Seventh International Symp. on Magnetic Bearings, August 23-25 , 2000, ETH Zurich

A FREQUENCY-DOMAIN MODEL OF ELECTROMAGNETS COMPOSED OF SOLID IRON CORES

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ABSTRACT

A dynamic model in the frequency domain is considered for electromagnets composed of solid iron cores with a U-shaped stator and an I-shaped moving core. Fringing effect and leakage flux, which were neglected, are considered to improve a model presented before in a simple form with a half-power of frequency. The numerical frequency response of incremental magnetic flux to magnet-coil current is compared with the experimental response. We conclude that the fringing effect is significant, but the effect of leakage flux is small in the improvement of the model.

INTRODUCTION

In the application of solid iron core to electromagnets, eddy current effects degrade the dynamic characteristics in a complicated way. Several approaches have been presented to modeling the eddy current effects. The theoretical models are distributive and described by transcendental functions of Laplace transform variable. Lumped parameter models may be obtained by a truncation of an infinite sum in lower frequencies[1] or by a parameter identification method for a prescribed system in a range of frequency[2]. It seems difficult, in general, to give a simple lumped parameter model.

On the other hand, Feeley[3] presented a simple approximation to a one-dimensional distributive model with a half-power of frequency. This form is equivalent to the description with a half-power of Laplace transform variable, $\sqrt{s}$. In addition, Britcher et al. [4] recommended an alternative form with a term such as $\sqrt{1+\tau s}$, where τ is a time constant. These forms have difficulties in their realization, but there are few problems in the frequency-domain analysis.

In a similar approach, the author[5] presented a simple model similar to Feeley's for two-dimensional iron cores, and checked it with experimental results. The agreement was good in lower frequencies (below about 200Hz) but becomes poorer with increasing frequency.

In the present paper, the author tries improving the model by considering factors neglected in the modeling: fringing effect and leakage flux.

SYMBOLS AND NOTATIONS

A **:** Cross-sectional area of iron core.
A_a **:** Cross-sectional area of working air-gap with fringing effect.
a : Width of iron core.
b **:** Thickness of lamination.
b' : Thickness with two-dimensional core ($b' = b/(1+b/a)$).
c_a, c_b : Fringing coefficients.
I : Magnet-coil current.
i : Increment of I .
j : $=\sqrt{-1}$.
l_a : Air-gap length.
l : Mean length of iron core.
N_1, N_2 : Magnet-coil turns.
R : Magnetic reluctance of iron core.
R_0 : Static magnetic reluctance of iron core.
R_{a1}, R_{a2} : Magnetic reluctances of working air-gaps.
R_k : Magnetic reluctance of iron core.
R_{kL} : Leakage magnetic reluctance.
s : Laplace transform variable.
Φ : Main magnetic flux
ϕ : Increment of main magnetic flux.
φ : Phase lag with static magnetic hysteresis.
μ : Permeability of iron core.
μ_R : Magnitude of complex permeability with iron core.
μ_0 : Permeability of free space ($4\pi \times 10^{-7}$ H/m).
θ : Argument of complex permeability.
σ : Conductivity of iron core.

A MODEL OF ELECTROMAGNETS WITH U-SHAPED STATOR

Figure 1 illustrates a configuration of electromagnets composed of a U-shaped stator and an I-shaped moving core. Two coils are set on the two pole legs and connected in series. The magnet iron cores are made of solid iron; hence eddy currents induced in the iron cores have a large effect on the dynamic characteristics.

The magnetic flux is distributive with leakage along the flux path; hence it may be difficult to give a simple model. For simplicity of analysis, we consider a magnetic circuit model shown in Fig. 2. In the figure, $N_k I$ are the magnetomotive forces, R_{ak} the magnetic reluctances of working air-gap, R_k the reluctances of iron core. The iron core is composed of four parts numbered by 1 and 2 for the pole legs, by 3 for a part of the stator connecting the pole legs and by 4 for the moving core. The notations R_{1L}, R_{2L} and R_{3L} are the magnetic reluctances of leakage flux; the former two are for shortcuts around the pole legs themselves, and the last for a bypath between the pole legs.

