NON-LINEAR BEHAVIOR OF A MAGNETICALLY SUPPORTED ROTOR

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ABSTRACT
Non-linearity is an inherent and essential characteristic of active magnetic bearings. For simplicity, only the non-linearity between force, current and displacement of the electromagnets is considered while other nonlinearities are neglected. The nonlinear response of a rotor in active magnetic bearings is investigated for the case of a primary resonance. The method of multiple scales is used to obtain four first-order ordinary differential equations that describe the modulation of the amplitudes and phases of oscillations in the horizontal and vertical directions. The steady state response and its stability are obtained numerically from the reduced equations. In the regime of multiple coexisting solutions, two stable solutions are found. Finally, the analytical results are verified by integrating the governing equations.

1. INTRODUCTION

Most of the components of AMBs are non-linear, therefore the entire system becomes inherently non-linear [1]. In simulations of the dynamic behavior of magnetically suspended rotors, usually the components are modeled linearly with the non-linearities neglected for simplicity. However, the non-linear properties of AMBs can lead to a different behavior of the rotor system than predicted by a linear model. The nonlinear response of rotor-active magnetic bearing systems has been studied by a number of people [2-6].

In this paper, only the dominant non-linearity between force, current and displacement of the electromagnets is considered while other nonlinearities are neglected. The independent axis control strategy is used to control the vibration of the rotor, and the PD control is used for the feedback control system. Further for simplicity, the rotor is assumed to be a rigid body in AMBs. Thus the model consists of one mass with two degree-of-freedom in the x and y directions. The fundamental resonance of the system is examined by using a perturbation method.

2. ROTOR–AMB SYSTEM MODEL

An AMB is shown schematically in Figure 1. The stator has eight pole pairs. For simplicity, the saturation and the hysteresis of the magnetic core material, the eddy current loss, and all other secondary effects are neglected. All magnets are assumed to have identical structure and the same number of windings.

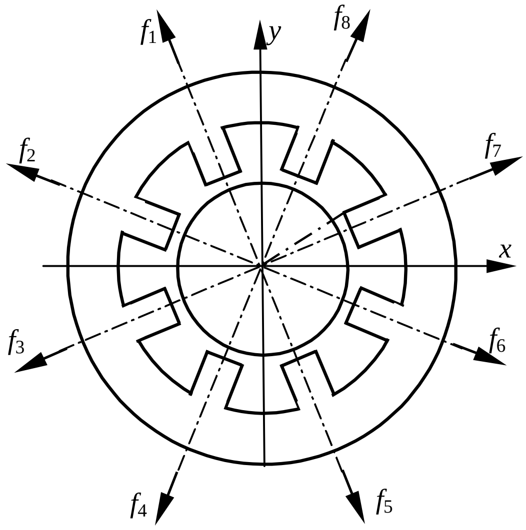


FIGURE 1: Schematic for modeling magnetic forces acting on the rotor.

According to the electromagnetic theory, the electromagnetic force f_i produced by every pair of electromagnets can be expressed as [7]

$$f_i = \frac{1}{4}\mu_0 N^2 A \frac{I_i^2}{\delta_i^2}\cos\varphi \, , i = 1,2,\cdots,8 \qquad (1)$$

where μ_0 is the permeability, A is the effective area of the cross section of one electromagnet, N is the number of winds around the core, I_i is the coil current that is equal to the sum of the bias current of the electromagnet and the control current, δ_i is the radial clearance between the stator and the rotor, and φ is the corresponding half angle of the radial electromagnetic circuit, respectively.

For an AMB, when the rotor deviation from the center of the bearings is x and y, the radial clearance between the electromagnets and the rotor can be written as

$$\delta_i = c_0 \pm x\sin\alpha \mp y\cos\alpha\, ,\ i = 1,5$$
$$\delta_i = c_0 \pm x\sin\alpha \pm y\cos\alpha\, ,\ i = 4,8$$
$$\delta_i = c_0 \pm x\cos\alpha \mp y\sin\alpha\, ,\ i = 2,6$$
$$\delta_i = c_0 \pm x\cos\alpha \pm y\sin\alpha\, ,\ i = 3,7 \qquad (2)$$

where c_0 is the steady state air gap and α is the corresponding angle of a radial electromagnetic circuit.

