A UNIFYING POINT OF VIEW OF CONTROLLERS FOR ACTIVE MAGNETIC BEARINGS

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ABSTRACT
This paper investigates the fundamental properties of active magnetic bearing controllers from the standpoint of output regulation in the presence of deterministic disturbances. The treated disturbances are stepwise and unbalance forces acting on the rotor. A transfer function approach is used in the analysis. The general structures of controllers achieving displacement or current regulation for stepwise force and achieving displacement, current or force regulation for unbalance force are derived. The mutual relations and equivalence between the independently developed controllers are clarified. A direct synthesis method of constructing controllers achieving specified performances is also shown based on these analyses.

INTRODUCTION
Various control methods of achieving unique performances, which are impossible for mechanical bearings, have been proposed and developed for active magnetic bearings. For example, in order to reduce a steady-state position error to zero for unknown constant disturbances, integral action is usually incorporated into the feedback loop. The virtually zero-power control has been used in space instruments [1, 2] and a magnetically levitated carrier system [3] in which permanent magnets provide bias flux. It makes the control current converges to zero for any static disturbances, which means zero power dissipation under steady loads.
A number of controllers compensating the effects of rotor unbalance have been proposed. Pioneering works were carried out by Dr. Habermann *et al*. [4, 5]. They patented two control circuits which

[H.1] amplify synchronous components of the displacement sensor signals [4],
[H.2] filter out synchronous components from the displacement sensor signals [5].

Both are characterized by the use of coordinate conversion between fixed and rotating reference systems. Another approach is observer-based compensation [6, 7]. Unbalance forces acting on the rotor are estimated by an observer; synchronous components are eliminated from rotor displacement, coil current or electromagnetic force by adjusting control input based on the estimation.
The author has proposed to apply a transfer function approach to clarify the essential properties of these controllers [8, 9]. This paper presents a unifying point of view to the controllers based on these analyses so as to show a perspective of the control of magnetic bearings. A direct synthesis method of constructing controllers achieving specified performances is also presented.

CLASSIFICATION AND NOTATION
The controllers will be classified according to:

(1) disturbance to be cancelled ($\boldsymbol{w}$):
stepwise force $\boldsymbol{w}_{step}$ or *unbalance force* $\boldsymbol{w}_{unb}$,

(2) variable to be regulated ($\boldsymbol{y}$):
displacement $\boldsymbol{x}$, *current* $\boldsymbol{i}$ or *bearing force* $\boldsymbol{f}$,

(3) equation in calculating the control input:
state equation or *convolution integral.*

We introduce a notation:

$$\boldsymbol{y}/\boldsymbol{w} \to 0, \qquad (1)$$

which means that output $\boldsymbol{y}$ converges to zero as $t \to \infty$ even in the presence of disturbance $\boldsymbol{w}$.

BASIC MODELS
Figure 1 shows a single-degree-of-freedom-of-motion model, which will be used in discussing regulation problems for stepwise force. The equation of motion is given by

$$m\ddot{z}(t) = f_z(t) + w_z(t)\,, \tag{2}$$

where z is the displacement of the rotor, m is the mass of the rotor, f_z is electromagnetic force acting on the rotor and w_z is disturbance force acting on the rotor. The electromagnetic force is approximately given by

$$f_z(t) = k_s z(t) + k_i i_z(t)\,, \tag{3}$$

where k_i and k_s are the gap- and current-force coefficients of the electromagnet.
Assuming that the disturbance is stepwise, the dynamics are represented with transfer functions as