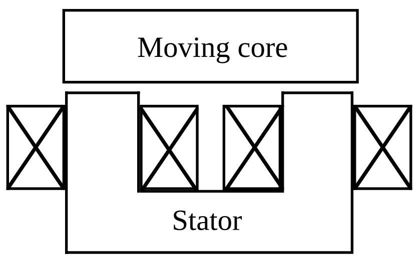

FIGURE 1: Configuration of Electromagnet

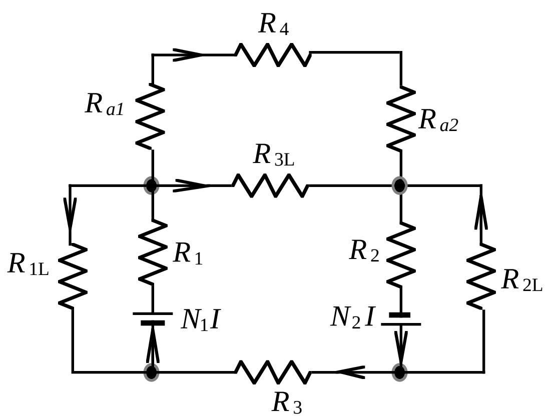

FIGURE 2: Magnetic Circuit Model

From the circuit theory, we obtain the relation

$$\Phi = \frac{1}{1+(1+\gamma_3\delta_{3L})\gamma_c+\gamma_3(1-\gamma_3)\delta_{3L}\gamma_c^2}\frac{F_1+F_2}{R_{a12}} \quad (1)$$

where Φ is the magnetic flux passing through the working air-gaps, and where

$$R_{a12} = R_{a1} + R_{a2},$$

$$R_C' = \frac{R_1}{1+\delta_{1L}} + \frac{R_2}{1+\delta_{2L}} + R_3 + R_4 \quad (2)$$

$$F_1 = \frac{N_1 I}{1+\delta_{1L}}, \quad F_2 = \frac{N_2 I}{1+\delta_{2L}},$$

$$\gamma_3 = \frac{R_3}{R_C'}, \quad \gamma_C = \frac{R_C'}{R_{a12}},$$

$$\delta_{3L} = \frac{R_{a12}}{R_{3L}}, \quad \delta_{1L} = \frac{R_1}{R_{1L}}, \quad \delta_{2L} = \frac{R_2}{R_{2L}} \quad (3)$$

MAGNETIC RELUCTANCE AND HYSTERESIS

We give the magnetic reluctance of working air-gap as

$$R_a = \frac{l_a}{\mu_0 A_a}, \quad A_a = (a+c_a l_a)(b+c_b l_a) \quad (4)$$

where A_a is the effective area of the working air-gap with the fringing effect; c_a and c_b are coefficients.
An expression for the magnetic reluctance of iron core was presented in [5] with the Laplace transforms by a simple form as

$$R(s) = R_0 r(s) \quad (5)$$

where

$$R_0 = \frac{l}{\mu A}, \quad r(s) = 1+\alpha\frac{b'}{2},$$

$$b' = \frac{b}{1+b/a}, \quad \alpha = \sqrt{\mu\sigma s} \quad (6)$$

where b is the thickness of the iron core, a the width, A the cross-sectional area, l the mean length, μ the permeability and σ the conductivity.
For the leakage magnetic reluctance between the exciting pole legs, by referring to [6], we give the equation

$$R_{3L} = \frac{\mu_0 h}{2}\left[\frac{b}{L}+0.264+0.732\log_{10}\left(1+\frac{2a}{L}\right)\right] \quad (7)$$

where h is the height of the pole legs and L is the distance between them. For the leakage flux around the exciting pole legs themselves, we guess its value from an experimental result.
About the permeability, Aspden[7] proposed an conception of complex permeability to model magnetic hysteresis characteristics with

$$\mu = \mu_R \; ^{-j\ \theta} \quad (8)$$

TRANSFER FUNCTION OF INCREMENTAL FLUX TO COIL CURRENT

We consider the electromagnet system whose dimensions are symmetric and the iron core is uniform along the flux path. For simplicity, we make an approximation in eq. (2) as

$$R_C' \cong R_C = R_1 + R_2 + R_3 + R_4 \qquad (9)$$

This approximation gives a larger reluctance for the iron core; hence, it should lead to a smaller magnetic flux. When the working air-gap is fixed, from eq. (1) with the magnetic reluctances above we obtain the following transfer function between the increments of magnetic flux and coil current.

