the existence of coordinate transformation to realize exact linearization are as follows:

(1) $\left\{ ad_{\boldsymbol{f}}^0\boldsymbol{g} \quad ad_{\boldsymbol{f}}^1\boldsymbol{g} \quad ad_{\boldsymbol{f}}^2\boldsymbol{g} \quad ad_{\boldsymbol{f}}^3\boldsymbol{g} \right\}(\boldsymbol{x})$ are linearly independent.

(2) $\left\{ ad_{\boldsymbol{f}}^0\boldsymbol{g} \quad ad_{\boldsymbol{f}}^1\boldsymbol{g} \quad ad_{\boldsymbol{f}}^2\boldsymbol{g} \right\}(\boldsymbol{x})$ are involutive.

The derived 4th order model dose not satisfy the above conditions. Then, the state space equation cannot be exactly linearized.

The derived 4th order model however can be exactly linearized from input to output, so-called input-output linearization. First, define the output of the AMB system as a new state variable. Second, define the derivative of the new state variable as a second new state variable. Repeat the operation until the input appears.

$$\xi_1 = x \tag{39}$$

$$\frac{d\xi_1}{dt} = \dot{x} = \xi_2 \tag{40}$$

$$\frac{d\xi_2}{dt} = -g + \frac{Q}{2M}\left(\frac{I_1+\tilde{i}_1}{X_W-x}\right)^2 - \frac{Q}{2M}\left(\frac{I_2-\tilde{i}_2}{X_W+x}\right)^2 = \xi_3 \tag{41}$$

$$\begin{aligned} \frac{d\xi_3}{dt} &= \frac{Q}{M}\frac{I_1+\tilde{i}_1}{X_W-x}\left(\frac{\dot{\tilde{i}}_1}{X_W-x} + \frac{I_1+\tilde{i}_1}{(X_W-x)^2}\dot{x}\right) \\ &\quad + \frac{Q}{M}\frac{I_2-\tilde{i}_2}{X_W+x}\left(\frac{\dot{\tilde{i}}_2}{X_W+x} + \frac{I_2-\tilde{i}_2}{(X_W+x)^2}\dot{x}\right) \end{aligned} \tag{42}$$

$$\begin{aligned} &= \frac{Q\dot{x}(I_1+\tilde{i}_1)^2}{M(X_W-x)^3} - \frac{Q}{M}\frac{I_1+\tilde{i}_1}{X_W-x}\frac{R\tilde{i}_1 + \frac{Q}{(X_W-x)^2}\dot{x}(I_1+\tilde{i}_1)}{Q+L_0(X_W-x)} \\ &\quad + \frac{Q\dot{x}(I_2-\tilde{i}_2)^2}{M(X_W+x)^3} - \frac{Q}{M}\frac{I_2-\tilde{i}_2}{X_W+x}\frac{R\tilde{i}_2 + \frac{Q}{(X_W+x)^2}\dot{x}(I_2-\tilde{i}_2)}{Q+L_0(X_W+x)} \\ &\quad + \left(\frac{Q}{M}\frac{I_1+\tilde{i}_1}{X_W-x}\frac{1}{Q+L_0(X_W-x)} \right. \\ &\quad \left. + \frac{Q}{M}\frac{I_2-\tilde{i}_2}{X_W+x}\frac{1}{Q+L_0(X_W+x)}\right)e. \end{aligned} \tag{43}$$

Using the input transformation

$$\begin{aligned} e &= \left(v - \left(\frac{Q\dot{x}(I_1+\tilde{i}_1)^2}{M(X_W-x)^3} - \frac{Q}{M}\frac{I_1+\tilde{i}_1}{X_W-x}\frac{R\tilde{i}_1 + \frac{Q}{(X_W-x)^2}\dot{x}(I_1+\tilde{i}_1)}{Q+L_0(X_W-x)}\right.\right. \\ &\quad \left.\left. + \frac{Q\dot{x}(I_2-\tilde{i}_2)^2}{M(X_W+x)^3} - \frac{Q}{M}\frac{I_2-\tilde{i}_2}{X_W+x}\frac{R\tilde{i}_2 + \frac{Q}{(X_W+x)^2}\dot{x}(I_2-\tilde{i}_2)}{Q+L_0(X_W+x)}\right)\right) \\ &\quad \Big/ \left(\frac{Q}{M}\frac{I_1+\tilde{i}_1}{X_W-x}\frac{1}{Q+L_0(X_W-x)} \right. \\ &\quad \left. + \frac{Q}{M}\frac{I_2-\tilde{i}_2}{X_W+x}\frac{1}{Q+L_0(X_W+x)}\right), \end{aligned} \tag{44}$$

