WIDE AREA STABILIZATION OF A MAGNETIC BEARING USING EXACT LINEARIZATION

Toru Namerikawa,[1] Masayuki Fujita,[1] Fumio Matsumura[2]

ABSTRACT

This paper deals with wide area stabilization of a magnetic bearing system via exact linearization method[Ishidori, 1989]. Feedback control is indispensable for a magnetic bearing, because it is essentially an unstable system. To design a feedback control system, a linear mathematical model of the plant is convenient, however, the force of electromagnet is highly nonlinear. Then usually a local linear approximation around the steady operating point is employed, but the obtained linearized model can not express the exact behavior of the system at any other operating points. In this paper, we apply exact linearization approach to a magnetic bearing to achieve wide area stabilization.

At first, we derive a nonlinear mathematical model of a magnetic bearing system, then we show this system belongs to a class of exactly linearizable nonlinear systems. Using exact linearization method, we transfer the nonlinear model of a magnetic bearing to a linear time invariant state space model in spite of a change of the operating point and the rotational speed of the rotor. Then we construct the state feedback control system by conventional LQ method. Finally, we evaluate the validity of our proposed method especially concerned with the change of the operating point and the rotational speed by several experiments.

INTRODUCTION

Characteristics of electromagnetic force are essentially expressed by nonlinear differential equations[Schweitzer, 1994]. But it is very difficult and complicated to design control systems for the original nonlinear model, a linear approximated model which is obtained by local linearization around the steady operating point is usually employed for design. This linearized model works well only around the origin, hence the available region in the operation space is very small. If the operating point moves from the origin, neglected nonlinearity sometimes makes the plant unstable. This is a serious problem for systems whose operating point move in wide areas, e.g., magnetic bearing spindle, etc. For strong nonlinear systems, control system design based on the original nonlinear model should be efficient against a locally linearized model. By the way, as nonlinear control system design, differential geometric approaches especially exact linearization method have been investigated and arranged for multi input/output

[1]Department of Electrical and Computer Engineering, Kanazawa University, 2-40-20 Kodatsuno, Kanazawa 920-8667, Japan, tel.: +81-76-234-4849, fax: +81-76-234-4870, e-mail: toru@t.kanazawa-u.ac.jp/fujita@t.kanazawa-u.ac.jp.
[2]Ishikawa National College of Technology, Tsubata, Ishikawa 929-0329, Japan, e-mail: pres@ishikawa-nct.ac.jp.

complicated systems[Ishidori, 1989], however there have not been so many application reports of the real systems[Charara, 1996] [Levin, 1996], experimental evaluation using real systems of the exact linearization method is now greatly expected.

In this paper, we apply the exact linearization method to a magnetic bearing system to achieve wide area stabilization. At first, we derive a nonlinear model of the magnetic bearing, then we prove that the model of the magnetic bearing is exactly linearizable. Next we derive an exactly linearized system according to state transformation and input transformation which does not lose any nonlinear information. For the obtained linear model, we design a state feedback control system based on LQ optimal regulator. Finally we perform control experiments which examine stability of the magnetic bearing controlled system against movement of the operating point and the change of the rotational speed of rotor, then we indicate the effectiveness of this proposed method.

EXPERIMENTAL MACHINE

The magnetic bearing system employed in this research is a 4-axis controlled horizontal shaft magnetic bearing with symmetric structure. The axial motion is not controlled actively. The diagram of the experimental machine is shown in Fig. 1. The diameter of the rotor is 96 mm and its span is 660 mm. A three-phase induction motor (1kW, four poles) is located at the center of the rotor. Around the rotor, four pairs of electromagnets are arranged radially. And four pairs of eddy-current type gap sensors are located on outside of the electromagnets. The steady state gap between the rotor and the electromagnet is 550[μm]. Further this system employs a tachometer in order to measure the rotational speed of the rotor. The experimental machine is controlled by a digital control system that consists of a 32-bit floating point Digital Signal Processor (DSP) DSP32C(AT&T), 12-bit A/D converters and 12-bit D/A converters. Using these components, the final discrete-time controllers are computed on the DSP.