$$\frac{\phi(s)}{i(s)} \cong \frac{k_1 e^{-j\varphi}}{1+\bar{r}_{01}\sqrt{\tau s}+\bar{r}_{3L}\tau s}\cdot\frac{1}{1+\bar{r}_{1L}\sqrt{\tau s}} \qquad (10)$$

where

$$k_1 = \frac{N_1}{(1+\bar{r}_0)R_{a1}}, \quad \bar{r}_0 = \frac{R_{C0}}{2R_{a1}}, \quad R_{C0} = \sum_{k=1}^{4} R_{k0}$$

$$\bar{r}_{01} = \frac{\bar{r}_0}{1+\bar{r}_0}(1+\gamma_3\delta_{3L} \quad , \quad \tau = \mu\sigma\left(\frac{b'}{2}\right)^2$$

$$\bar{r}_{3L} = \frac{\bar{r}_0^{\,2}}{1+\bar{r}_0}\gamma_3(1-\gamma_3)\delta_{3L}, \quad \bar{r}_{1L} = \frac{l_1}{\sum_{k=1}^{4} l_k}\bar{r}_0\frac{R_{a12}}{R_{1L}} \quad (11)$$

The phase lag φ in eq. (10) is estimated from the experimental loop of the static magnetic hysteresis[5].

EXPERIMENTAL RESULTS

Experimental Setup

Figure 3 shows the primary dimensions of the iron core. The moving core sticks out by 2.5mm from the pole legs on each side. The material of the iron cores is a soft iron. The working air-gap was set to 1mm. The bias magnet-coil current was given by 1.5A, generating a magnetic flux density of about 0.37T nearly equal to the theoretical value. The primary specifications and data of the electromagnet are summarized in Table 1.

Static Magnetic Hysteresis

Magnetic flux density was measured in the working air-gap with a gaussmeter in several cycles when the coil current was changed in a very low frequency with several amplitudes around the bias current The hysteresis loop was so small that we may neglect the static phase lag φ, $\varphi \cong 0$, for the present material. It seems to be not so easy to estimate the permeability of the iron core; hence we assume this value in the numerical analysis later.

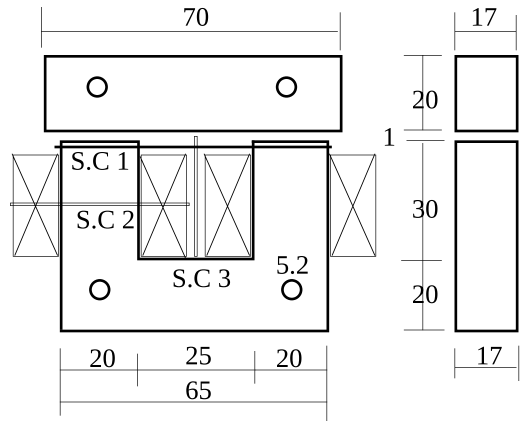

FIGURE 3: Experimental Setup

TABLE 1: Specifications and data of experimental setup

Thickness of iron core	b=17 ~10^{-3}	m
Width of iron core	a=20 ~10^{-3}	m
Air-gap length	l_a=1.0 ~10^{-3}	m
Permeability of iron core	μ_R=5 000 μ_0	
Conductivity of iron core	σ=1.0 ~10^{7} 1/ ¶¥m	
Magnet coil: Turns	$N_1 = N_2$=200	
Mean length of iron core:		
Pole legs	$l_1 = l_2$=30 ~10^{-3}	m
Other parts	$l_3 = l_4$=65 ~10^{-3}	m
Phase of permeability	$\theta = \pi/8$	rad.
Static phase shift	φ=0	rad.

Frequency Response of Main Incremental Flux

A biasing sinusoidal voltage was input to the power amplifier driving the magnet coils. First, the frequency response of the induced voltage between the ends of a search coil was measured to the coil current. Then, the response of the incremental magnetic flux was obtained with the numerical operation of dividing the frequency response by pure complex numbers $j\omega$. The response of the main flux was detected with a search coil wound around two pole legs near the pole faces, one turn in each, denoted by S.C 1 in Fig. 3. The amplitude of the input voltage was given by values of about 25, 50 and 75% of the bias voltage.

