FAULT TOLERANT, DECOUPLING CONTROL OF MAGNETIC BEARINGS INCLUDING MATERIAL PATH RELUCTANCES

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ABSTRACT
An equivalent magnetic circuit of 8-pole heteropolar magnetic bearing including path reluctances is developed with non-dimensional forms of flux, flux density, and magnetic force equations. The results show that fluxes and magnetic forces are considerably reduced for the magnetic circuit including relatively large path reluctances. A Lagrange Multiplier optimization method is used to determine current distribution matrices for the magnetic bearing including large path reluctances. Optimizing this cost function yields distribution matrices calculated for certain combination of 5 poles failed out of 8 poles.

Control gains are determined based on the conditions that the closed loop dynamic properties of a failed magnetic bearing should be the same as those of an unfailed magnetic bearing. The cross coupled stiffnesses due to uneven flux distribution in case of a failed magnetic bearing are effectively canceled out by use of the cross feedback control.

NOMENCLATURE
A: Pole face area
b_{sat}: Saturation flux density
g: Air gap distance
I: Current vector
V_c: Input voltage vector
K : Current map matrix
k_p, k_d : Control gains
N: Coil turn matrix
n: Number of coil turns
q: Number of active poles
$\hat{T}$: Reduced distribution matrix
f: Magnetic flux vector
k : Power amplifier DC gain
x: Sensor sensitivity
l : Lagrange multiplier
m_0: Permeability of air

INTRODUCTION
Critical applications of magnetic bearings benefit from a fail-safe control approach. Without this many advantages of a magnetic bearing over conventional bearings such as oil film or rolling element bearings may be diminished. Fault-tolerant control seeks to provide continued operation of the bearing when power amplifiers or coils suddenly fail. The strong coupling property of a heteropolar magnetic bearing and redefined remaining coil currents make it possible to produce desired force resultants in the x and y directions even when some coils fail.

Lyons et al. [1] used a three control axis radial bearing structure with control algorithms for redundant force control and rotor position measurement. Therefore, if one of the coils fails, its control axis can be shut down while maintaining control. A bias current linearization method to accommodate the fault tolerance of magnetic bearings was developed, so the redistribution matrix which linearizes control forces can be obtained even if one or more coils fail [2], [3]. The fault tolerant magnetic bearing system was demonstrated on a large flexible-rotor test rig [4]. Na and Palazzolo [5] developed an optimization method to realize fault-tolerant magnetic bearings up to 5 poles failed out of 8 pole heteropolar magnetic bearing.

Material path reluctances are usually neglected for the analysis of a small magnetic bearing. However, path reluctances can affect the magnetic forces and their linearization for a large magnetic bearing or a magnetic bearing with low permeability material.

BEARING MODEL
Magnetic Circuit Model
The magnetic and electric fields of a magnetic bearing can be generally described by using Maxwell's equations. There exist some discrepancies between Maxwell's equations and one-dimensional magnetic circuit mainly due to flux leakage, fringing, and eddy current effects. Finite material path permeability may be

included in the magnetic force calculation to better predict the current-force relation [6], [7]. Figure 1 shows the equivalent magnetic circuit of an 8-pole heteropolar magnetic bearing including material path reluctances.

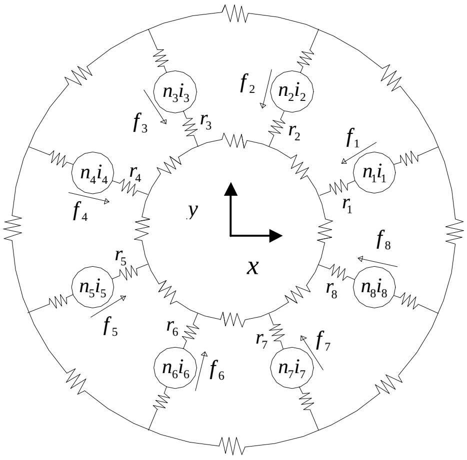

FIGURE 1 Equivalent Magnetic Circuit

Reluctances in the air gap are described as;

