MODEL BASED DECOUPLING CONTROL OF A DISC ROTOR ON ACTIVE MAGNETIC BEARINGS

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ABSTRACT
Under certain conditions a coupled system can be decoupled by means of feedback control. In this paper a method is presented to control a hard disc drive prototype on AMBs with a decoupling state feedback. In addition a feedforward matrix for the tracking problem is presented.
The method is based on a linear analytical model of the system which is adapted by using frequency domain measurements. The derivation of the model, the design of the feedback controller and the feedforward is shown. Simulations are compared with measurements on the experimental setup. The current work focuses on improving the performance of the deocupling controllers.

INTRODUCTION
A rotor on active magnetic bearings (AMBs) is a strongly coupled system especially due to the gyroscopic effects. But also at standstill the two radial bearings of a conventional AMBs are coupled in one plane. A disturbance entering on one radial bearing will also have an impact on the other radial bearing. The consequence of these effects is that single input single output (SISO) controllers often used to control AMBs are difficult to tune since the individual loops interfere with each other. The couplings get worse for disc rotors (length/diameter < 1). Methods to tune the SISO controllers under consideration of the other loops are protracted and do not always lead to a satisfying result. Decoupling filters can be used instead but they can lead to high order controllers or the filter can get unstable [1].
The idea of decoupling control is to control a variable of a fully coupled plant without influencing the other variables. Decoupling controllers are known already for a long time and have been used successfully on various application fields [1],[2]. Also for magnetic bearings decoupling controllers have been used. [3] describes decoupling control by a modal transformation of the differential equations.
The method presented in this paper is a decoupling method in state space. The states of the plant are the radial displacements of the rotor in a sensor reference frame and the corresponding speeds of these displacements. When the rotor is spinning the system is fully coupled.
The classical application of decoupling control is definitely the tracking problem where the output should follow a given trajectory. Dealing with magnetic bearings one is most often confronted with the disturbance rejection problem. Nevertheless there are applications for active magnetic bearings in tracking problems [4],[5]. Therefore the tracking problem is also treated in this paper.

THE SYSTEM
The system on which the developed controllers are tested is a hard disc drive rotor on active magnetic bearings. The rotor consists of a short hub (∅ 25 mm, L=30 mm) with an aluminum disc (∅ 85 mm) mounted on it. The mass of the rotor is about 100 g and the ratio of the inertial moments is $J_p / J_x \cong 2$.
The magnetic actuators are heteropolar reluctance type actuators. The iron cores consist of laminated Fe-Si. The bearing has an inside stator configuration (tubular rotor). The outer diameter of the actuators is 20 mm. The actuator coils are driven by linear amplifiers.

THE MODEL
The plant consists of the amplifier, the electromagnets, the rotor, the sensors and the antialiasing filter. To iden-

tify the mass and stiffness matrices of the system we consider the rotor at standstill. Then the two planes *xz* and *yz* are decoupled. Thus a 2 DOF model is sufficient to identify mass and stiffness. The inputs of this model are two voltages (vi_{xa}, vi_{xb}) corresponding to the references for the currents. The outputs are two displacements (y_{xa}, y_{xb}) expressed in the sensor reference system. In order to keep the order of the controller low a simple model is used first. The dynamics of the power amplifier and the filter is necleted. The inputs of this model are two currents (i_{xa}, i_{xb}), the outputs are the same as before (y_{xa}, y_{xb}). The model is linear, containing the mass matrix of the rotor and the force matrices of the magnetic actuators. The sensor is being modeled as a static gain. This simple approach will allow the use of a differentiator to obtain the state. FIGURE [1] shows a block scheme of the plant.

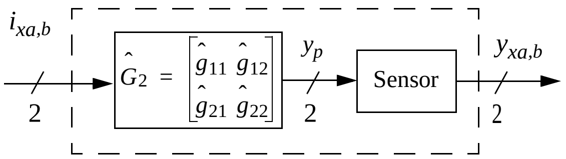

FIGURE 1: Open loop model of the AMB system.

The transfer function matrix G_2 describes the dynamics of the bearing with the two diagonal terms g_{11} and g_{22} and the cross coupling terms g_{12} and g_{21}. G_2 is not explicitly measurable but can be reconstructed by means of closed loop measurements. This method is described in [6].

The parameters of the model $\hat{G}_2$ are adapted by a least square optimization algorithm [6]. Since the real sensor dynamic is included in the measurement and the parameters of G_2 are fitted to these measurements one can argue that the model G_2 contains the sensor. FIGURE [2] shows the measurement G_2 and the model G_2.