The pre-magnetization current I_0 is usually sent through all coils, and the control currents are superimposed on the pre-magnetization current. Thus currents flowing in the coils are given by

$$I_1 = I_8 = I_0 - i_y\, ,$$
$$I_4 = I_5 = I_0 + i_y\, ,$$
$$I_6 = I_7 = I_0 - i_x\, ,$$
$$I_2 = I_3 = I_0 + i_x\, , \qquad (3)$$

The magnetic force acting on the rotor in each direction is the difference between the attractive forces of both magnets fixed on opposite sides. Therefore the total electromagnetic forces in the horizontal and vertical directions can be derived as

$$f_x = f_x(linear) + f_x(cubic) + 0(4)\, ,$$
$$f_y = f_y(linear) + f_y(cubic) + 0(4)\, , \qquad (4)$$

where 0(4) denotes the terms of order greater than four. Here, for the sake of brevity, the simple notations $f_x(linear)$ and $f_y(linear)$ are used to denote the linear terms, and $f_x(cubic)$ and $f_y(cubic)$ represent the cubic nonlinear terms, respectively.

For magnetically suspended rotors various control techniques have been used to achieve various aims. However in this article, only the current PD control is considered

$$i_x = k_p x + k_d \dot{x}\, ,$$
$$i_y = k_p y + k_d \dot{y}\, , \qquad (5)$$

where k_p and k_d are the proportional and derivative gains respectively, and the controllers' PD gains for all eight pole–pairs are taken to be same.

As the focus of the work is on the effect of non-linearity of AMBs on the nonlinear response of a rotor, the rotor is assumed to be a rigid body in AMBs for simplicity. Thus the model consists of one mass with two degree-of-freedom. The non-dimensional equations of motion governing the unbalance of the rotor can be derived as

$$\ddot{x} + 2\mu\dot{x} + \omega^2 x - (\alpha_1 x^3 + \alpha_2 xy^2 + \alpha_3 x^2\dot{x} + \alpha_4\dot{x}y^2 + \alpha_5 x\dot{y}^2 + \alpha_6 x\dot{x}^2 + \alpha_7 xy\dot{y}) = 2f\cos\Omega t\, ,$$
$$\ddot{y} + 2\mu\dot{y} + \omega^2 y - (\alpha_1 y^3 + \alpha_2 x^2 y + \alpha_3 y^2\dot{y} + \alpha_4 x^2\dot{y} + \alpha_5\dot{x}^2 y + \alpha_6 y\dot{y}^2 + \alpha_7 x\dot{x}y) = 2f\sin\Omega t\, , \qquad (6)$$

where μ, ω, Ω, f and the coefficients of the nonlinear terms α_i are defined in Appendix A. The rotor weight is neglected in the present analysis. The closed form of the solutions for equation (6) cannot be found. Hence, approximate solutions are sought by using the method of multiple scales (MMS) [8].

3. PERTURBATION ANALYSIS BY USING MMS

The MMS [8] is employed to obtain four first order amplitude– and phase–modulated equations. To achieve this, the small dimensionless parameter ε is introduced as a book-keeping device to indicate the smallness of damping (derivative gain), non-linearities and excitation (unbalance). Assuming the amplitude of motion is small, equation (6) can be expressed as

$$\ddot{x} + \varepsilon 2\mu\dot{x} + \omega^2 x - \varepsilon(\alpha_1 x^3 + \alpha_2 xy^2 + \alpha_3 x^2\dot{x} + \alpha_4\dot{x}y^2 + \alpha_5 x\dot{y}^2 + \alpha_6 x\dot{x}^2 + \alpha_7 xy\dot{y}) = \varepsilon 2f\cos\Omega t\, ,$$
$$\ddot{y} + \varepsilon 2\mu\dot{y} + \omega^2 y - \varepsilon(\alpha_1 y^3 + \alpha_2 x^2 y + \alpha_3 y^2\dot{y} + \alpha_4 x^2\dot{y} + \alpha_5\dot{x}^2 y + \alpha_6 y\dot{y}^2 + \alpha_7 x\dot{x}y) = \varepsilon 2f\sin\Omega t \qquad (7)$$

where $|\varepsilon| << 1$, $x = \varepsilon^{\frac{1}{2}}\bar{x}$, $y = \varepsilon^{\frac{1}{2}}\bar{y}$, $\mu = \bar{\mu}\varepsilon^{-1}$, $2\varepsilon\bar{f} = 2f\varepsilon^{-\frac{1}{2}}$. Here, for the sake of brevity, the superscripts "bar" have been omitted.