$$Z(s) = \frac{1}{t_o(s)}(b_0 I_z(s) + d_0 W_z(s))\,, \tag{4}$$

$$W_z(s) = \frac{A_0}{s}\,, \tag{5}$$

where each Laplace-transformed variable is denoted by its capital, and

$$t_o(s) = s^2 - a_0\,, \tag{6}$$

$$a_0 = \frac{k_s}{m},\quad b_0 = \frac{k_i}{m},\quad d_0 = \frac{1}{m}.$$

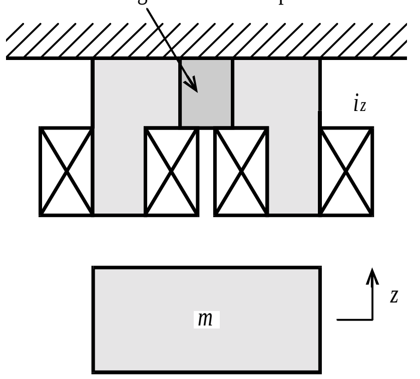


FIGURE 1: Basic model with single degree of freedom of motion

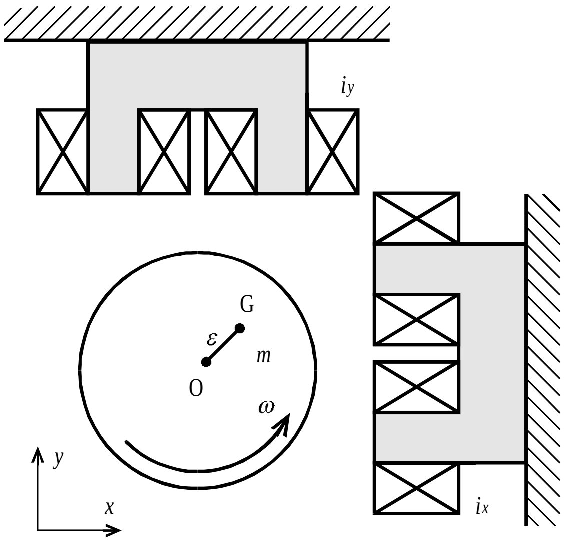


FIGURE 2: Basic model with two degrees of freedom of motion

Figure 2 shows a two-degree-of-freedom-of-motion model related to translation [8]. The equations of motion are

$$m\ddot{x}(t) = k_s x(t) + k_i i_x(t) + m\varepsilon\omega^2 \cos\omega t\,, \tag{7}$$

$$m\ddot{y}(t) = k_s x(t) + k_i i_y(t) + m\varepsilon\omega^2 \sin\omega t\,, \tag{8}$$

where x and y are the displacements of the geometric center O of the rotor in the radial directions, i_x and i_y are the control currents in the x- and y-directions, ω is the rotational speed of the rotor, and ε is the amount of static unbalance of the rotor.
Complex variables are introduced for simplifying analysis.

$$x_c(t) = x(t) + jy(t)\,, \tag{9}$$

$$i_c(t) = i_x(t) + ji_y(t)\,, \tag{10}$$

$$w_c(t) = w_x(t) + jw_y(t) = m\varepsilon\omega^2 e^{j\omega t}\,. \tag{11}$$

The transfer function representation of the dynamics given by (4) and (5) becomes

$$X_c(s) = \frac{1}{t_o(s)}(b_0 I_c(s) + d_0 W_c(s))\,, \tag{12}$$

$$W_c(s) = \frac{A_\omega}{s - j\omega}\,, \tag{13}$$

where

$$A_\omega = m\varepsilon\omega^2\,. \tag{14}$$

REGULATION FOR STEPWISE FORCE

The current is treated as control input in this paper. When linear control laws are applied, control input is generally represented as

$$I_z(s) = -\frac{h(s)}{g(s)} Z(s)\,, \tag{15}$$

where $g(s)$ and $h(s)$ are polynomials satisfying

(A.1) They are coprime.
(A.2) The closed-loop system becomes stable.