The responses of the flux are shown in Fig. 4, depending on the input amplitudes. The dependency is small in low frequencies as had been expected from the static hysteresis loop, but significant in frequencies over about 20Hz.

Evaluation of Leakage Flux

A search coil of two turns around the magnet coil, denoted by S.C 2 in Fig. 3, was used to estimate the leakage flux of shortcut around the magnet coil. The frequency characteristics were similar to that of the foregoing response, and so omitted here. In the case of amplitude 0.4V, the gain was about 20% larger than that of the main flux at low frequencies and about 30% at around 1kHz. The gain decay of the main flux due to this leakage is estimated to be about 1dB at around 1kHz. This decay is much smaller than the difference between in the experiment and in the numerical analysis later shown.

Leakage flux between the pole legs was measured with a search coil in a rectangular shape of 30-by-50mm, located between the two magnet coils, S.C 3 in Fig. 3. The gain characteristic was close to that of the main flux and estimated to be 7~8% of it in the case of amplitude 0.4V. This measurement may suggest that this leakage has a negligible effect on the frequency characteristics of the main flux.

Input-dependency similar to the main flux was observed for the responses. The results above may give a conclusion that the leakages have small effects on the dynamic characteristics of the electromagnet.

NUMERICAL RESULTS

In the static gain of the frequency response, the leakage flux around the magnet coil is about 20% larger than the main flux, as shown above. From this we may estimate this leakage reluctance to be about five times that of the working air-gap of 1mm length, i.e, $R_{1L} = R_{2L} \cong 5R_{a1}$. The experimental static gain is about 1.7dB larger than the numerical value based on eq. (1) without fringing effect. From this results we may estimate the static fringing effect. The gain shift of 1.7dB is obtained with $c_a = c_b$=2 in eq. (4).

Effects of Permeability

The numerical analysis confirms that a smaller permeability brings a larger decay for the response. In fact, with decreasing permeability, the gain decay increases but the phase lag has a limit in high frequencies. Also, complex permeability has significant effects on the numerical result. With increasing argument the gain decays less, and the phase lag becomes larger. Assuming an appropriate value for the argument, we can close the numerical phase lag to the experimental one; however, the gain becomes farther.

From the several results compared with the experimental response of amplitude 0.4V, the author selected a value of $\theta = \pi / 8$ for the argument with the permeability of μ_R=5 000.

Numerical Frequency Responses

Figure 4 shows the frequency responses of the main flux to the coil current based on eq. (1) with the data obtained above. To check the effects of fringing and leakage, the responses are given in case of (1) without fringing effect nor leakage, denoted by "Non-fringing," (2) with fringing effect but without leakage, by "Fringing effect", and (3) with both fringing effect and leakage, by "Leakage." The gain in case of (1) is shifted 1.7dB up for comparison.

We see that the fringing effect is significant in the gain but small in the phase lag, and that the leakage effect is small below a frequency of lkHz. The latter fact agrees with the experimental result of the leakage.

The response of case (3) is compared with the experimental results in Fig. 3. Thus, we improve the model, but the improvement is still unsatisfactory in higher frequencies.

CONCLUSIONS

We tried improving a dynamic model of electromagnets composed of solid iron, presented before in a simple form in the frequency-domain, by considering the fringing effect and leakage fluxes. We saw that the leakage effect is small but the fringing effect is significant in the gain characteristics.

The improvement may be still unsatisfactory in higher frequencies. This is probably due to complicated phenomena in the magnetic system. Qualitatively we can understand the discrepancy in higher frequencies as follows. The flux takes a shortcut in the iron cores and its distribution is non-uniform with saturation in some parts of the iron cores. Such a situation becomes more as the magnetic reluctance of the iron core increases with frequency. Then, the equivalent permeability would become lower, which leads to the decay of the characteristics.

The induced voltage in search coil 1 was monitored with the coil current. The distortion of the signal wave form was seen to be small; hence, we may neglect non-linearity effects.

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