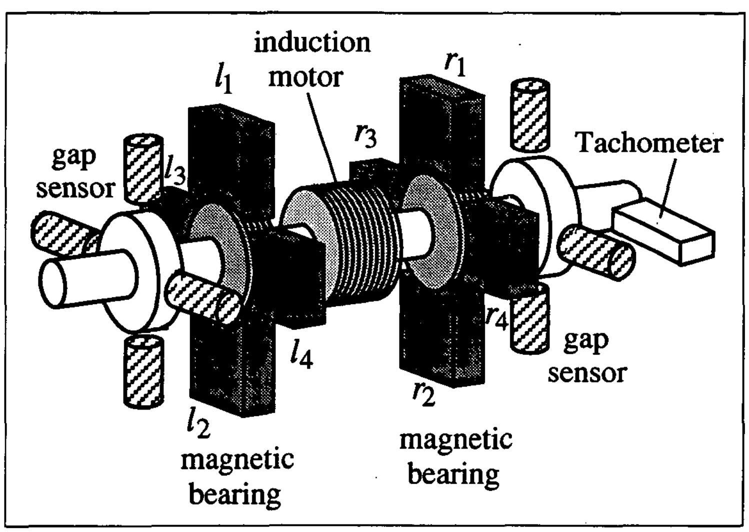

Figure 1: 4-Axis Controlled-type Magnetic Bearing

MODELING

We derive the state equation of a magnetic bearing system with the following assumptions:

(A1) The rotor is rigid and has no unbalance.

(A2) All electromagnets(EM) are identical.

(A3) Attractive force of an EM is in proportion to (electric current / gap length)^2.

(A4) The resistance and inductance of the EM are constant and independent of the gap length.

Using the steady-state current: I_j, gap W: and small deviations from them: i_j and g_j, the equations of electromagnetic force are given by

$$f_j = k\left(\frac{I_j + i_j(t)}{W + g_j(t)}\right)^2, \tag{1}$$

where, k is a constant and $j(= l1, l2, l3, l4, r1, r2, r3, r4)$ indicate the number of electromagnets.
The rotor dynamics is expressed by the following equations [Matsumura, 1990],

$$\begin{aligned} \frac{d}{dt}g_f &= \dot{g}_f && (2)\\ \frac{d}{dt}\dot{g}_f &= A_{03}A_{01}P_{00}A_{03}^{-1}\dot{g}_f + A_{03}A_{02}f_t && (3)\\ \frac{d}{dt}i &= -\frac{R}{L}I_4 i + \frac{1}{L}I_4 e, && (4)\end{aligned}$$

where

$$g_f = [\,g_{l1}\ g_{r1}\ g_{l3}\ g_{r3}\,]^T,\quad i = [\,i_{l1}\ i_{r1}\ i_{l3}\ i_{r3}\,]^T,\quad e = [\,e_{l1}\ e_{r1}\ e_{l3}\ e_{r3}\,]^T$$

$$P_{00} = \begin{bmatrix} 0 & 0 & p & 0\\ 0 & 0 & 0 & -p\\ -p & 0 & 0 & 0\\ 0 & p & 0 & 0 \end{bmatrix},\ A_{01} = \begin{bmatrix} 0 & 0 & 0 & 0\\ 0 & \frac{J_x}{J_y} & 0 & 0\\ 0 & 0 & 0 & 0\\ 0 & 0 & 0 & \frac{J_x}{J_y} \end{bmatrix},\ A_{02} = \begin{bmatrix} -\frac{1}{m} & -\frac{1}{m} & 0 & 0\\ \frac{l_l}{J_y} & -\frac{l_r}{J_y} & 0 & 0\\ 0 & 0 & \frac{1}{m} & \frac{1}{m}\\ 0 & 0 & \frac{l_l}{J_y} & -\frac{l_r}{J_y} \end{bmatrix}$$

$$A_{03} = \begin{bmatrix} 1 & -l_l & 0 & 0\\ 1 & l_r & 0 & 0\\ 0 & 0 & -1 & -l_l\\ 0 & 0 & -1 & l_r \end{bmatrix},\ f_t = \begin{bmatrix} f_{lv} - \frac{l_r}{l_l + l_r}mg\\ f_{rv} - \frac{l_l}{l_l + l_r}mg\\ f_{lh}\\ f_{rh} \end{bmatrix}$$

$$f_{lv} = f_{l1} - f_{l2},\quad f_{rv} = f_{r1} - f_{r2},\quad f_{lh} = f_{l3} - f_{l4},\quad f_{rh} = f_{r3} - f_{r4}$$

g_j : deviations from the steady gap lengths between the electromagnets and the rotor
i_j : deviations from the steady currents of the electromagnets
e_j : deviations from the steady voltages of the electromagnets
p : rotational speed $\qquad$ g : gravity $\qquad$ $(j = l1, r1, l3, r3.)$

