THE DYNAMICS OF A CRACKED ROTOR WITH ACTIVE FEEDBACK CONTROL SYSTEM

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ABSTRACT
The dynamical characteristics of a cracked rotor with an active feedback control system(AFCS) are theoretically analyzed in this paper. The effects of control algorithms and controller parameters on dynamical characteristics of the cracked rotor are discussed. It is shown that the dynamical characteristics of the cracked rotor with AFCS are obviously complex than that of the traditional cracked rotor system. The fault characteristics of the cracked rotor, which are often used to diagnose the crack, will depend on the control algorithm used. Therefore, it is very difficult to diagnose the crack in the rotor system with AFCS. If the effect of the crack is not considered in designing the controller, the rotor system will lose stability in some cases when cracks appear.

INTRODUCTION
Active vibration control technology of rotating machinery, especially a series of successful applications of active magnetic bearings, provides a possibility to design high speed rotating machinery with small vibration and high stability. With the further development of the active vibration control technology, the smart machine technology, which integrates active vibration control, diagnosis, prognosis and correction, can guarantee an optimal condition of the machines with respect to higher performance and higher reliability for any state of the operation during the machines life period. This advanced technology will become a new research focus in the future.

Nowadays, almost research still focuses on the active vibration control, which includes developing new actuators and sensors, dynamics and modeling of active rotor system, control algorithms and fault detection of actuators and sensors. A few papers study the fault diagnosis in the active rotor system with active feedback control system(AFCS).

The diagnosis of faults in the active rotor system is a key stage in the smart machine technology. Only if the faults can be correctly diagnosed, the system can take a correct correction procedure. In order to correctly diagnose faults, the basic dynamic behaviour of the active rotor system with faults should first be studied, since there exist many differences in the fault characteristics of rotor systems between without and with AFCS.

It is well known that cracks may appear in the rotating shaft due to material fatigue at some time during machines life period and can result in calamitous accidents if undetected. It is very interesting to know what will happen when the cracks appear in rotor systems with AFCS.

There are two kinds of problem to be studied in the cracked rotor with AFCS. First, the AFCS, generally, is a time variant system; the stiffness, damping and mass of the active rotor system may be changed with time or/and rotational speed according with control algorithms. Therefore, the dynamics of the cracked rotor with AFCS will be different to that with invariant parameters. What are the main differences in the dynamic characteristics between the cracked rotor with AFCS and the general cracked rotor? Is it possible to use the methods used before to detect cracks in active rotor system? If not, how to detect the crack in the active rotor system?

Secondly, cracks in the rotating shaft make the stiffness of the rotor system periodically time-variant due to the effect of the opening and closing of the crack, which will result in the potential instability of the rotor system. However, the conventional controller design does not consider this problem, therefore there are many problems to be studied, for example, whether will the original AFCS continue to provide stable control when a crack appears? Can and how will the AFCS suppress the instability of the cracked rotor? How to design a stable controller for rotor systems with periodic time-variable parameters?

The dynamic characteristics of the cracked rotor with AFCS are studied using a simple rotor model in this paper. First, we summarize in brief the basic dynamic characteristics of the uncontrolled cracked rotor. Then, the effects of uncoupled PD feedback and optimal control algorithms and the controller parameters on the dynamics of the active cracked rotor are analyzed. Finally, the conclusions and the problems to be studied in the future are given.

SYSTEM MODEL OF CRACKED ROTOR

Rotor model

The rotor system is a massless flexible shaft with a middle disk, which is supported on two identical rigid bearings as shown in **FIGURE 1**. An active actuator(for example, active magnetic bearing) is located in the disk position as a damper in order to control the vibration of the disk. There is a transversal crack close the middle disk. The stiffness of uncracked rotor system is symmetric and the damping at the middle disk due to the air dynamical effect is viscous.