Displacement Regulation

Substituting (15) into (4) gives

$$Z(s) = \frac{g(s)}{t(s)} d_0 W_z(s) = \frac{g(s)}{t(s)} \cdot \frac{d_0 A_0}{s}\,, \tag{16}$$

where

$$t(s) = (s^2 - a_0)g(s) + b_0 h(s)\,. \tag{17}$$

To achieve

$$\boldsymbol{x} / \boldsymbol{w}_{step} \to 0\,, \tag{18}$$

the controller must have a pole at origin, *that is*

$$g(s) = s\tilde{g}(s)\,, \tag{19}$$

where $\tilde{g}(s)$ is an appropriate polynomial. Figure 3 shows a general form of the controller. Integral action is included in the compensation as well known.

Current Regulation

Substituting (16) into (15) gives

$$I_z(s) = -\frac{h(s)}{t(s)} d_0 W_z(s)\,. \tag{20}$$

To achieve

$$\boldsymbol{i} / \boldsymbol{w}_{step} \to 0\,, \tag{21}$$

the controller must have a zero at origin, *that is*

$$h(s) = s\tilde{h}(s)\,, \tag{22}$$

where $\tilde{h}(s)$ is an appropriate polynomial. A general form of the controller is presented by Fig.4 (a). It implies that an approach of achieving (21) is

- *Feeding back the velocity signal of the rotor* [1],

because velocity is the time derivative of displacement. The block diagram in Fig.4(a) can be modified as shown by Fig.4(b) because

$$I_z(s) = -\frac{s\tilde{h}(s)}{g(s)} Z(s)$$

$$= -\frac{1}{1+\frac{\tau}{s}} \cdot \frac{(s+\tau)\tilde{h}(s)}{g(s)} Z(s)\,, \tag{23}$$

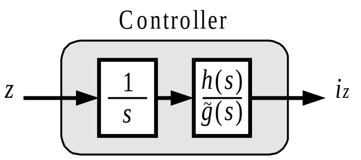
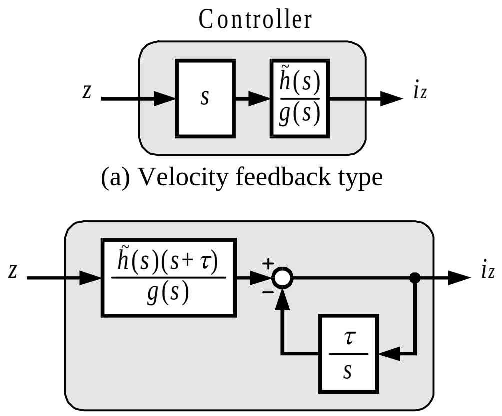


Figure 3: Contoller for displacement regulation



(a) Velocity feedback type

(b) Current feedbak type

Figure 4: Contoller for current regulation

where τ is a parameter introduced for the transformation. Figure 4(b) implies that another approach of achieving (21) is

- *Feeding back the integral of current* [2][3].

Force Regulation

It is clear from Newtons̊ second law of motion that

$$\boldsymbol{f} / \boldsymbol{w}_{step} \to 0\,, \tag{24}$$

is impossible because the forces acting on the rotor must be balanced for the rotor to remain at rest.

REGULATION FOR UNBALANCE FORCE

The process of analysis in this section is similar to that in the previous one. When a linear controller is used, it can be generally represented as

$$I_c(s) = -\frac{h_c(s)}{g_c(s)} X_c(s)\,, \tag{25}$$

where $g_c(s)$ and $h_c(s)$ are coprime polynomials with *complex* coefficients, which are selected for the closed-loop system to be stable.