To study its fundamental resonance, the external detuning parameter σ is introduced as $\omega^2 = \Omega^2 + \varepsilon\sigma$.

According to the MMS, an approximation to the solution of equation (7) can be expanded in the form

$$x(t;\varepsilon) = x_0(T_0,T_1) + \varepsilon x_1(T_0,T_1) + \cdots,$$
$$y(t;\varepsilon) = y_0(T_0,T_1) + \varepsilon y_1(T_0,T_1) + \cdots, \qquad (8)$$

where $T_0 = t$, $T_1 = \varepsilon t \cdots$ are different time scales.

Following the procedures of the MMS, the so–called solvability conditions for the first-order are derived as

$$-2i\Omega A_1' - (\sigma + i2\mu\Omega)A_1 + (3\alpha_1 + \Omega^2\alpha_6)A_1^2\bar{A}_1 + 2(\alpha_2 + \alpha_5\Omega^2)A_1A_2\bar{A}_2$$
$$+(\alpha_2 - \alpha_5\Omega^2)\bar{A}_1A_2^2 + i\alpha_3\Omega A_1^2\bar{A}_1 + i2\alpha_4\Omega A_1A_2\bar{A}_2 + i(\alpha_7 - \alpha_4)\Omega\bar{A}_1A_2^2 + f = 0,$$
$$-2i\Omega A_2' - (\sigma + i2\mu\Omega)A_2 + (3\alpha_1 + \Omega^2\alpha_6)A_2^2\bar{A}_2 + 2(\alpha_2 + \alpha_5\Omega^2)A_1\bar{A}_1A_2$$
$$+(\alpha_2 - \alpha_5\Omega^2)A_1^2\bar{A}_2 + i\Omega\alpha_3A_2^2\bar{A}_2 + i\alpha_4\Omega A_1\bar{A}_1A_2 + i(\alpha_7 - \alpha_4)\Omega A_1^2A_2 - if = 0, \qquad (9)$$

where the prime denotes differentiation with respect to T_1. Introducing the polar transformation

$$A_n = \frac{1}{2}a_n\exp(i\beta_n), \ \bar{A}_n = \frac{1}{2}a_n\exp(-i\beta_n), \ n=1,\ 2. \qquad (10)$$

where a_n and β_n are real functions of time, substituting equation (10) into equation (9) and separating the real and imaginary parts, the following equations for a_n and β_n are obtained

$$a_1' = -\mu a_1 + b_1a_1a_2^2\sin 2\phi + b_2a_1^3 + b_3a_1a_2^2\cos 2\phi + b_4a_1a_2^2 - f_1\sin\beta_1$$
$$a_1\beta_1' = \sigma_1a_1 - b_5a_1^3 - b_6a_1a_2^2 - b_1a_1a_2^2\cos 2\phi + b_3a_1a_2^2\sin 2\phi - f_1\cos\beta_1,$$
$$a_2' = -\mu a_2 - b_1a_1^2a_2\sin 2\phi + b_2a_2^3 + b_3a_1^2a_2\cos 2\phi + b_4a_1^2a_2 - f_1\cos\beta_2$$
$$a_2\beta_2' = \sigma_1a_2 - b_5a_2^3 - b_6a_1^2a_2 - b_1a_1^2a_2\cos 2\phi - b_3a_1^2a_2\sin 2\phi + f_1\sin\beta_2, \qquad (11)$$

where $\phi = \beta_2 - \beta_1$, the coefficients σ_1, f_1 and b_i (i=1,6) are defined in Appendix B. These averaged equations are first-order approximations of the original system (6), and describe the modulation of the amplitudes and phases of the fundamental resonance of the system on a slow time scale. Equation (11) can be reduced to a set of nonlinear algebraic equations by imposing the condition of stationarity, namely, $a_{1,2}' = 0$ and $\beta_{1,2}' = 0$. Then the steady-state responses are obtained from the non-linear algebraic equations by using the Newton–Raphson method.