The rotor system can be considered as Jeffcott rotor, the equations of motion of the rotor system in stationary Cartesian co-ordinates can be written as:

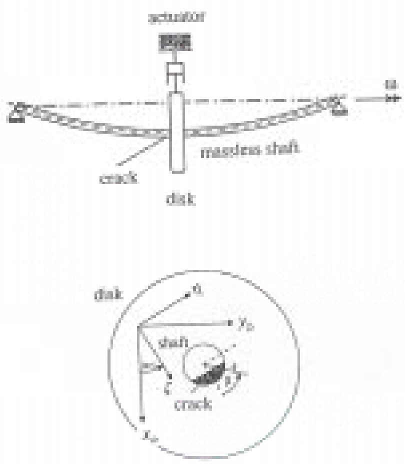


FIGURE 1: Cracked rotor model with AFCS

$$\begin{bmatrix} m_D & 0 \\ 0 & m_D \end{bmatrix}\begin{Bmatrix} \ddot{x}_D \\ \ddot{y}_D \end{Bmatrix}+\begin{bmatrix} c & 0 \\ 0 & c \end{bmatrix}\begin{Bmatrix} \dot{x}_D \\ \dot{y}_D \end{Bmatrix}+\boldsymbol{K}\begin{Bmatrix} x_D \\ y_D \end{Bmatrix}=\begin{Bmatrix} m_D g \\ 0 \end{Bmatrix}+m_D e_\mu \omega^2\begin{Bmatrix} \cos(\omega t+\phi) \\ \sin(\omega t+\phi) \end{Bmatrix} \quad (1)$$

Where x and y are the displacements of the disk in the stationary coordinate system. $\boldsymbol{K}$ is the stiffness matrix of the shaft in the disk position. m_D is the equivalent disk mass, c is the viscous damping coefficient, e_μ is the unbalance eccentricity between the centre of the gravity and the geometric centre of the disk, ϕ is the positional anguler of the imbalance with respect to the centre direction of the crack(i.e., the minimum stiffness direction ζ).

Dividing both sides by $m_D\omega^2\delta_{st}$, we obtain the non-dimensional equations of motion of the cracked rotor system as follows:

$$\begin{Bmatrix} X_D'' \\ Y_D'' \end{Bmatrix}+\frac{1}{\Omega}\begin{bmatrix} 2\xi & 0 \\ 0 & 2\xi \end{bmatrix}\begin{Bmatrix} X_D' \\ Y_D' \end{Bmatrix}+\bar{\boldsymbol{K}}(\tau)\frac{1}{\Omega^2}\begin{Bmatrix} X_D \\ Y_D \end{Bmatrix}=\frac{1}{\Omega^2}\begin{Bmatrix} 1 \\ 0 \end{Bmatrix}+U\begin{Bmatrix} \cos(\tau+\phi) \\ \sin(\tau+\phi) \end{Bmatrix} \quad (2)$$

Where $X_D = x_D/\delta_{st}, Y_D = y_D/\delta_{st}$. $U = e_\mu/\delta_{st}$ is the rotor unbalance parameter, $\xi = c/2m_D\omega_{cr}$ is the viscous damping ratio, $\Omega = \omega/\omega_{cr}$ is the rotating speed ratio. δ_{st} is the corresponding deformation of the shaft due to the weight of the rotor. $\omega_{cr} = \sqrt{k_o/m_D}$ is the critical speed of the uncracked rotor. k_0 is the stiffness of the uncracked rotor. $\tau = \omega t$ is nondimensional time. $\bar{\boldsymbol{K}}(\tau)$ is the nondimensional stiffness matrix of the cracked rotor system. Dot and prime refer to differentiation with respect to t and τ, respectively.

Crack Model

When there is no crack in the rotating shaft, $\bar{\boldsymbol{K}}(\tau)$ will be an invariant symmetrical diagonal matrix in which the diagonal element just is the stiffness of the uncracked rotor. When there exists a crack in the rotor, $\bar{\boldsymbol{K}}(\tau)$ is nonlinear and time varying during operation due to the effect of the crack.