$$r_j = \frac{g_0 \hat{g}_j(\hat{x},\hat{y})}{m_0 A},\ j=1,2,..,8 \qquad (1)$$

where the nondimensional air gap equations are;

$$\hat{g}_j = 1 - \hat{x}\cos q_j - \hat{y}\sin q_j$$

$$\hat{x} = \frac{x}{g_0},\ \hat{y} = \frac{y}{g_0}$$

Material path reluctances for back iron, pole leg, and journal iron segments are described as;

$$r_i = \frac{l_i}{m_{rel} m_0 A_i},\ i=P,B,and\ J \qquad (2)$$

where the cross sectional areas, length, and length between poles are $A_P = r_P A$, $A_B = r_B A$, $A_J = r_J A$, $l_P = kg_0$, $l_B = kk_B g_0$, and $l_J = kk_J g_0$.
Apply Ampere's loop law, Gauss's law, and conservation law of fluxes of the magnetic circuit to obtain a matrix relation;

$$\frac{g_0}{m_0 A}\left[R(\hat{g}_j) + a\Re(h)\right]\Phi = NI \qquad (3)$$

where $a = \frac{k}{m_{rel} r_P}$, $h = \frac{r_P(r_J k_B + r_B k_J)}{r_B r_J}$

The magnetic fluxes in the gap is then described as;

$$\Phi = \frac{m_0 An}{g_0}\hat{R}^{-1}\hat{N}I, \qquad (4)$$

where $\hat{R} = R + a\Re$ (5)

The second term of Eq. (5) represents the intensity of the material path reluctance. This term may not be neglected for a large magnetic bearing or a magnetic bearing with low permeability material. The flux density in the air gap is also substantially reduced due to the leakage and fringing effects. The leakage and fringing were investigated by some researchers [8], [9], and [10]. Allaire [10] showed that the flux leakage and fringing effects can be approximated by a simple scaling factor. The flux density vector in the air gap is scaled by the leakage and fringing factor s ;

$$B = \frac{sm_0 n}{g_0}\hat{V}I, \qquad (6)$$

where $\hat{V} = \hat{R}^{-1}\hat{N}$

Magnetic Forces

The current-force relation including material path reluctances and leakage and fringing effects is then described as;

$$F_j = -\frac{s^2 m_0 An^2}{2g_0^2} I^T QI, \qquad (7)$$

where $Q(j,a,h) = \hat{V}^T \frac{\partial \hat{D}}{\partial \hat{j}}\hat{V} = \hat{V}^T \frac{\partial(diag(\hat{g}_j))}{\partial \hat{j}}\hat{V}$

The parameter j represents either x or y. Empirically determined value of s for a typical homopolar magnetic bearing ranges from 0.75 to 0.9. The currents distributed to the bearing are related to the control voltage vector with the distribution matrix [2], [5].

$$I = kTV_c = kK\hat{T}V_c, \qquad (8)$$

where $T = \left[T_b\, T_x\, T_y\right]$, $V_c = \left[v_b, v_{cx}, v_{cy}\right]^T$

The bias flux density should be set equal to $b_{sat}/2$ to obtain maximum magnetic forces. The bias voltage for obtaining the maximum magnetic force is then set as;

$$v_b = \frac{g_0 b_{sat}}{2km_0 n\left|\hat{V}T_b\right|_\infty} \qquad (9)$$

A distribution matrix for an unfailed 8-pole heteropolar magnetic bearing is;

$$T_1 = \begin{bmatrix} 1 & \cos\left(\frac{p}{8}\right) & \sin\left(\frac{p}{8}\right) \\ -1 & -\cos\left(\frac{3p}{8}\right) & -\sin\left(\frac{3p}{8}\right) \\ 1 & \cos\left(\frac{5p}{8}\right) & \sin\left(\frac{5p}{8}\right) \\ -1 & -\cos\left(\frac{7p}{8}\right) & -\sin\left(\frac{7p}{8}\right) \\ 1 & \cos\left(\frac{9p}{8}\right) & \sin\left(\frac{9p}{8}\right) \\ -1 & -\cos\left(\frac{11p}{8}\right) & -\sin\left(\frac{11p}{8}\right) \\ 1 & \cos\left(\frac{13p}{8}\right) & \sin\left(\frac{13p}{8}\right) \\ -1 & -\cos\left(\frac{15p}{8}\right) & -\sin\left(\frac{15p}{8}\right) \end{bmatrix}$$