the linearized state space equation is

$$\frac{d}{dt}\begin{bmatrix} \xi_1 \\ \xi_2 \\ \xi_3 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}\begin{bmatrix} \xi_1 \\ \xi_2 \\ \xi_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} v. \tag{45}$$

There exists a state variable which is not linearized. However, it dose not affect the output of the system. The input-output exact linearization is now achieved.

4 MIMO INPUT-OUTPUT LINEARIZATION

In this section, using the result of subsection **3.2**, the non-linear control system is derived for AMB rotor system. Based on the AMB MIMO model considering rotor modal characteristic, an input-output exact linearization is carried out. First of all, define the output x_G and θ_G as new state variables ξ_1 and ξ_2. Then, also define the derivatives as new state variables and repeat the operation until the input of the system appears in the output as follows:

$$x_G = \xi_1, \tag{46}$$

$$\theta_G = \xi_2, \tag{47}$$

$$\dot{\xi}_1 = \dot{x}_G = \xi_3, \tag{48}$$

$$\dot{\xi}_2 = \dot{\theta}_G = \xi_4, \tag{49}$$

$$\dot{\xi}_3 = \ddot{x}_G = \alpha(\boldsymbol{x}) = \xi_5, \tag{50}$$

$$\dot{\xi}_4 = \ddot{\theta}_G = \beta(\boldsymbol{x}) = \xi_6. \tag{51}$$

The transformed state space equation is then

$$\frac{d}{dt}\begin{bmatrix} \xi_1 \\ \xi_2 \\ \xi_3 \\ \xi_4 \\ \xi_5 \\ \xi_6 \end{bmatrix} = \begin{bmatrix} \xi_3 \\ \xi_4 \\ \xi_5 \\ \xi_6 \\ f_5(x) \\ f_6(x) \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ g_{15}(x) & g_{25}(x) \\ g_{25}(x) & g_{26}(x) \end{bmatrix} \begin{bmatrix} \tilde{e}_l \\ \tilde{e}_r \end{bmatrix}, \tag{52}$$

$$\dot{\xi}_7 = *, \tag{53}$$

$$\dot{\xi}_8 = *, \tag{54}$$

where

$$\begin{aligned} f_5(x) &= \frac{2k}{M}\frac{I_1+\tilde{i}_{l1}}{\left(X_W - x_G + l_b\theta_G\right)^2}\left(-\frac{R\tilde{i}_{l1} + \frac{Q(\dot{x}_G - l_b\dot{\theta}_g)}{(X_W - x_G + l_b\theta_G)^2}(I_1+\tilde{i}_{l1})}{\frac{Q}{X_W - x_G + l_b\theta_G} + L_0}\right) \\ &+ \frac{2k}{M}\frac{(I_1+\tilde{i}_{l1})^2}{\left(X_W - x_G + l_b\theta_G\right)^3}\left(\dot{x}_G - l_b\dot{\theta}_G\right) \\ &+ \frac{2k}{M}\frac{I_2-\tilde{i}_{l2}}{\left(X_W + x_G - l_b\theta_G\right)^2}\left(-\frac{R\tilde{i}_{l2} + \frac{Q(\dot{x}_G - l_b\dot{\theta}_G)}{(X_W + x_G - l_b\theta_G)^2}(I_2-\tilde{i}_{l2})}{\frac{Q}{X_W + x_G - l_b\theta_G} + L_0}\right) \\ &+ \frac{2k}{M}\frac{(I_2-\tilde{i}_{l2})^2}{\left(X_W + x_G - l_b\theta_G\right)^3}\left(\dot{x}_G - l_b\dot{\theta}_G\right) \\ &+ \frac{2k}{M}\frac{I_1+\tilde{i}_{r1}}{\left(X_W - x_G - l_b\theta_G\right)^2}\left(-\frac{R\tilde{i}_{r1} + \frac{Q(\dot{x}_G + l_b\dot{\theta}_G)}{(X_W - x_G - l_b\theta_G)^2}(I_1+\tilde{i}_{r1})}{\frac{Q}{X_W - x_G - l_b\theta_G} + L_0}\right) \\ &+ \frac{2k}{M}\frac{(I_1+\tilde{i}_{r1})^2}{\left(X_W - x_G - l_b\theta_G\right)^3}\left(\dot{x}_G + l_b\dot{\theta}_G\right) \\ &+ \frac{2k}{M}\frac{I_2-\tilde{i}_{r2}}{\left(X_W + x_G + l_b\theta_G\right)^2}\left(-\frac{R\tilde{i}_{r2} + \frac{Q(\dot{x}_G + l_b\dot{\theta}_G)}{(X_W + x_G + l_b\theta_G)^2}(I_2-\tilde{i}_{r2})}{\frac{Q}{X_W + x_G + l_b\theta_G} + L_0}\right) \\ &+ \frac{2k}{M}\frac{(I_2-\tilde{i}_{r2})^2}{\left(X_W + x_G + l_b\theta_G\right)^3}\left(\dot{x}_G + l_b\dot{\theta}_G\right), \end{aligned}$$