The variations of the stiffness of the rotor are continued when a cracked rotor rotates slowly under the load of its own weight, the crack will open and close once per revolution. The closing and opening of the crack will be determined by the tension state of the crack area. The periodic closing and opening of the crack is called "*breathing*" action[1,2]. Since an exact model of the "*breathing*" crack is quite complicated, the variation of stiffnesses of the cracked shaft in the rotating coordinate system is often considered as the following form:

$$\begin{bmatrix} k_\zeta \\ k_\eta \end{bmatrix}=\begin{bmatrix} k_{m\zeta}+\Delta k_\zeta\cos\tau \\ k_{m\eta}+\Delta k_\eta\cos\tau \end{bmatrix} \quad (3)$$

where k_ζ and k_η are the stiffnesses of the rotor in the minimum and maximum stiffnss directions in the rotating coordinate system, ζ and η, in **FIGURE 1.** $k_{m\zeta}$ (or $k_{m\eta}$) and Δk_ζ (or Δk_η) are the average stiffness and the variations of stiffness of the cracked rotor in the ζ (or η) direction, respectively.

Transferring from the rotating coordinates to the stationary coordinate and nondimensionalizing, we obtain the nondimensional stiffness matrix of the cracked rotor as follows:

$$\bar{\boldsymbol{K}}(\tau)=\begin{bmatrix} \bar{k}_m+\Delta\bar{k}\cos 2\tau & \Delta\bar{k}\sin 2\tau \\ \Delta\bar{k}\sin 2\tau & \bar{k}_m-\Delta\hat{k}\cos 2\tau \end{bmatrix} \quad (4)$$

where $\bar{k}_m = \frac{1}{4}\{[(1+\bar{k}_\eta)+(1+\bar{k}_\zeta)]+[(1-\bar{k}_\eta)+(1-\bar{k}_\zeta)]\cos\tau\}$,

$\Delta\bar{k}=\frac{1}{4}\{[(1+\bar{k}_\eta)-(1+\bar{k}_\zeta)]+[(1-\bar{k}_\eta)-(1-\bar{k}_\zeta)]\cos\tau\}$. $\bar{k}_\eta$ and $\bar{k}_\zeta$ are the ratios of the stiffnesses of the cracked rotor in the η and ζ directions to the stiffness of the uncracked rotor k_0, respectively, and depend on the crack depth.

Eq.(2) with Eq.(4) are the general equations of motion of the cracked Jeffcott rotor in the steady state case. It is a linear system with periodic stiffness coefficient and an exact solution is not possible. The numerical methods can be used to get the solutions in order to analyse the unbalance responses of the cracked rotor system.

Control Model

The state equations of the cracked rotor system in Eq.(2) can be written as

$$q' = \begin{bmatrix} 0 & \boldsymbol{I} \\ -\boldsymbol{M}^{-1}\boldsymbol{K}(\tau) & -\boldsymbol{M}^{-1}\boldsymbol{C} \end{bmatrix} \boldsymbol{q} + \begin{bmatrix} 0 \\ \boldsymbol{M}^{-1} \end{bmatrix} \boldsymbol{u} + \boldsymbol{f}_e = \boldsymbol{A}\boldsymbol{q} + \boldsymbol{B}\boldsymbol{u} + \boldsymbol{f}_e \quad (5)$$

Where $\boldsymbol{A}$ is the system matrix, $\boldsymbol{q}=[\boldsymbol{x}\ \boldsymbol{x}']^T$ is the state vector of the rotor, $\boldsymbol{u}=[u_x\ u_y]^t$ is the control force matrix. $\boldsymbol{f}_e$ is the external force vector which includes the rotor imbalance and the gravity force. $\boldsymbol{x}=[X_D\ Y_D]^T$. $\boldsymbol{I}$ is the unit matrix. $\boldsymbol{K}(\tau)=\bar{\boldsymbol{K}}(\tau)/\Omega^2$. $\boldsymbol{C}=diag(2\xi/\Omega)$. $\boldsymbol{M}=\boldsymbol{I}$.