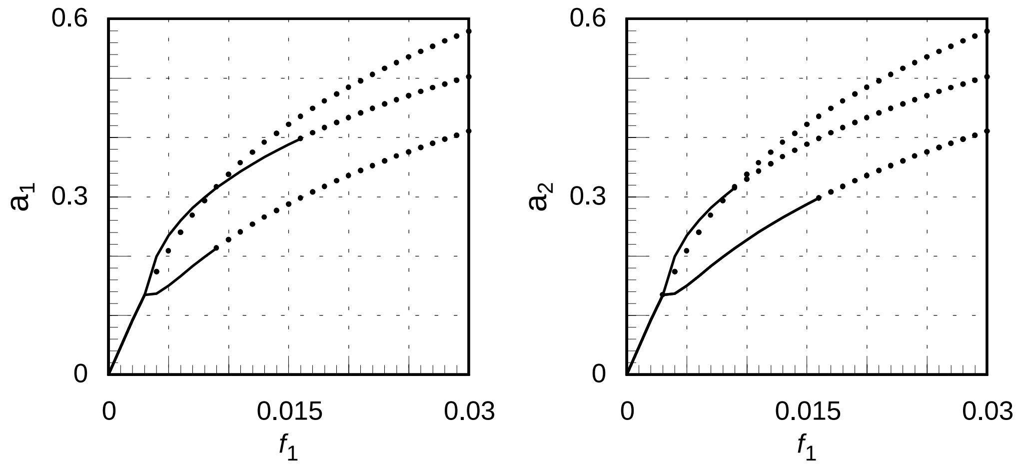
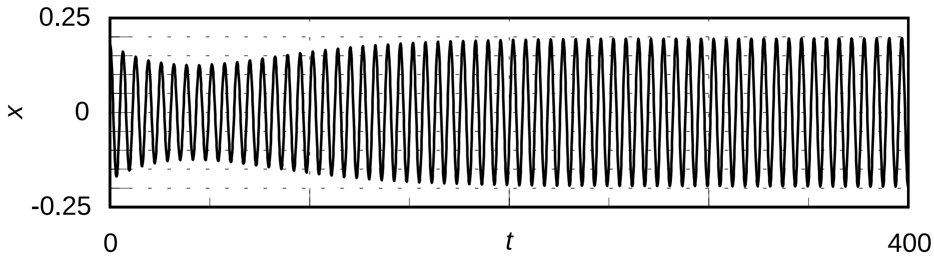


FIGURE 2: Forced response curve for positive external detuning, *p*=1.22, *d*=0.005, *c*=0.001.



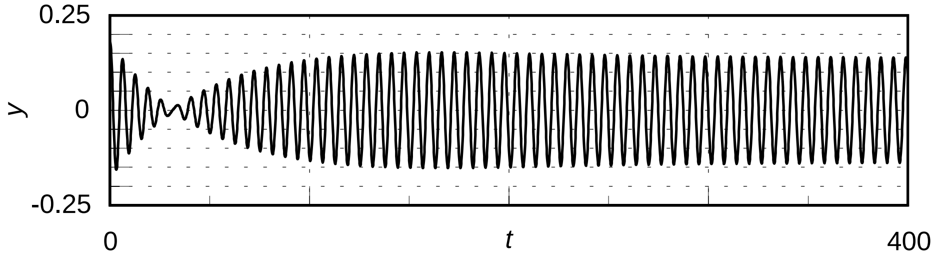
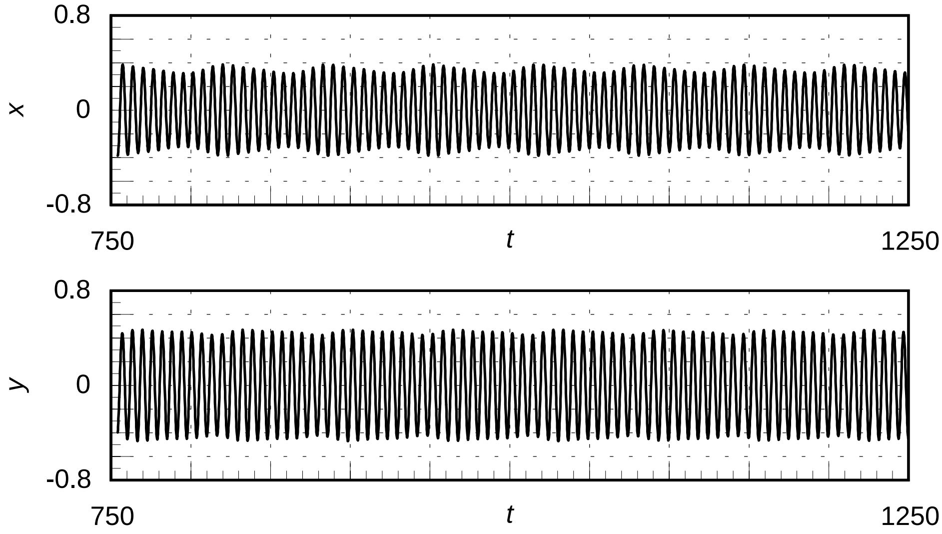
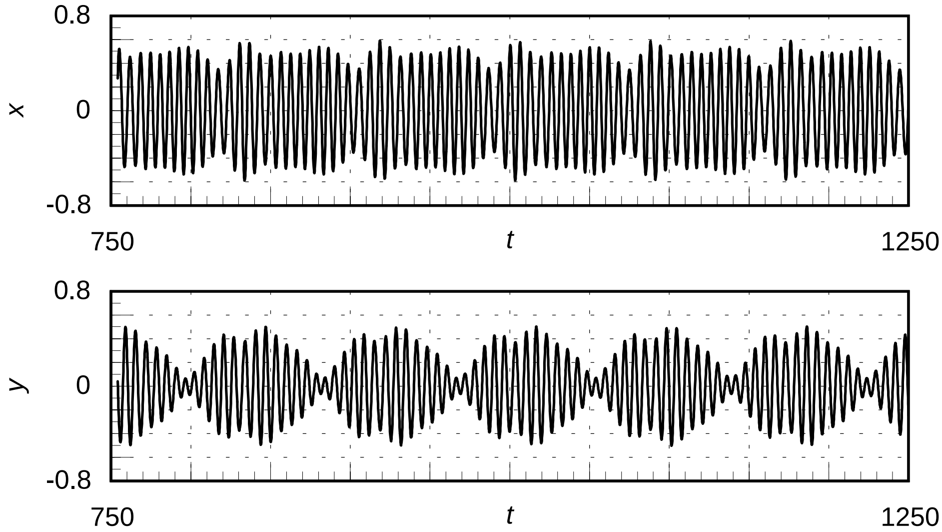