The magnetic force along the j direction becomes a maximum when v_{cj} is equal to v_b. The bias current i_b becomes kv_b for an unfailed bearing. The maximum magnetic force of the 8-pole heteropolar bearing along the j direction calculated without material path reluctances is simplified as;

$$F_j^{\max} = \frac{4s^2 m_0 A n^2 i_b^2}{g_0^2} \qquad (10)$$

FAULT-TOLERANT MAGNETIC BEARINGS

Calculation of Distribution Matrices

The coil current distribution of bias currents, x control currents, and y control currents must be redefined in the case of single or multiple coil failures in order to produce the same force resultants. The optimization method for obtaining distribution matrices can be applied on a heteropolar magnetic bearing including path reluctances. The necessary conditions for the bias linearization are [2], [5];

$$\hat{T}^T G_j \hat{T} - M_j = \underline{0} \qquad (11)$$

where $$G_j = -\frac{m_0 A n^2}{2g_0^2} K^T \hat{V}^T \frac{\P \hat{D}}{\P \hat{j}} \hat{V} K,$$

$$M_x = \begin{bmatrix} 0 & 0.5 & 0 \\ 0.5 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \quad M_y = \begin{bmatrix} 0 & 0 & 0.5 \\ 0 & 0 & 0 \\ 0.5 & 0 & 0 \end{bmatrix}$$

The reduced distribution matrix to be determined is;

$$\hat{T} = \left[\hat{T}_b, \hat{T}_x, \hat{T}_y\right] \qquad (12)$$

where $\hat{T}_b = \left[t_1, t_2, \ldots, t_q\right]^T$, $\hat{T}_x = \left[t_{q+1}, t_{q+2}, \ldots, t_{2q}\right]^T$

$$\hat{T}_y = \left[t_{2q+1}, t_{2q+2}, \ldots, t_{3q}\right]^T$$

A cost function is defined in a manner that the Euclidean norm of flux density vector B is weighted with a diagonal matrix P:

$$J(\hat{T}) = B(\hat{T})^T P B(\hat{T}) \qquad (13)$$

The weighting matrix P can be assigned so that the load capacity in a specific direction is increased. Twelve equality constraint equations are also derived from Eq. (11) [5].

$$h_j(\hat{T}) = 0 \qquad (14)$$

The Lagrange Multiplier method can be applied to the basic problem to solve for $\hat{T}$ that satisfies Eq. (11). Define:

$$\hat{L}(\hat{T}) = B(\hat{T})^T P B(\hat{T}) + \sum_{j=1}^{12} l_j h_j(\hat{T}) \qquad (15)$$

Partial differentiation of Eq. (15) with respect to t_i and l_j leads to $3q + 12$ nonlinear algebraic equations to solve for t_i and l_j.

$$w_i = \frac{\P \hat{L}}{\P t_i} = 0, \quad i = 1, 2, \ldots, 3q \qquad (16)$$

$$w_{(j+3q)} = h_j(\hat{T}) = 0, \quad j = 1, 2, \ldots, 12 \qquad (17)$$

A vector form of $3q + 12$ nonlinear algebraic equations is;

$$W(t, l) = \begin{bmatrix} w_1(t, l) \\ w_2(t, l) \\ . \\ . \\ . \\ w_{(3q+11)}(t, l) \\ w_{(3q+12)}(t, l) \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ . \\ . \\ . \\ 0 \\ 0 \end{bmatrix} \qquad (18)$$

Distribution matrices for a heteropolar magnetic bearing including path reluctances are obtained by solving the system of nonlinear algebraic equations shown in Eq. (18). A least square iterative method (MATLAB) was used to solve the system of nonlinear algebraic equations, which yields multiple solutions. Various initial guess of t_i and l_j may be tested in order to obtain converged solutions.