Open-loop PD control First, we use simple open-loop uncoupled proportional (P) and derivative (D) feedback control. For the simple open-loop PD feedback control, it is assumed that the AFCS produces a force direct according to measuring position and velocity of the disk. Then, the force is fed into the system with constant gain negative feedback[5]. So the control force can be written as:

$$\boldsymbol{u}=-\frac{\boldsymbol{K}_F}{\Omega^2}\boldsymbol{q}-\frac{\boldsymbol{C}_F}{\Omega}\boldsymbol{q}' \quad (6)$$

Where $\boldsymbol{K}_F$ and $\boldsymbol{C}_F$ are P and D gain matrices which are independent on the rotational speed.

Optimal control without the crack Generally, the effect of the crack on the AFCS, especially controller, is not considered at initial design stage. The controller is designed only according with the uncracked rotor system. In this case, $\boldsymbol{K}(\tau)$ in Eq.(5) is a time invariant matrix and equals the stiffness of the uncracked rotor, i.e., $\boldsymbol{K}(\tau)=\mathrm{diag}(1/\Omega^2)$.

Consider the quadratic performance index given by

$$\boldsymbol{J}=\int_0^\infty(\boldsymbol{q}^T\boldsymbol{Q}\boldsymbol{q}+\boldsymbol{u}^T\boldsymbol{R}\boldsymbol{u})dt \quad (7)$$

where $\boldsymbol{Q}$ and $\boldsymbol{R}$ are the positive semidefinite and positive definite weighting symmetric matrices, respectively, then the solution to minimization of $\boldsymbol{J}$ is the optimal control law given by

$$\boldsymbol{u}=-\boldsymbol{R}^{-1}\boldsymbol{B}^T\boldsymbol{P}\boldsymbol{q} \quad (8)$$

where $\boldsymbol{P}$ is the solution of the following seady state algebraic Riccati matrix equation.

$$\boldsymbol{P}\boldsymbol{A}+\boldsymbol{A}^T\boldsymbol{P}-\boldsymbol{P}\boldsymbol{B}\boldsymbol{R}^{-1}\boldsymbol{B}^T\boldsymbol{P}+\boldsymbol{Q}=\boldsymbol{0} \quad (9)$$

If the system [$\boldsymbol{A}$,$\boldsymbol{B}$] is controllable and [$\boldsymbol{A}$,$\boldsymbol{D}$] is completely observable, where $\boldsymbol{D}$ is any matrix such that $\boldsymbol{D}\boldsymbol{D}^T=\boldsymbol{Q}$, the positive definite solution matrix $\boldsymbol{P}$ always exists and the controlled system is asymptotically stable and the performance index can be reached. As the result, the optimal control force vector $\boldsymbol{u}_{opt}$, can be written using the feedback gain matrices as

$$\boldsymbol{u}_{opt}=-\boldsymbol{R}^{-1}\boldsymbol{B}^T\boldsymbol{P}\boldsymbol{q}=-[\boldsymbol{K}_{opt}\quad \boldsymbol{C}_{opt}]\boldsymbol{q}=-\{\boldsymbol{K}_{opt}\boldsymbol{q}+\boldsymbol{C}_{opt}\boldsymbol{q}'\} \quad (10)$$

Where $[\boldsymbol{K}_{opt}\boldsymbol{C}_{opt}]=\boldsymbol{R}^{-1}\boldsymbol{B}^T\boldsymbol{P}$, $\boldsymbol{K}_{opt}$ and $\boldsymbol{C}_{opt}$ are optimal feedback gain matrices. In fact, the optimal control can also be considered as a general PD control in which the feedback gains depend on the rotational speed.

Finally, the equations of motion of the active cracked rotor system with AFCS can be written in the following general form:

$$\begin{Bmatrix} X_D'' \\ Y_D'' \end{Bmatrix}+[\boldsymbol{C}+\boldsymbol{C}_c]\begin{Bmatrix} X_D' \\ Y_D' \end{Bmatrix}+[\boldsymbol{K}(\tau)+\boldsymbol{K}_c]\begin{Bmatrix} X_D \\ Y_D \end{Bmatrix}=\frac{1}{\Omega^2}\begin{Bmatrix} 1 \\ 0 \end{Bmatrix}+U\begin{Bmatrix} \cos(\tau+\phi) \\ \sin(\tau+\phi) \end{Bmatrix} \quad (11)$$

Where C_c and K_c are feedback gain matrices of the controller and depend on the control algorithms. Eq.(11) is the general equations of motion of the active cracked rotor system. The steady state unbalance responses are obtained by numerically solving these equations.