From eqs. (2), (3) and (4), the nonlinear state space equation of this magnetic bearing is as.

$$\dot{x} = f(x) + g(x)u, \qquad y = h(x) \tag{5}$$

where

$$x(t) = [\,g_f\ \dot{g}_f\ i\,]^T,\ u(t) = [\,e_{l1}\ e_{r1}\ e_{l3}\ e_{r3}\,]^T,\ g(x) = \left[\,0_4\ 0_4\ \frac{1}{L}I_4\,\right]^T,\ h(x) = [\,g_{l1}\ g_{r1}\ g_{l3}\ g_{r3}\,]^T$$

Theorem 1 Necessary and sufficient condition to solve the exact linearization problem*[Ishidori, 1989]*

Suppose the matrix $g(x^0)$ has rank m (u, $y \in R^m$, x^0 is an initial state). Then, the state space exact linearization problem is solvable if and only if

(1) *for each $(0 \le i \le n-1)$, the distribution G_i has constant dimension near x^0;*

(2) *the distribution G_{n-1} has dimension n;*

(3) *for each $(0 \le i \le n-2)$, the distribution G_i is involutive.*

Here we check the above conditions in the case of $i = 0$, where $m = 4$, $n = 12$, $x^0 = 0_{12\times1}$.

$$G_0 = span\{g_1,\ g_2,\ g_3,\ g_4\} \tag{6}$$

The distribution G_0 has constant dimension four near x^0.

$$\begin{aligned}[g_1 \cdot g_1] &= [g_1 \cdot g_2] = [g_1 \cdot g_3] = [g_1 \cdot g_4] = [g_2 \cdot g_2] = [g_2 \cdot g_3] \\ &= [g_2 \cdot g_4] = [g_3 \cdot g_3] = [g_3 \cdot g_4] = [g_4 \cdot g_4] = 0_{12\times1}\end{aligned} \tag{7}$$

Then the distribution G_0 is involutive. Similar calculations show that the above conditions are satisfied in the case $i = 1\sim11$. This magnetic bearing system satisfies the hypotheses of Theorem 1, then the state space exact linearization problem is solvable for this plant.

PROCEDURE OF THE EXACT LINEARIZATION

Suppose the system is expressed by the following nonlinear state space equation.

$$\dot{x} = f(x) + g(x)u, \quad y_1 = h(x) \tag{8}$$

Derivative of $y_1(t)$ with respect to t is given by

$$\dot{y}_1(t) = \frac{\partial y_1}{\partial x}\frac{dx}{dt} = \frac{\partial y_1}{\partial x}(f(x) + g(x)u) = \frac{\partial y_1}{\partial x}f(x) + \frac{\partial y_1}{\partial x}g(x)u. \tag{9}$$

If $\frac{\partial y_1}{\partial x}g(x) = L_g y_1(x) = 0_4$, is satisfied, then, we define $y_2 = \frac{\partial y_1}{\partial x}f(x)$ and $\dot{y}_1 = y_2 = \frac{\partial y_1}{\partial x}f(x) = L_f y_1(x)$. Similar calculations of $y_2, \cdots, y_{n-m}$ bring the following results.

$$\frac{\partial y_i}{\partial x}g(x) = L_g L_f^{i-1} y_1(x) = 0_4, \quad \dot{y}_i = y_{i+1} = \frac{\partial y_i}{\partial x}f(x) = L_f^i y_1(x) \quad (i = 1, \cdots, n-m) \tag{10}$$

Next, a number of m of differentials are shown by the new inputs $v_1, \cdots, v_m$ as,

$$\begin{cases} \dot{y}_i = y_{i+1} + v_{i+m-n} = L_f^i y_1(x) + (L_g L_f^{i-1} y_1(x))u \quad (i = n-m+1, \cdots, n-1) \\ \dot{y}_n = v_m = L_f^n y_1(x) + (L_g L_f^{n-1} y_1(x))u \end{cases} \tag{11}$$

Then, we have obtained the following linearized state space equation.