Examples

The 8-pole heteropolar magnetic bearing used in this analysis has g_0 of 0.000508 m, A of 0.000602 m^2, l_P of 0.03 m, l_B of 0.065 m, l_J of 0.021 m, and n of 50. A_P, A_B, and A_J are assumed to be identical to A. m_{rel} of 500 is used for calculation of distribution matrices. $\hat{T}$ for 4 adjacent poles failed (5-6-7-8th) magnetic bearings is;

$$\hat{T}_4 = \begin{bmatrix} 2.94442 & 0.0770 & -1.2560 \\ 1.83753 & -0.1083 & -1.3101 \\ 1.83753 & 0.1083 & -1.3101 \\ 2.94444 & -0.077 & -1.2560 \end{bmatrix}$$

$\hat{T}$ for 2-4-6-7-8th poles failed magnetic bearing is shown as;

$$\hat{T}_5 = \begin{bmatrix} 8.06183 & 0.0775 & -0.2336 \\ 3.71891 & -0.0363 & 0.0418 \\ 8.06183 & 0.0402 & -0.2491 \end{bmatrix}$$

CONTROL DESIGN AND SIMULATIONS

Linearized Forces

The nonlinear magnetic forces with path reluctances can be linearized about the bearing center position and the zero control voltages by using Taylor series expansion.

$$F_x \approx -K_{pxx}x - K_{pxy}y + K_{vxx}v_{cx} + K_{vxy}v_{cy} \qquad (19)$$

$$F_y \approx -K_{pyx}x - K_{pyy}y + K_{vyx}v_{cx} + K_{vyy}v_{cy} \qquad (20)$$

Position stiffnesses and voltage stiffnesses are defined as;

$$K_{pwj} = -\frac{s^2 m_0 A n^2 v_b^2}{g_0^3} T_b^T \hat{U}_{wj0} T_b \qquad (21)$$

$$K_{vwj} = \frac{s^2 m_0 A n^2 v_b}{g_0^2} T_b^T \hat{U}_{w0} T_j \, , \qquad (22)$$

where $\hat{U}_j = -\hat{V}^T \frac{\partial \hat{D}}{\partial \hat{j}} \hat{V}$, $\hat{U}_{wj} = -\hat{V} \frac{\partial \hat{D}}{\partial \hat{w}} \left(\frac{\partial \hat{V}}{\partial \hat{j}} \right)^T$

Equations (19) and (20) are reduced to;

$$F_x \approx -K_{pxx}x - K_{pxy}y + K_v v_{cx} \qquad (23)$$

$$F_y \approx -K_{pyx}x - K_{pyy}y + K_v v_{cy} \qquad (24)$$

The calculated position stiffnesses and voltage stiffnesses are shown in Table 1.

TABLE 1 The Calculated Stiffnesses

	T_4	T_5
K_{pxx} (N/m)	-1221422	-580702
K_{pxy} (N/m)	0	-183119
K_{pyx} (N/m)	0	-183119
K_{pyy} (N/m)	-46624	-214463
K_v (N/V)	5	1.09
v_b (V)	5	1.09

Control Law

The simple PD control with low pass filters are used to design the closed loop system for unfailed bearings. The same closed loop stiffnesses and dampings may be maintained before and after coil failure if control gains are switched to appropriate values. The decoupled linearized forces for an unfailed bearing are;

$$F_j^{\,N} = -K_{pjj}^{\,N} j + K_{vjj}^{\,N} v_{cj}^{\,N} \qquad (25)$$

where $v_{cj}^{\,N} = -k_{pj}^{\,N} v_{sj} - k_{dj}^{\,N} \dot{v}_{sj}$, $v_{sj} = xj$