FLOQUET STABILITY THEORY

Either the cracked rotor system without or with AFCS is a linear periodic time varying system, the Floquet theory should be used in order to analyse the stability of the system. From Eq.(11), we obtain the pertubation equations of motion of the cracked system with AFCS as follows:

$$\begin{Bmatrix} \Delta X_D'' \\ \Delta Y_D'' \end{Bmatrix}+[\boldsymbol{C}+\boldsymbol{C}_c]\begin{Bmatrix} \Delta X_D' \\ \Delta Y_D' \end{Bmatrix}+[\boldsymbol{K}(\tau)+\boldsymbol{K}_c]\begin{Bmatrix} \Delta X_D \\ \Delta Y_D \end{Bmatrix}=0 \quad (12)$$

So, the first order state pertubation equations of motion of the system can be obtained and written in the matrix form as.

$$\delta\boldsymbol{q}'=\begin{bmatrix} 0 & \boldsymbol{I} \\ -\boldsymbol{M}^{-1}(\boldsymbol{K}(\tau)+\boldsymbol{K}_c) & -\boldsymbol{M}^{-1}(\boldsymbol{C}+\boldsymbol{C}_c) \end{bmatrix}\delta\boldsymbol{q}=[\boldsymbol{A}(\tau)]\delta\boldsymbol{q} \quad (13)$$

Where $\delta\boldsymbol{q}=\{\Delta X_D\ \Delta Y_D\ \Delta X_D'\ \Delta Y_D'\}^T$. $\boldsymbol{A}(\tau)=\boldsymbol{A}(\tau+2\pi)$ is a periodic coefficient matrix with period 2π.

From the Floquet stability theory, we should first have to calculate the transition matrix of the periodic time varying system over a period $T=2\pi$, $\boldsymbol{\Gamma}(\tau)$. This matrix will tell us how the state vecror $\boldsymbol{q}(\tau)$ of the system has changed after one period T. The relation of the state of the system after one period T with the initial state can be expressed as

$$\boldsymbol{q}(\tau+T)=\boldsymbol{\Gamma}(\tau)\boldsymbol{q}(\tau) \quad (14)$$

It is clear that if the transition matrix $\boldsymbol{\Gamma}(\tau)$ is known, the stability of the system can be determined from the eigenvalue of the following equation:

$$\left|\Gamma(\tau)-\mu I\right|=0 \tag{15}$$

Where μ refers the Floquet multiplier and gives the conditions of the stability. If the modulus of every eigenvalue $|\mu|$ is less than unity, the system is stable, otherwise the system is unstable. If fact, $|\mu|$ is the factor of increase or decrease of the vibration amplitude of the cracked rotor system after one period *T*.

Although the stability of the periodic time varying system can be determined from the transition matrix of the system, unfortunately, there is no general analytical method for calculating the transition matrix $\Gamma(\tau)$ for multi-variable systems. Therefore, the problem for determining the system stability becomes to numerically calculate the transition matrix $\Gamma(\tau)$.

To obtain the $\Gamma(\tau)$, the period *T* is divided in a number *n* of intervals of length *h=T/n*, in such a way that it is possible to consider that $A(\tau_i)$ is a constant on each small interval [*ih (i+1)h*], and so, to calculate elementary transition matrices $\Gamma(\tau_i)$ over the different intervals by using following formula given in [3]:

$$\Gamma_i = I + \frac{h}{2}\left(A(\tau_i)+A(\tau_{i+1})\right)\left(I+\frac{h}{2}A(\tau_{i+1})\right) \tag{16}$$

The finial transition matrix over one period is given by

$$\Gamma(T)=\Gamma(n-1)\Gamma(n-2).....\Gamma(i-1)\Gamma(i).....\Gamma(1)\Gamma(0) \tag{17}$$

So, we can easily obtain numerically the transition matrix over a period *T*, $\Gamma(\tau)$, the stability of the cracked rotor with or without AFCS can be analysed by the Floquet method.