$$\begin{aligned} f_6(x) &= \frac{1}{J}\Bigg(-2k\frac{I_1+\tilde{i}_{l1}}{\left(X_W - x_G + l_b\theta_G\right)^2}\left(-\frac{R\tilde{i}_{l1} + \frac{Q(\dot{x}_G - l_b\dot{\theta}_g)}{(X_W - x_G + l_b\theta_G)^2}(I_1+\tilde{i}_{l1})}{\frac{Q}{X_W - x_G + l_b\theta_G} + L_0}\right) \\ &-2k\frac{(I_1+\tilde{i}_{l1})^2}{\left(X_W - x_G + l_b\theta_G\right)^3}\left(\dot{x}_G - l_b\dot{\theta}_G\right) \\ &-2k\frac{I_2-\tilde{i}_{l2}}{\left(X_W + x_G - l_b\theta_G\right)^2}\left(-\frac{R\tilde{i}_{l2} + \frac{Q(\dot{x}_G - l_b\dot{\theta}_G)}{(X_W + x_G - l_b\theta_G)^2}(I_2-\tilde{i}_{l2})}{\frac{Q}{X_W + x_G - l_b\theta_G} + L_0}\right) \\ &-2k\frac{(I_2-\tilde{i}_{l2})^2}{\left(X_W + x_G - l_b\theta_G\right)^3}\left(\dot{x}_G - l_b\dot{\theta}_G\right) \\ &+2k\frac{I_1+\tilde{i}_{r1}}{\left(X_W - x_G - l_b\theta_G\right)^2}\left(-\frac{R\tilde{i}_{r1} + \frac{Q(\dot{x}_G + l_b\dot{\theta}_G)}{(X_W - x_G - l_b\theta_G)^2}(I_1+\tilde{i}_{r1})}{\frac{Q}{X_W - x_G - l_b\theta_G} + L_0}\right) \\ &+2k\frac{(I_1+\tilde{i}_{r1})^2}{\left(X_W - x_G - l_b\theta_G\right)^3}\left(\dot{x}_G + l_b\dot{\theta}_G\right) \\ &+2k\frac{I_2-\tilde{i}_{r2}}{\left(X_W + x_G + l_b\theta_G\right)^2}\left(-\frac{R\tilde{i}_{r2} + \frac{Q(\dot{x}_G + l_b\dot{\theta}_G)}{(X_W + x_G + l_b\theta_G)^2}(I_2-\tilde{i}_{r2})}{\frac{Q}{X_W + x_G + l_b\theta_G} + L_0}\right) \\ &+ 2k\frac{(I_2-\tilde{i}_{r2})^2}{\left(X_W + x_G + l_b\theta_G\right)^3}\left(\dot{x}_G + l_b\dot{\theta}_G\right)\Bigg) l_b, \end{aligned}$$