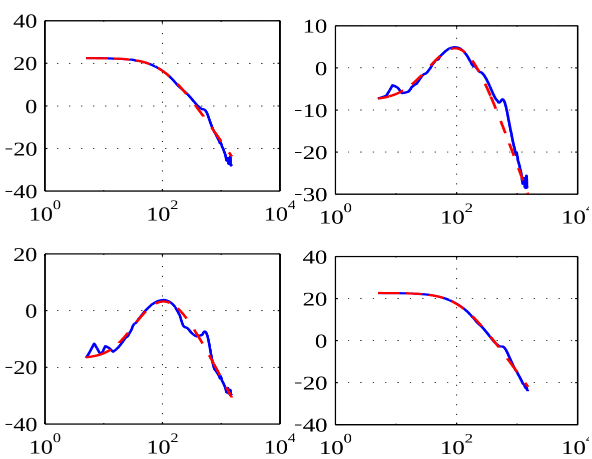

FIGURE 2: Open loop transfer functions g_{11}..g_{22} measured (solid) and simulated (dash).

To take into account the rotation the gyroscopic matrix is estimated using an FEM model of the rotor. This leads to the fully coupled 4x4 model matrix G_4.

DECOUPLING CONTROL

The method which is used in this paper is a model based decoupling in state space [7]. The states of the model are chosen as the displacements of the rotor in sensor coordinates and the corresponding velocities. The displacements are measured directly and the velocities can be obtained by means of a simple differentiator. The state and the plant are given by

$$\underline{x} = \begin{bmatrix} y_{xa} & y_{xb} & y_{ya} & y_{yb} & \dot{y}_{xa} & \dot{y}_{xb} & \dot{y}_{ya} & \dot{y}_{yb} \end{bmatrix}^T$$

$$\dot{\underline{x}} = A\underline{x} + B\underline{u}$$

$$\underline{y} = C\underline{x} = \begin{bmatrix} \underline{c}_1 & \underline{c}_2 & \underline{c}_3 & \underline{c}_4 \end{bmatrix}^T \underline{x}$$

The control law is a state feedback K_b with a feedforward matrix K_f:

$$\underline{u} = -K_b\underline{x} + K_f\underline{w}$$

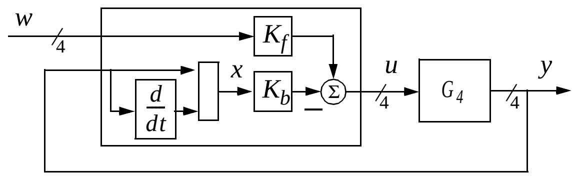

FIGURE 3: State feedback and feedforward matrix

Under the assumption that:

$$\underline{c}_i^T A^{\delta_i - k} B = \underline{0}^T \quad , \quad i = 1 \ldots 4, \quad k = 2 \ldots \delta_i$$

we can write:

$$\frac{d^{\delta_i} y_i}{dt} = (\underline{c}_i^T A^{\delta_i} - \underline{c}_i^T A^{\delta_i - 1} B K_b)\underline{x} + \underline{c}_i^T A^{\delta_i - 1} B K_f \underline{w}$$

$$i = 1 \ldots 4$$

where δ_i is the lowest derivation order of y_i that is directly influenced by *u*. δ_i is called the difference order of the plant relative to y_i [7]. The decoupling feedforward matrix is then given by:

$$K_f = \begin{bmatrix} \underline{c}_1^T A^{\delta_1 - 1} B \\ \underline{c}_2^T A^{\delta_2 - 1} B \\ \underline{c}_3^T A^{\delta_3 - 1} B \\ \underline{c}_4^T A^{\delta_4 - 1} B \end{bmatrix}^{-1} \begin{bmatrix} k_1 & 0 & 0 & 0 \\ 0 & k_2 & 0 & 0 \\ 0 & 0 & k_3 & 0 \\ 0 & 0 & 0 & k_4 \end{bmatrix} = D^{*-1} K$$

on condition that D* is not singular. The $\underline{c}_i^T$ are the transposed column vectors of C. k_i represent gains for each feedforward channel and can be chosen freely. The condition that D* has to be invertible is the decoupling condition.
The decoupling feedback matrix K_b is given by:

$$K_b = \begin{bmatrix} \underline{c}_1^T A^{\delta_1 - 1} B \\ \underline{c}_2^T A^{\delta_2 - 1} B \\ \underline{c}_3^T A^{\delta_3 - 1} B \\ \underline{c}_4^T A^{\delta_4 - 1} B \end{bmatrix}^{-1} \begin{bmatrix} \underline{c}_1^T A^{\delta_1} + \sum_{\nu}^{\delta_1 - 1} q_{1\nu} \underline{c}_1^T A^{\nu} \\ \underline{c}_2^T A^{\delta_2} + \sum_{\nu}^{\delta_2 - 1} q_{2\nu} \underline{c}_2^T A^{\nu} \\ \underline{c}_3^T A^{\delta_3} + \sum_{\nu}^{\delta_3 - 1} q_{3\nu} \underline{c}_3^T A^{\nu} \\ \underline{c}_4^T A^{\delta_4} + \sum_{\nu}^{\delta_4 - 1} q_{4\nu} \underline{c}_4^T A^{\nu} \end{bmatrix}$$

$q_{i\nu}$ are any constants you like and are used to determine the closed loop poles of the control system. With this *Kb* the closed loop differential equations can be written as:

$$\frac{d^{\delta_i} y_i}{dt} = -\sum_{\nu}^{\delta_i - 1} q_{i\nu} \frac{d^{\nu} y_i}{dt} + k_i w_i \qquad , i = 1 \ldots 4$$

The equation for y_i is decoupled from y_j $(i \neq j)$ and w_i influences only y_i. The transfer function from w_i to y_i is given by:

$$Y_i(s) = \frac{k_i}{s^{\delta i} + q_{i,\delta_i - 1} s^{\delta i - 1} + \ldots + q_{i1} s + q_{i0}} W_i(s)$$

$$i = 1 \ldots 4$$

SIMULATION
Several time domain simulation of the closed loop system have been performed. In the simulations a model is used that contains not only the bearing dynamics (including the sensor) but also the measured dynamics of the power amplifiers and the antialiasing filter. Thus an additional phase loss is introduced into the simulation. Note that the model used for the design of the controller does not contain the amplifier and the filter. The rotor spins at 7200 [rpm]. First the step response of y_{xa} and y_{xb} after a reference step of 10 [μ m] of w_{xa} is simulated (tracking problem).

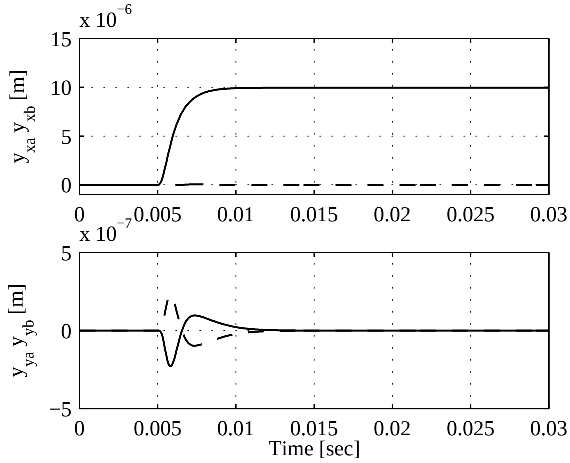

FIGURE 4: Step response of y_{xa} (solid) y_{xb} (dash) and *yya* (solid) and *yyb* (dash) respectively.

The two bearing planes *yxa* and y_{xb} are well decoupled. The displacement of y_{xb} is very small. However the gyroscopic matrix is not completely decoupled *yya* and *yyb* are not zero. The corresponding currents i_{xa} and i_{xb} are shown in FIGURE [5].

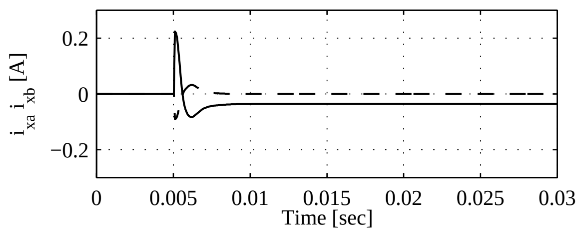

FIGURE 5: Currents i_{xa} and i_{xb} with decoupling control.

Note that the current saturates at 1.2 [A].
To have a comparison the same simulation is done using a PD feedback controller. The responses of y_{xa} and y_{xb} are shown in FIGURE [6].