RESULTS AND ANALYSE

The Dynamics of the Cracked Rotor without AFCS

Before discussing the stability and unbalance response of the cracked rotor with AFCS we briefly summarise the main results of dynamic behaviour of cracked rotor systems without AFCS.

(1). There are many super-harmonics in the vibration signals and subcritical resonances in the vibration speed response curves due to the crack breathing action. The former is produced by the periodic time-variant system and the reason for the latter is that one of the super-harmonics resonates at the subcritical speed. Note that the 1/*n*-th order subcritical resonance is caused by the *n*-th order super-hamonic vibration component.

(2).The crack mainly results in the additional resonances at ½, 1/3, 1/4…of main critical speed except the main resonsnce, or 2X, 3X, 4X…revolution super-harmonic components, especially for the 1/2 of the crirical speed or the 2X component. In some cases, the 2X vibration amplitude is greater than the 1X vibration amplitude.

(3).The motions of the cracked rotor are quite complicated due to existence of the 1X, 2X, 3X…frequency of the rotational speed in the vibration signals, but the period of these motions is equal to the period of the rotor motion.

(4). With the increase of the damping, the resonant amplitudes in both the main critical speed and the subcritical speeds will decrease greatly.

(5). For a given crack depth, the 1X revolution amplitude is associated with the rotor unbalance and the position of the unbalance relative to the crack direction. It is maximun when the unbalance is in phase with the crack, and minimum when out of phase with the crack. With the increase of the rotor unbalance, only the 1X amplitude increases, the other 2X, 3X… amplitudes do not change at all since the subcritical resonances are caused by the gravity.

With the increase of the crack depth, the 2X and 3X amplitudes or the 1/2 and 1/3 sub-critical resonant peaks increase obviously, as shown in **FIGURE 2**. The main resonant speed of the cracked rotor decreases with the increase of the crack depth, but its change is very small and it is impossible to use this change to detect the existence of a crack.

The damping ratio remains virtually constant during operation of machines, the 2X and 3X amplitudes change in the run-up or run-down operation can be used to as indexes for the detection of the cracks. When the rotor is running at a constant speed for a long period, the 2X and 3X amplitudes increase with the crack growth. This information can be used to detect the cracks as[4].

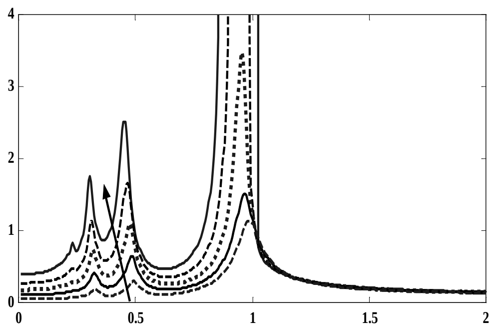


FIGURE 2: Effect of Crack Depth on Rotor Vibration

(6).The existence of the crack produces more unstable regions except in the vicinity of the main critical speed. The unstable regions near the rotational speeds at the 1/2, 1, 2 of the critical speed will expend with the increase of the crack depth when the external damping is relative small. Besides the three larger unstable regions, there exist other regions at lower speeds, but these regions are very narrow. If the damping of the rotor system is large, the unstable regions will disappear.

The Dynamics of the Cracked Rotor with AFCS

Open-loop PD control system The effects of the open-loop uncoupled PD feedback control on the vibration of the cracked rotor and system stability are shown in **FIGURES** 3 and 4, respectively. In fact, the negative and positive feedback P controls are possibly used to adjust the locations of resonant speeds, only the negative feedback D control is used to reduce the rotor vibration or to improve the system stability.