The parameter x is the sensor sensitivity. In general, the linearized forces for the failed bearing have undesirable cross-coupled position stiffnesses, and the direct position stiffnesses along the x and y axes are usually not symmetric. Cross feedback control forces are added in the linearized force equations of the failed bearing in order to cancel out the cross coupled position stiffnesses. The linearized forces for the failed bearing are;

$$F_x^{\,F} = -K_{pxx}^{\,F} x - K_{pxy}^{\,F} y + K_{vxx}^{\,F} (v_{cx}^{\,F} + \hat{v}_{cy}) \qquad (26)$$

$$F_y^{\,F} = -K_{pyx}^{\,F} x - K_{pyy}^{\,F} y + K_{vyy}^{\,F} (v_{cy}^{\,F} + \hat{v}_{cx}) \qquad (27)$$

where $v_{cj}^{\,F} = -k_{pj}^{\,F} v_{sj} - k_{dj}^{\,F} \dot{v}_{sj}$

$\hat{v}_{cx} = -k_{yc} k_{px}^{\,F} v_{sx}$, $\hat{v}_{cy} = -k_{xc} k_{py}^{\,F} v_{sy}$

The necessary conditions for the same closed loop stiffnesses and dampings before and after failure are;

$$F_x^{\,N} = F_x^{\,F}, \quad F_y^{\,N} = F_y^{\,F} \qquad (28)$$

This yields control gains for the failed bearing operation;

$$k_{px}^{\,F} = \frac{-K_{pxx}^{\,F} + K_{pxx}^{\,N} + xK_{vxx}^{\,N} k_{px}^{\,N}}{xK_{vxx}^{\,F}} \qquad (29)$$

$$k_{py}^{\,F} = \frac{-K_{pyy}^{\,F} + K_{pyy}^{\,N} + xK_{vyy}^{\,N} k_{py}^{\,N}}{xK_{vyy}^{\,F}} \qquad (30)$$

$$k_{dx}^{\ F} = \frac{K_{vxx}^{\ N}}{K_{vxx}^{\ F}} k_{dx}^{\ N} \tag{31}$$

$$k_{dy}^{\ F} = \frac{K_{vyy}^{\ N}}{K_{vyy}^{\ F}} k_{dy}^{\ N} \tag{32}$$

The cross feedback gains to eliminate the cross-coupled position stiffnesses are;

$$k_{xc} = -\frac{K_{pxy}^{\ F}}{x k_{py}^{\ F} K_{vxx}^{\ F}} \tag{33}$$

$$k_{yc} = -\frac{K_{pyx}^{\ F}}{x k_{px}^{\ F} K_{vyy}^{\ F}} \tag{34}$$

Simulations

Fault tolerant control system of a horizontal rigid rotor supported on magnetic bearings is constructed. A symmetric horizontal rigid rotor has mass of 10.7 kg, polar moment of inertia of 0.008 kgm^2, transverse moment of inertia about the mass centor of 0.36 kgm^2, and bearing locations of 0.22 m on each side of the mass center. Unbalances of eccentricity of 2.5 E-6 are applied on two bearing locations with a relative phase angle 90°. The sensor sensitivity z is 7874 V/m. The power amplifier gain k is 1 Amp/Volt. The control law was designed with simple PD control and low pass filters.

The following system dynamics simulation illustrates the transient response of a rotor supported by magnetic bearings during a coil failure event. A distribution matrix of T_1 is used to distribute currents to the unfailed bearings. The parameters K_p, K_v, and v_b with T_1 at $a = 0.1181 (m_{rel} = 500)$ for unfailed bearings are –1152000 N/m, 116.78 N/Volt, and 5.836 respectively. The designed PD control gains k_p and k_d for the unfailed bearings are 10 and 0.03 respectively. A new distribution matrix and control gains should be provided to produce desired force resultants when some coils in a magnetic bearing fail suddenly. The transient response from normal operation to fault-tolerant control with 5-6-7-8th coils failed for both bearings was simulated for nonlinear bearings with path reluctances at 10,000 RPM. The distribution matrix of T_1 was switched to T_{5678} when 4 adjacent coils failed at 0.1 second. The PD control gains k_{px}, k_{py}, and k_d for the failed bearings were adjusted as 235.3, 205.5 and 0.7 respectively. Transient response of the orbit at bearing A is shown in Fig. 2. Transient response of the current inputs to bearing A for the 5-6-7-8th poles failed case is shown in Fig. 3. This shows that large currents are required to maintain similar dynamic properties before and after failure. Transient response of the flux densities in Bearing A is shown in Fig. 4. The adjusted distribution matrix of T_{5678} yields the required inactive pole fluxes so that the bearing has the necessary forces to maintain stability. The load capacity is considerably reduced though in the failed bearing.