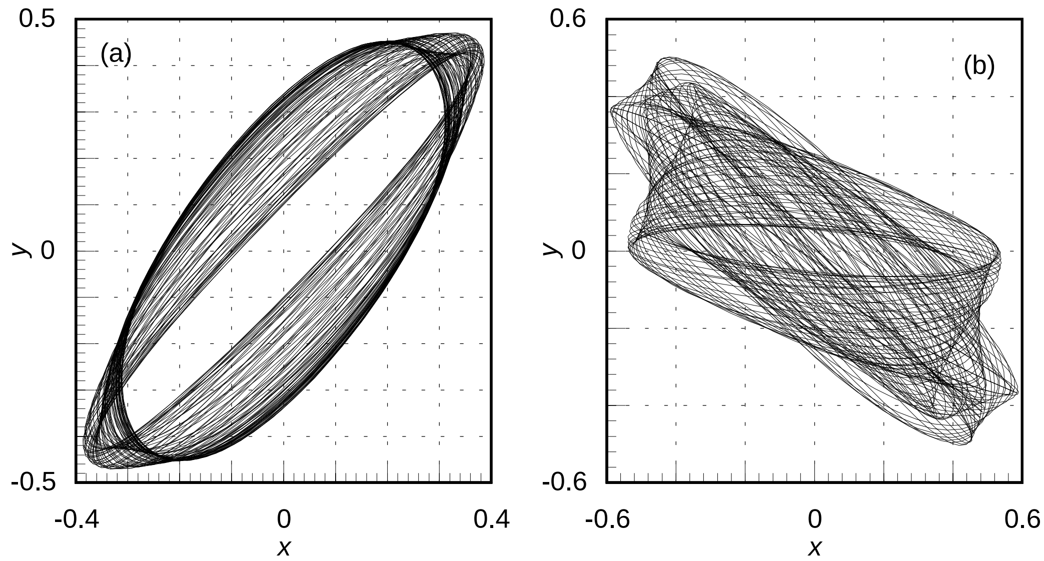
FIGURE 3: Numerical integration: transition from an unstable steady state motion to a stable one. Under p=1.22, d=0.005, f_1=0.004. With initial conditions x(0)=0.18, y(0)=0.18 and zero initial velocities.



(a) Amplitude modulated motions corresponding to the lower branch of Figure 2.



(b) Amplitude modulated motions corresponding to the upper branch of Figure 2.



(c) The orbit of the rotor.

FIGURE 4: Amplitude modulated motions under *p*=1.22, *d*=0.005, f_1=0.021.