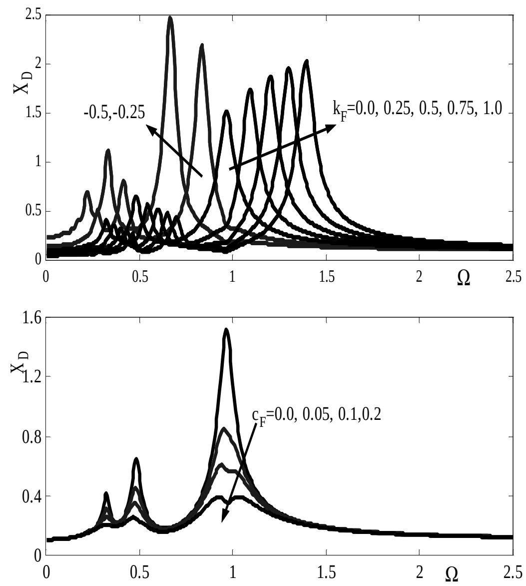


FIGURE 3: Effect of PD Gains on Rotor Vibration

It is shown that the effect of the uncoupled negative P feedback control alone on the rotor system is to increase the resonant speeds and decrease the 2X and 3X amplitudes, but the positive P feedback acts in the opposite effect. The negative D feedback control only changes the vibration amplitude in resonant regions, but does not greatly change the critical speeds. Although the 2X and 3X amplitudes change with the P or D gains, the sub-critical resonance also appears at the 1/*n* of the equivalent main resonant speed of the system. The 2X and 3X amplitudes will depend on the original damping of the rotor system and PD gains. Only if the uncoupled P or D control is fed to the system alone and the gain is known, is it possible to use the traditional method to detect the crack in the active rotor system. If the combination PD control is used, the problem becomes much more complex since there are three possible factors which can make the 2X and 3X amplitudes increase. In fact, the traditional diagnosis methods of cracks can be considered as a point since these methods just use the information in a fixed point, the introduction of the AFCS will make the point become to an area or set.

Since the uncoupled P and D controls act as additional stiffness and damping effects, respectively, therefore, with the increase of the negative D feedback gain, the unstable regions of the cracked rotor system narrow or even disappear. The unstable regions narrow and move towards the high speed with the increase of the negative P feedback gain, but widen and move towards the lower speed. The reason for this is that the equivalent stiffness of the rotor system changes with the P gain.