$$\begin{aligned} g_{15}(x) &= \Bigg(\frac{2k}{M}\frac{I_1+\tilde{i}_{l1}}{\left(X_W - x_G + l_b\theta_G\right)^2}\frac{1}{\frac{Q}{X_W - x_G + l_b\theta_G} + L_0} \\ &+ \frac{2k}{M}\frac{I_2-\tilde{i}_{l2}}{\left(X_W + x_G - l_b\theta_G\right)^2}\frac{1}{\frac{Q}{X_W + x_G - l_b\theta_G} + L_0}\Bigg), \end{aligned} \tag{55}$$

$$\begin{aligned} g_{25}(x) &= \Bigg(\frac{2k}{M}\frac{I_1+\tilde{i}_{r1}}{\left(X_W - x_G - l_b\theta_G\right)^2}\frac{1}{\frac{Q}{X_W - x_G - l_b\theta_G} + L_0} \\ &+ \frac{2k}{M}\frac{I_2-\tilde{i}_{r2}}{\left(X_W + x_G + l_b\theta_G\right)^2}\frac{1}{\frac{Q}{X_W + x_G + l_b\theta_G} + L_0}\Bigg), \end{aligned} \tag{56}$$

$$\begin{aligned} g_{16}(x) &= \frac{1}{J}\Bigg(-2k\frac{I_1+\tilde{i}_{l1}}{\left(X_W - x_G + l_b\theta_G\right)^2}\frac{1}{\frac{Q}{X_W - x_G + l_b\theta_G} + L_0} \\ &- 2k\frac{I_2-\tilde{i}_{l2}}{\left(X_W + x_G - l_b\theta_G\right)^2}\frac{1}{\frac{Q}{X_W + x_G - l_b\theta_G} + L_0}\Bigg) l_b, \end{aligned} \tag{57}$$

$$\begin{aligned} g_{26}(x) &= \frac{1}{J}\Bigg(2k\frac{I_1+\tilde{i}_{r1}}{\left(X_W - x_G - l_b\theta_G\right)^2}\frac{1}{\frac{Q}{X_W - x_G - l_b\theta_G} + L_0} \\ &+ 2k\frac{I_2-\tilde{i}_{r2}}{\left(X_W + x_G + l_b\theta_G\right)^2}\frac{1}{\frac{Q}{X_W + x_G + l_b\theta_G} + L_0}\Bigg) l_b. \end{aligned} \tag{58}$$

The input transformation for input-output exact linearization which preserve the dynamical characteristics at the equilibrium point is

$$\begin{bmatrix} \tilde{e}_l \\ \tilde{e}_r \end{bmatrix} = \begin{bmatrix} g_{15}(x) & g_{25}(x) \\ g_{25}(x) & g_{26}(x) \end{bmatrix}^{-1} \Bigg[\begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} & a_{15} & a_{16} \\ a_{21} & a_{22} & a_{23} & a_{24} & a_{25} & a_{26} \end{bmatrix} \cdot \begin{bmatrix} \xi_1 \\ \xi_2 \\ \xi_3 \\ \xi_4 \\ \xi_5 \\ \xi_6 \end{bmatrix} + \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix}\begin{bmatrix} v_l \\ v_r \end{bmatrix} - \begin{bmatrix} f_5(x) \\ f_6(x) \end{bmatrix}\Bigg] \tag{59}$$

where

$$a_{11} = \frac{4k}{M}\frac{{I_1}^2 + {I_2}^2}{(X_W)^3}\frac{R}{L_C} \tag{60}$$

$$a_{12} = 0 \tag{61}$$

$$a_{13} = -\frac{2k}{M}\frac{{I_1}^2 + {I_2}^2}{(X_W)^3}\left(\frac{Q\,(I_1 + I_2)}{(X_W)\,L_C} - 2\right) \tag{62}$$

$$a_{14} = 0 \tag{63}$$

$$a_{15} = -\frac{R}{L_C} \qquad (64)$$

$$a_{16} = 0 \qquad (65)$$

$$a_{21} = 0 \qquad (66)$$

$$a_{22} = \frac{4kl_b{}^2}{J}\frac{I_1{}^2+I_2{}^2}{(X_W)^3}\frac{R}{L_C} \qquad (67)$$