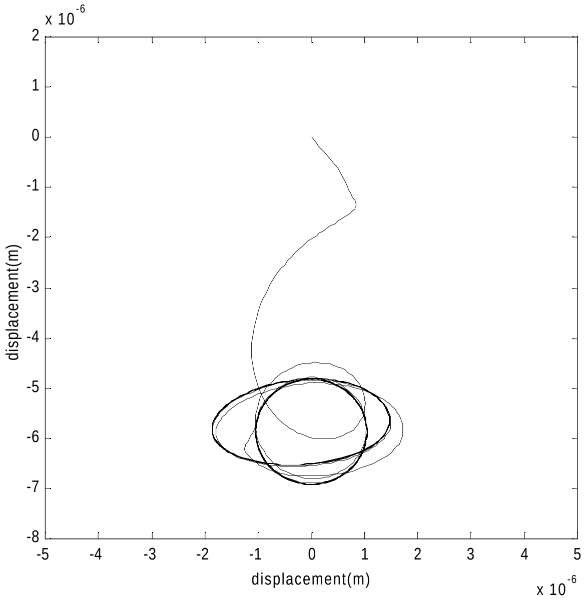

FIGURE 2 Orbit Plot for Normal Operation to the 5-6-7-8th Poles Failed Operation

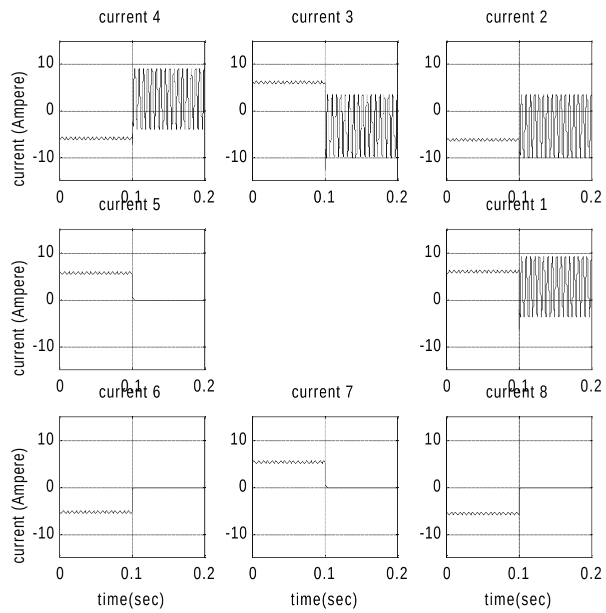

FIGURE 3 Current Inputs for Normal Operation to the 5-6-7-8th Poles Failed Operation

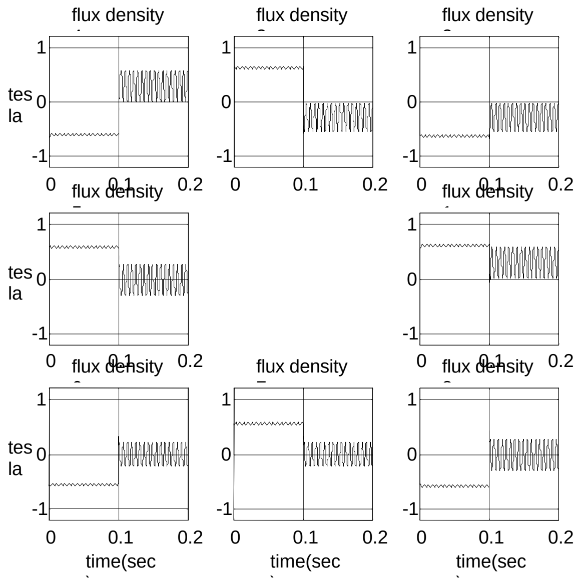

FIGURE 4 Flux Densities for Normal Operation to the 5-6-7-8th Poles Failed Operation

CONCLUSIONS

Material path reluctances are usually neglected for the calculation of fluxes and magnetic forces, however, they may significantly influence fluxes and magnetic forces for a large magnetic bearing or a magnetic bearing with low relative permeability. Therefore, material path reluctances should be included in the calculation of distribution matrices for fault-tolerant control.