$$\dot{y} = \begin{bmatrix} 0_{(n-1)\times1} & I_{n-1} \\ 0 & 0_{1\times(n-1)} \end{bmatrix} y + \begin{bmatrix} 0_{(n-m)\times m} \\ I_m \end{bmatrix} v, \tag{12}$$

$$y = \begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_n \end{bmatrix} = \begin{bmatrix} L_f^0\phi(x) \\ L_f^1\phi(x) \\ \vdots \\ L_f^{n-1}\phi(x) \end{bmatrix}, \quad v = \begin{bmatrix} v_1 \\ \vdots \\ v_{m-1} \\ v_m \end{bmatrix} = \begin{bmatrix} 0 \\ \vdots \\ 0 \\ L_f^n y_1(x) \end{bmatrix} + \begin{bmatrix} L_g L_f^{n-m} y_1(x) \\ \cdots \\ L_g L_f^{n-2} y_1(x) \\ L_g L_f^{n-1} y_1(x) \end{bmatrix} u$$

LINEARIZED STATE-SPACE FORM

We apply the procedure of the exact linearization to the real magnetic bearing system (5), then we have obtained the following linear state space formula. Further state transform functions and input transform functions are given by equations (14) and (15), respectively. The block diagram of the exactly linearized system is shown in Fig. 2.

Linearized State-Space Form

$$\frac{d}{dt}\begin{bmatrix}\xi_1\\ \xi_2\\ \xi_3\end{bmatrix} = \begin{bmatrix}0_4 & I_4 & 0_4\\ 0_4 & 0_4 & I_4\\ 0_4 & 0_4 & 0_4\end{bmatrix}\begin{bmatrix}\xi_1\\ \xi_2\\ \xi_3\end{bmatrix} + \begin{bmatrix}0_4\\ 0_4\\ I_4\end{bmatrix} v, \qquad y = [\, I_4 \quad 0_4 \quad 0_4 \,]\begin{bmatrix}\xi_1\\ \xi_2\\ \xi_3\end{bmatrix} \tag{13}$$

State Transformation

$$\xi_1 = \begin{bmatrix} g_{l1}\\ g_{r1}\\ g_{l3}\\ g_{r3}\end{bmatrix}, \quad \xi_2 = \begin{bmatrix} \dot g_{l1}\\ \dot g_{r1}\\ \dot g_{l3}\\ \dot g_{r3}\end{bmatrix}, \quad \xi_3 = \begin{bmatrix} -a\dot g_{lr3} + (-\frac{1}{m} - \frac{l_l^2}{J_y}) f_{lv} + (-\frac{1}{m} + \frac{l_l l_r}{J_y}) f_{rv} + g \\ b\dot g_{lr3} + (-\frac{1}{m} + \frac{l_l l_r}{J_y}) f_{lv} + (-\frac{1}{m} - \frac{l_r^2}{J_y}) f_{rv} + g \\ a\dot g_{lr1} + (-\frac{1}{m} - \frac{l_l^2}{J_y}) f_{lh} + (-\frac{1}{m} + \frac{l_l l_r}{J_y}) f_{rh} \\ -b\dot g_{lr1} + (-\frac{1}{m} + \frac{l_l l_r}{J_y}) f_{lh} + (-\frac{1}{m} - \frac{l_r^2}{J_y}) f_{rh} \end{bmatrix} \tag{14}$$

Input Transformation

$$v = \dot\xi_3 = \frac{\partial \xi_3}{\partial x} f(x) + \frac{\partial \xi_3}{\partial x} g(x) u \tag{15}$$

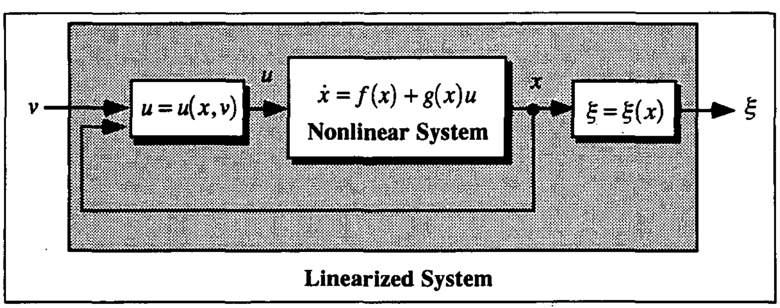

Figure 2: Structure of Exactly Linearized System

CONTROL SYSTEM DESIGN BY LQ OPTIMAL REGULATOR

In order to linearize the nonlinear system exactly, the state variable $x(t)$ have to be measured, which is used to calculate state and input transformation. Hence we employ the LQ optimal state feedback to stabilize the linearized system, not by output feedback.