$$a_{23} = 0 \qquad (68)$$

$$a_{24} = -\frac{2kl_b{}^2}{J}\frac{I_1{}^2+I_2{}^2}{(X_W)^3}\left(\frac{Q\,(I_1+I_2)}{(X_W)\,L_C}-2\right) \qquad (69)$$

$$a_{25} = 0 \qquad (70)$$

$$a_{26} = -\frac{R}{L_C} \qquad (71)$$

$$b_{11} = \frac{2k}{M}\frac{I_1{}^2+I_2{}^2}{(X_W)^2}\frac{1}{L_C} \qquad (72)$$

$$b_{12} = \frac{2k}{M}\frac{I_1{}^2+I_2{}^2}{(X_W)^2}\frac{1}{L_C} \qquad (73)$$

$$b_{21} = -\frac{2kl_b}{J}\frac{I_1{}^2+I_2{}^2}{(X_W)^2}\frac{1}{L_C} \qquad (74)$$

$$b_{22} = \frac{2kl_b}{J}\frac{I_1{}^2+I_2{}^2}{(X_W)^2}\frac{1}{L_C}. \qquad (75)$$

5 EXPERIMENTAL RESULTS

In this section, the results of rotor levitation experiments with input-output exact linearization technique are shown. The physical parameters of the experimental rig are M: 13[kg], J: 0.07[kg·m^2], g: 9.81[m/s^2], R: 0.81[Ω], X_0: 0, I_{1l}: 3.50[A], I_{2l}: 1.99[A], I_{1r}: 3.50[A], I_{2r}: 2.29[A], k_l: 7.98×10^{-5}, k_r: 9.47×10^{-5}, X: 2.77×10^{-3}[m], W: 4.5×10^{-4}[m], Q: 1.62×10^{-5}[m·H] and L_0: 8.55×10^{-4}[H]. The linear part of the control system is PID controller whose parameters are P: 7400, I: 100, and D: 50.

Fig.4 shows the experimental result of PID controller. Fig.5 shows the result of PID controller with non-linear controller for input-output exact linearization. The reference gap goes down step by step in the experiments. In Fig.4, since the performance of PID controller is poor, the response of the control system begins to oscillate after the first step reference input. On the other hand, the control system using nonlinear controller for input-output exact linearization does not however oscillate. By these experimental results, the effectiveness of the input-output exact linearization to enlarge the stability region is confirmed.

6 CONCLUSION

In this paper, a control system design method for AMB system was proposed. The design method is based on the natural model of the push-pull type AMB system considering rotor modal characteristics. Using input-output exact linearization technique, nonlinear control system was designed. In the case of poor linear controller, it was confirmed that nonlinear controller supported the linear controller and tremendously improved the control system performance.

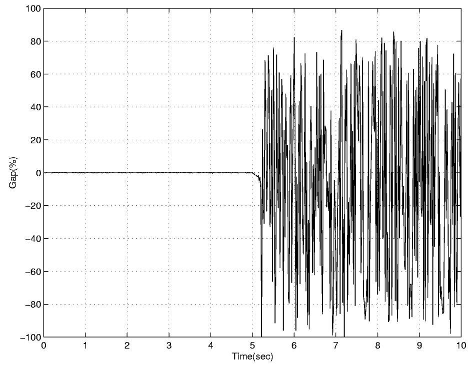

Figure 4: PID controller

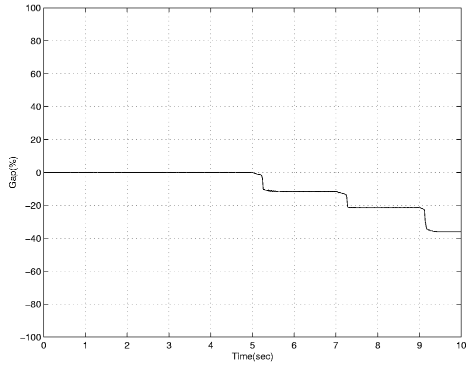

Figure 5: PID controller with input-output exact linearization

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