A Lagrange Multiplier optimization method is used to determine distribution matrices for the magnetic bearing including large path reluctances. A cost function is defined in a manner that represents load capacity in a specific direction. The distribution matrices are calculated up to certain combination of 5 poles failed out of 8 poles. Nondimensional form of the position stiffnesses and voltage stiffnesses are calculated for the fault-tolerant magnetic bearings.

Control gains are determined based on the conditions that the closed loop dynamic properties of a failed magnetic bearing should be the same as those of an unfailed magnetic bearing. Orbits after failure can be maintained close to the orbit before failure if appropriate control gains are selected after failure to maintain the same closed loop dynamic properties. Relatively large increase in currents and flux densities may be required to maintain the same closed loop dynamic properties after failure. Therefore, disturbance levels from imbalance, runout or sideloads should be maintained at low levels for success of this approach.

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REFERENCES

1. Lyons, J. P., Preston, M. A., Gurumoorthy, R., and Szczesny, P. M., 1994, "Design and Control of a Fault-Tolerant Active Magnetic Bearing System for Aircraft Engine," Proceedings of the Fourth International Symposium on Magnetic Bearings, ETH Zurich, pp. 449-454.

2. Maslen, E. H. and Meeker, D. C., 1995, "Fault Tolerance of Magnetic Bearings by Generalized Bias Current Linearization," IEEE Transactions on Magnetics, Vol. 31, No. 3, pp. 2304 – 2314.

3. Meeker, D. C., 1996, "Optimal Solutions to the Inverse Problem in Quadratic Magnetic Actuators," Ph. D. Dissertation, Univ. of Virginia, Mechanical Engineering

4. Maslen, E. H., Sortore, C. K., Gillies, G. T., Williams, R. D., Fedigan, S. J., and Aimone, R. J., 1997, "A Fault Tolerant Magnetic Bearing System," Proceedings of MAG'97, Industrial Conference and Exibition on Magnetic Bearings, pp. 231-240.

5. Na, U.J. and Palazzolo, A.B., 1999 *Proceedings of the 1999 ASME Design Engineering Technical Conferences*, September 12-15, Las Vegas, Nevada. Paper VIB-8258. Optimized Realization of Fault-Tolerant Heteropolar Magnetic Bearings for Active Vibration Control, To appear in Jul., 2000, *ASME Journal of Vibration and Acoustics*

6. Knight, J.D., Xia, Z., McCaul, E., and Hacker, H., Jr., 1992, "Determination of Forces in a Magnetic Bearing Actuator: Numerical Computation With Comparison to Experiment," Journal of Tribology, Vol. 114, pp. 796-801.

7. Allaire, P.E., Fittro, R.L., Maslen, E.H., and Wakefield, W.C., 1997, "Measured Force/Current Relations in Solid Magnetic Thrust Bearings," Journal of Engineering for Gas Turbines and Power, Vol. 119, pp. 131-142.

8. Meeker, D.C., Maslen, E.H., and Noh, M.D., "A Augmented Circuit Model for Magnetic Bearings Including Eddy Currents, Fringing, and Leakage," IEEE Transactions on Magnetics, Vol. 32, No. 4, pp. 3219-3227.

9. Hsiao, F. and Lee, A., 1994, "An Investigation of the Characteristics of Electromagnetic Bearings Using the Finite Element Method," Journal of Tribology, Vol. 116, pp. 710-719.

10. Allaire, P.E., 1989, "Design and Test of a Magnetic Thrust Bearing," Journal of the Franklin Institute, Vol. 326, No. 6, pp. 831-847.