4. NUMERICAL RESULTS AND DISCUSSION

The modal amplitudes a_1 and a_2 of the periodic solutions as functions of the forcing amplitude f_1 are shown in Figure 2. To obtain the numerical results, the values for the system parameters are chosen as follows: $\Omega = 1$, $\alpha = 0.3926991$, $c_1 = 0.001$, $p = 1.22$, $d = 0.005$. It can be seen that the response curves of the two modal amplitudes are similar and topologically equivalent. There are three types of solution; namely, $a_1 = a_2$, $a_1 > a_2$, and $a_1 < a_2$. When the forcing amplitude is small, the system admits a stable solution $a_1 = a_2$. This stable solution loses its stability via saddle-node (SN) bifurcation at $f_1 = 0.003$, and a jump from this unstable steady motion to a stable one occurs. In Figure 3, a jump from an unstable steady motion to a stable one is presented. The numerical simulation is with parameters that are the same as those of Figure 2, $f_1 = 0.004$ and initial conditions *x*(0)=0.18, *y*(0)=0.18 and zero initial velocities (corresponding to an unstable solution). It can be seen that after some initial transients the motion settles down to the theoretically predicted stable steady state response, which corresponds to the upper stable branch of Figure 2. As $f_1 > 0.003$, a total of three solutions exist, but only two stable branches occur. The upper stable branch of a_1 corresponds to the lower branch of a_2, and vise versa. As the amplitude of excitation increases, these two stable branches lose their stability by Hopf bifurcation (HB) occurring at $f_1 = 0.009$ and $f_1 = 0.016$ respectively, and amplitude-modulated motions are generated. The HB is expected to lead to amplitude modulation of the steady state responses. Figure 4 shows the amplitude-modulated motions for $f_1 = 0.021$. It can be noted that their responses are different. This indicates that the response is dependent on the initial conditions.

5. CONCLUSIONS

It was shown that the non-linear properties of AMBs can lead to phenomena that are not described by a linear model, indicating the importance of taking non-linearities into account. A variety of interesting phenomena include bifurcation, jump, sensitivity to initial conditions, coexistence of multiple solutions, and amplitude-modulated motion. The results obtained by the perturbation method and numerical integration are in good agreement. The results presented are expected to be useful in the design of a controller to reduce the vibration amplitude of rotor-AMB systems.

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Appendix A

Expressions for the coefficients of equation (6)

$$p = \frac{k_p c_0}{I_0},$$

$$d = \frac{k_d c_0}{I_0 B},$$

$$B^2 = \frac{4mc_0^3}{\mu_0 N^2 A I_0^2},$$

$$c_1 = \frac{cc_0}{B},$$

$$2\mu = 8d\cos\alpha + c_1,$$

$$\omega^2 = 8(p\cos\alpha - 1),$$

$$\alpha_1 = 16(\cos^4\alpha + \sin^4\alpha) - 24p\cos^3\alpha + 8p^2\cos^2\alpha,$$

$$\alpha_2 = 96\cos^2\alpha\sin^2\alpha - 24p\cos\alpha\sin^2\alpha + 8p^2\sin^2\alpha - 48p\sin^2\alpha\cos\alpha,$$

$$\alpha_3 = -24d\cos^3\alpha + 16pd\cos^2\alpha,$$

$$\alpha_4 = -24d\cos\alpha\sin^2\alpha,$$

$$\alpha_5 = 8d^2\sin^2\alpha,$$

$$\alpha_6 = 8d^2\cos^2\alpha,$$

$$\alpha_7 = 16pd\sin^2\alpha - 48d\sin^2\alpha\cos\alpha.$$

Appendix B

Expressions for the coefficients of equation (11)

$$b_1 = \frac{1}{8\Omega}(\alpha_2 - \Omega^2\alpha_5),$$

$$b_2 = \frac{1}{8}\alpha_3,$$

$$b_3 = \frac{1}{8}(\alpha_7 - \alpha_4),$$

$$b_4 = \frac{2}{8}\alpha_4,$$

$$b_5 = \frac{1}{8\Omega}(3\alpha_1 + \Omega^2\alpha_6),$$

$$b_6 = \frac{1}{8\Omega}(2\alpha_2 + 2\Omega^2\alpha_5),$$

$$\sigma_1 = \frac{\sigma}{2\Omega},$$

$$f_1 = \frac{f}{\Omega}.$$