Displacement Regulation

Substituting (25) into (12) gives

$$X_c(s) = \frac{g_c(s)}{t_c(s)} d_0 W_c(s) = \frac{g_c(s)}{t_c(s)} \cdot \frac{d_0 A_\omega}{s - j\omega}\,, \tag{26}$$

where

$$t_c(s) = (s^2 - a_0) g_c(s) + b_0 h_c(s)\,. \tag{27}$$

To achieve

$$\boldsymbol{x} / \boldsymbol{w}_{unb} \to 0\,, \tag{28}$$

$g_c(s)$ must have a factor $(s - j\omega)$, *that is*

$$g_c(s) = (s - j\omega)\tilde{g}_c(s)\,, \tag{29}$$

where $\tilde{g}_c(s)$ is a polynomial with complex coefficients. A general form of the controller is shown by Fig.5 (a) where complex variables are used. Figure 5 (b) shows the rewritten block diagram with real variables where the internal model of disturbance has a state-space form. Figure 5 (c) shows another equivalent controller in which the control input is calculated by means of convolution integral instead of state-space equation [8]. In these figures, the following transfer functions are introduced.

$$\frac{h_c(s)}{\tilde{g}_c(s)} = G_s(s) + jG_c(s)\,. \tag{30}$$

The controller shown by Fig.5 (b) has a general form including the observer-based unbalance compensator [6] while the controller shown Fig.5 (c) has a general form including [H.1].

Current Regulation Control
Substituting (26) into (25) gives

$$I_c(s) = -\frac{h_c(s)}{t_c(s)} d_0 W_c(s) \, . \tag{31}$$

To achieve

$$\boldsymbol{i} / \boldsymbol{w}_{unb} \to 0 \, , \tag{32}$$

$h_c(s)$ must have a factor $(s - j\omega)$, *that is*

$$h(s) = (s - j\omega)\tilde{h}(s) \tag{33}$$

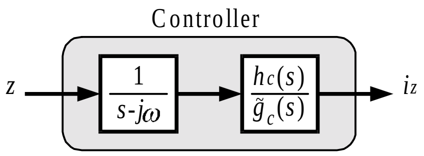
where $\tilde{h}(s)$ is a polynomial with complex coefficients. A general form of the controller is shown by Fig.6 (a) where complex variables are used. Figure 6 (b) shows a modified version with a *minor* feedback loop where a complex parameter $\alpha(=\alpha_s + j\alpha_c)$ is introduced for the transformation. The rewritten block diagram with real variables is shown by Fig.6 (c). Figure 6 (d) shows another equivalent controller in which the control input is calculated by means of convolution integral. In these figures, the following transfer functions are introduced.

$$\frac{\tilde{h}(s)(s + \alpha - j\omega)}{g(s)} = \tilde{G}_s(s) + j\tilde{G}_c(s) \, . \tag{34}$$

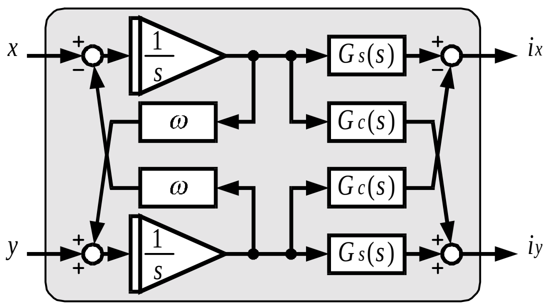
The controller shown by Fig.6 (c) has a general form including the observer-based current regulation controller [7] while the controller shown by Fig.6 (d) has a general form including [H.2].
In general, *the rotor does **not** rotate about its axis of inertia **exactly*** even if (32) is achieved. Such rotation is realized only when

$$a_0 << \omega^2 \, , \tag{35}$$

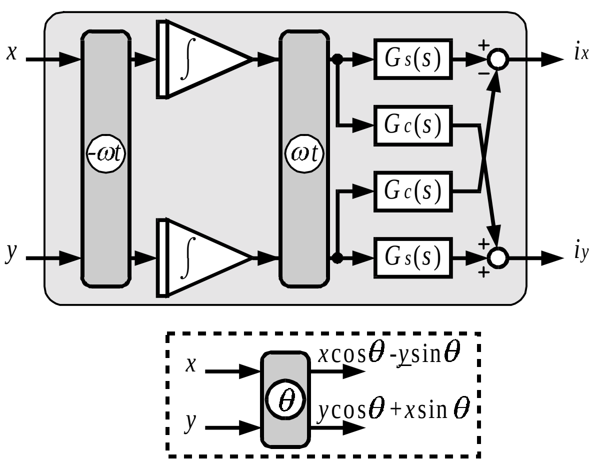
[8]. The condition (35) is satisfied when the rotation speed is very high or the bias flux is zero ($a_0 = 0$).