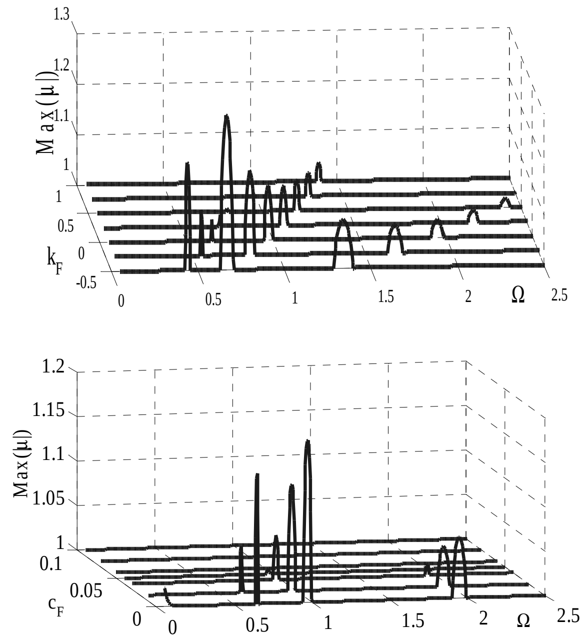


FIGURE 4: Effect of PD Gains on Rotor Stability

Close-loop optimal control system The optimal control based on the uncracked rotor system only can make the uncracked rotor system asymptotically stable and minimize the performance index. When the crack appears, the optimal control cannnot always guarantee the cracked rotor system is stable, the instability will occur in some cases as shown in **FIGURE 5**. The feedback gains of the optimal controller in the steady state, i.e., the additional stiffness and damping, are changed with the rotational speed, there possibly exist resonances in the vibration speed response curve, but the subcritical speeds are not exactly equal to a fraction of the resonant speed except the feedback gains are independent of the rotation speed. Whether the main and subcritical resonances appear or not in the active rotor system mainly depends on the weighting matrices ***Q*** and ***R***. **FIGURE 6** shows the effect of the ***R*** and ***Q*** on rotor unbalance response. Where the dotted line is for the cracked rotor wthout AFCS, the dashed line for the active cracked rotor with ***Q=I=I***, the lines above the dashed line are with ***Q***=I and ***R***=diag(r=5,10,20,50,100 and 500), the lines below the dashed line are with ***R=I*** and ***Q***=diag(5) or diag(10). It is shown that if ***R*** is much larger, i.e., we pay more attention to the controller and less to the rotor vibration, the effect of the controller or the additional damping and stiffness is very weak, the resonances will appear and locate at almost the same speed positions as that without AFCS. However it is still difficult to diagnose the crack since the damping and the stiffness of the rotor system vary with the rotational speed. When ***R*** is small and the ***Q*** is large, i.e., we pay more attention to the rotor vibration and less to the controller, the vibration of the rotor system is much smaller and the resonances do no appear at all. In this case with small rotor vibration, we can not get any information about the 2X and 3X vibrations from the unbalance response. Therefore, for the cracked rotor system with AFCS, it seems impossible to use the traditional method to detect the crack due to frequency variant parameters, new methods for detecting the crack in the active rotor system should be developed.

Since there still exists instability problems in the cracked rotor with optimal feedback control of the uncracked rotor, the controller design should be based on the cracked rotor system. In this way, the feedback control can make the cracked rotor stable and minimize the rotor vibration. The control problem of periodic time-varying systems is very complicated and has been studied[6,7]. The optimal control of the cracked rotor is in progress, the results will be reported in others.

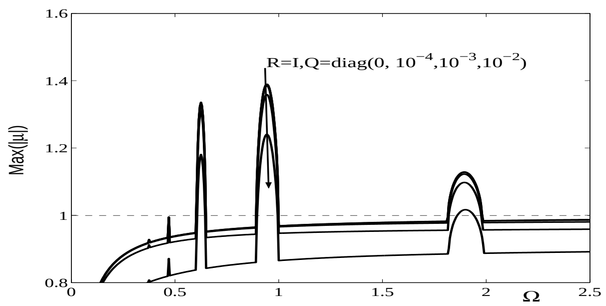
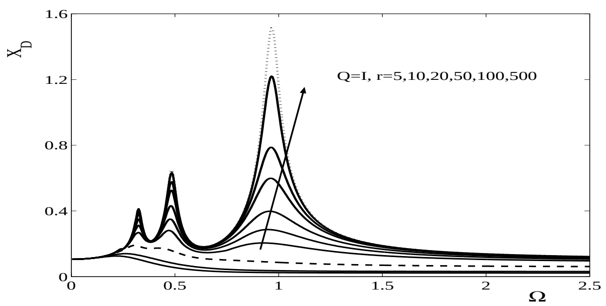


FIGURE 5: Stability of the Cracked Rotor with Optimal Control Gains of Uncracked Rotor System

FIGURE 6: Unbalance Response of the Cracked Rotor



with Optimal Control Gains of Uncracked Rotor System

CONCLUSIONS

The effect of the crack on the rotor system should be considered in designing the controller, otherwise the unstable motions in some regions of the rotational speeds possibly occur when crack appears.

The introduction of the AFCS in the rotor system will obviously change the dynamic characteristics of the cracked rotor system, make an index for diagnosing a crack from a point to a set or area. The reason for this is that the active rotor system with active feedback control is frequency variant. It is very necessary to develop new methods for diagnosing cracks in the active rotor systems.

The results also highlight many problems for further study, which include: (a)What are the differences in dynamic characteristics between the active rotor and uncontrolled rotor system when cracks appear in order to present a new fault index? (b)How to detect cracks in the active rotor system? (c) The controller design of the active rotor system with time or frequency variant parameters?

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