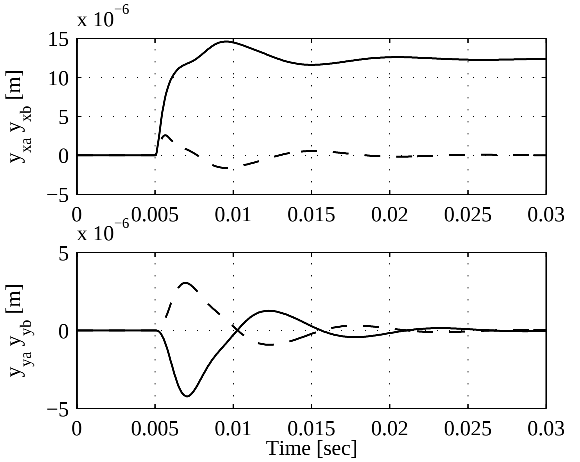

FIGURE 6: Step response of y_{xa} (solid), y_{xb} (dash) and *yya* (solid), *yyb* (dash) with PD controller.

In order to test the decoupling of the gyroscopic matrix the tilt coordinates α and β of the rotor are compared. An impulse is given on the acceleration $\ddot{\beta}$ of β (disturbance rejection). In FIGURE [7] the impulse responses of α and β are shown.

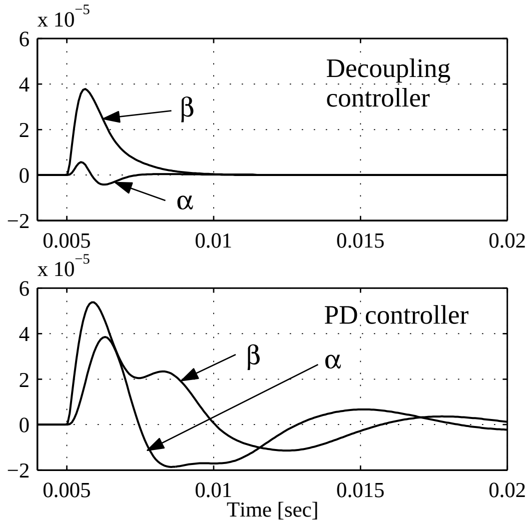

FIGURE 7: Simulated α and β after an impulse on $\ddot{\beta}$ (amplitude 1000 [rad/sec^2], duration 11 [ms]) once with decoupling control and once with PD.

The decoupling controller still works for this case although α doesn't remain close to zero anymore. With PD control the error of α is almost as important as the error of β .

EXPERIMENTAL RESULTS

The feedback controller and the feedforward matrix have been implemented on an experimental setup of a hard disc drive prototype on active magnetic bearings. The system is controlled by a control system based on a SHARC DSP of Analog Devices. The measurements with the spinning rotor are performed using a comb filter in order to get rid of the harmonics due to rotation.

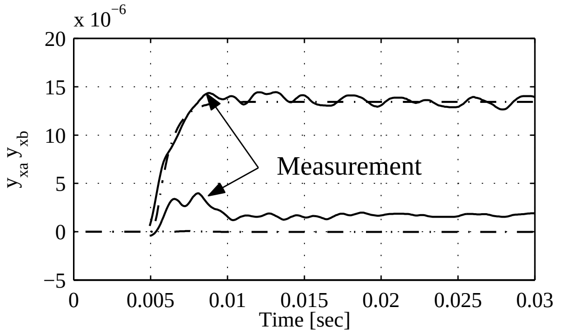

FIGURE 8: Measurement (solid) and simulation (dash-dot) of y_{xa} and y_{xb}.

FIGURE [8] shows the comparison of the measurement on one of the setups and the simulation presented in the preceding chapters. The measurement do not match the simulations completely. The amplitude of y_{xb} is eight times higher than expected.

DISCUSSION

A simple method to decouple an AMB system in state space was presented. Since the state of the model contains only the displacements of the rotor and the corresponding speeds, a differentiator could be used to estimate the state vector (speeds of the displacements in sensor coordinate frame).

The decoupling feedback controller performs well in simulation even when the simulation model an the model that was used to design the controller are not the same (the simulation model contains also the amplifiers and the filters).

The experimental result on the newest prototype did not fulfill all the expectations. The earlier implementation of the same controller on a setup with slightly different actuator-sensor configuration was very promising. There the decoupling worked more efficiently. The reason for this difference is probably a transmission zero in the cross coupling terms of the closed loop system which is more or less dominant. The present research focuses on taking this into account improving the decoupling control also for the latest prototype.

Including the dynamics of the power amplifier and the sensor into the design model and using a state observer of higher order might help to improve in addition.

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