(a) Complex-variable representation



(b) Real-variable representation (stace-space form)



(c) Real-variable representation (convolution-integral form)

FIGURE 5: Displacement regulation control for unbalance force

Force Regulation Control
From Eq.(3), the variable component of bearing force can be estimated by

$$f_c(t) = \frac{a_0 x_c(t) + b_0 i(t)}{d_0} \, . \tag{36}$$

Substituting (26) and (31) into the Laplace transformation of (36) gives

$$F(s) = \frac{a_0 g_c(s) - b_0 h_c(s)}{t_c(s)} W_c(s) \, . \tag{37}$$

To achieve

$$\boldsymbol{f} / \boldsymbol{w}_{unb} \to 0 \, , \tag{38}$$

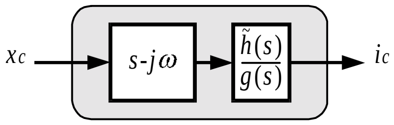
the numerator must have a factor $(s - j\omega)$, *that is*

$$a_0 g_c(s) - b_0 h_c(s) = (s - j\omega) k_c(s) \, , \tag{39}$$

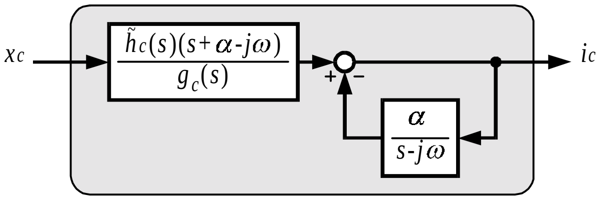
where $k_c(s)$ is a polynomial with complex coefficients. When the component synchronized with rotation is removed from bearing force, *the rotor rotates about its axis of inertia exactly* [8]. The control objective (38) is compatible with (32) if $a_0 = 0$.

RELATIONS BETWEEN THE CONTROLLERS

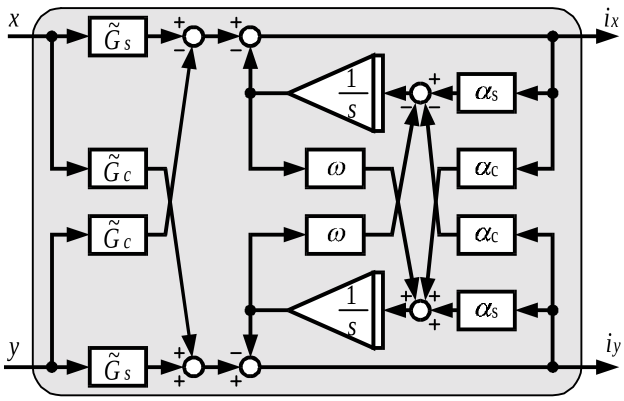
Table 1 gives a summary of the analyses. In the table, the state of "*automatic balancing*"is defined as the state where the rotor rotates about its axis of inertia.
We can interpret that regulation for stepwise force treats a DC component while regulation for unbalance force treats AC components; the latter controllers tend to the former as $\omega \to 0$ except in the case of force regulation.
Current regulation control and force regulation control have similar performances at high frequencies but *not* at low frequencies if $a_0 \neq 0$. Especially, force regulation control loses physical realizability as $\omega \to 0$ [9].



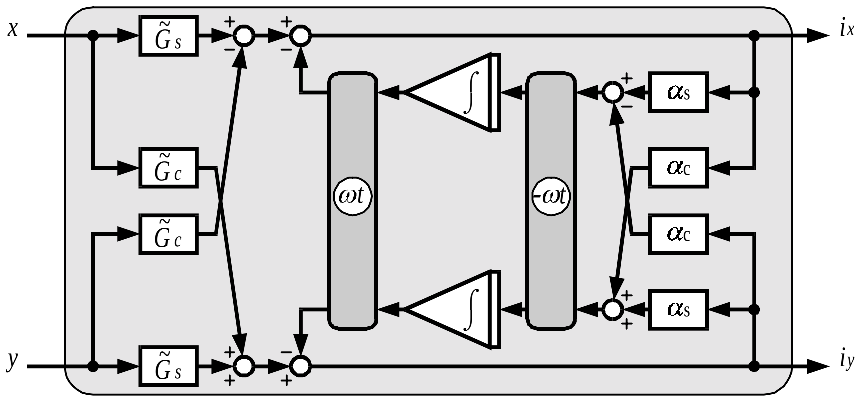
(a) Complex-variable representation



(b) Complex-variable representation of current feedback type



(c) Real-variable representation of (b)



(d) Convolution-integral form of (c)

FIGURE 6: Current regulation control for unbalance force

TABLE 1: Classification of the output regulation control

Variable to be regulated	Disturbance	
	Stepwise force	Unbalance force
Displacement	Zero static error Integral action	Zero whirling [H.1]
Current	Zero-power control Velocity feedback Integral feedback of current	Zero AC power consumption [H.2] *Approximate* state of "automatic balancing"
Force	*Impossible*	Zero transmitted vibration *Exact* state of "automatic balancing"

APPLICATION TO CONTROL SYSTEM DESIGN

We can directly construct controllers achieving specified control objectives based on the previous analyses. For example, a general form of controllers achieving the objective (21) is given by

$$I_z(s) = -\frac{s\tilde{h}(s)}{g(s)} Z(s). \tag{40}$$

Assuming that compensator is restricted to proper rational function, a second- or higher-order compensator is necessary for assigning the closed-loop poles arbitrarily. Thus, the minimal-order compensator that achieves both current regulation and pole assignment is given by

$$I_z(s) = -\frac{s(\tilde{h}_2 s + \tilde{h}_1)}{s^2 + g_1 s + g_0 s} Z(s). \tag{41}$$

Replacing the factor s in the numerator of the controller (41) by $(s - j\omega)$, we obtain a controller achieving (31) as

$$I_c(s) = -\frac{(s - j\omega)(\tilde{h}_2 s + \tilde{h}_1)}{s^2 + g_1 s + g_0 s} X_c(s). \tag{42}$$

Figure 7 shows a block diagram with real variables and coefficients of this controller. Such configuration has not been reported as a controller realizing an approximate state of automatic balancing.

It is to be mentioned that controllers for achieving other control objectives will be constructed in similar ways.

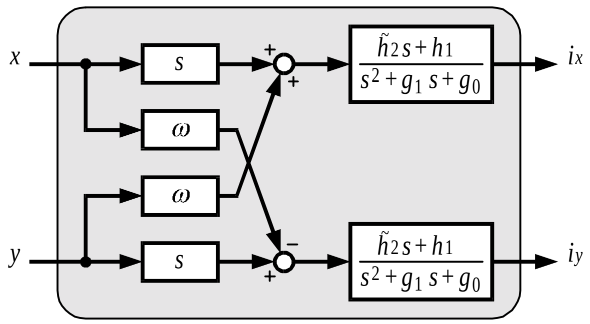


FIGURE 7: Real-variable representation of a current regulation controller for unbalance force

CONCLUSION

The general structures of the controllers achieving specified performances were derived using a transfer function approach. The analyses clarified the relation between the controllers compensating stepwise force and those compensating unbalance force, and the equivalence between the independently developed unbalance compensators. A synthesis method of constructing controllers with specified performances was also presented.

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