First we consider the following linear system which corresponds to equations (13).

$$\dot\xi = A_g \xi + B_g v, \qquad y = C_g \xi \tag{16}$$

We introduce a new state variable as $\dot z := y - r$ (r: reference signal), and now we assume $r = 0$, then the augmented plant with z is as follows. We design a LQ optimal regulator for this augmented plant.

$$\dot x = Ax + Bu, \qquad y = Cx \tag{17}$$

where,

$$x = [\xi, \ z]^T, \quad u = v, \quad A = \begin{bmatrix} A_g & 0_{12\times4} \\ C_g & 0_4 \end{bmatrix}, \ B = \begin{bmatrix} B_g \\ 0_4 \end{bmatrix}, \ C = [C_g \quad 0_4]$$

Next we define the following linear quadratic function (18), where $Q \geq 0$ and $R > 0$ are any symmetrical matrices. From equation (17), it is well known that the optimal control input $u(t)$ is given by equation (19) which minimize the quadratic function (18).

$$J = \int_0^\infty (x^T Qx + u^T Ru)dt, \tag{18}$$

$$u = -R^{-1}B^T Px, \tag{19}$$

where P is a unique positive definite solution of a following Riccati equation

$$PA + A^T P - PBR^{-1}B^T P + Q = 0. \tag{20}$$

After several experimental trial and error, we have decided the weighting matrices as

$$Q = \text{diag}\,[100I_4, \ 0.1I_4, \ 0_4, \ 10000I_4], \qquad R = 1 \times 10^{-12} I_4. \tag{21}$$

EXPERIMENTAL EVALUATION

In order to evaluate this linearization, we carry out several control experiments and compare two linearization methods as follows, where the design methods of the state feedback controllers are same, and sampling rates of digital implementation are 86[μs], 300[μs], respectively.

- **[LL]: Local Linearization around** x^0 + LQ optimal state feedback
- **[EL]: Exact Linearization via feedback** + LQ optimal state feedback

ROTATIONAL EXPERIMENTS

All through the experiments, a small weight(30[g]) is attached at the edge of the rotor side to increase unbalance of the rotor. In Fig. 1, the weight is located at the left edge of the rotor. Control experiments are performed in the condition that the rotor rotates round at 1000[rpm] with the following position reference signals, where a unit is μm.

- **[Exp1]:** $[g_{l1}, \ g_{l3}] = [0, \ 0]$ • **[Exp2]:** $[g_{l1}, \ g_{l3}] = [100, \ 100]$ • **[Exp3]:** $[g_{l1}, \ g_{l3}] = [200, \ 200]$

Three Orbits in 0.8 [s] of the physical center of the rotor, which are results of [Exp1], [Exp2] and [Exp3], are shown in Figs. 3 and 4, where a vertical and a horizontal axises show the deviations from the origin : g_{l1}[mm] and g_{l3}[mm], respectively. The steady state gaps W between electromagnets and rotor are all 550[μm], hence, these movements of the operating point influence the electromagnetic force dynamics. In Fig.3, experimental results with [LL] are shown. And in Fig. 4, results with [EL] are also shown. From these figures, in the case [LL], moving area(solid lines) of the physical center of rotor becomes bigger if the operation point is shifted from the origin to the end of work space. On the other hand in the case [EL], radiuses of the all moving areas are smaller than those of [LL], and deterioration in the control performance is restricted against the change of the operating point. These results can be seen that the nonlinearity is absorbed by state and input transformations, then the exact linearization method have brought the improvement of control performance.

$$f(x) = \begin{bmatrix} \dot{g}_{l1} \\ \dot{g}_{r1} \\ \dot{g}_{l3} \\ \dot{g}_{r3} \\ -a\dot{g}_{lr3} + (-\frac{1}{m} - \frac{l_l^2}{J_y})f_{lv} + (-\frac{1}{m} + \frac{l_l l_r}{J_y})f_{rv} + g \\ b\dot{g}_{lr3} + (-\frac{1}{m} + \frac{l_l l_r}{J_y})f_{lv} + (-\frac{1}{m} - \frac{l_r^2}{J_y})f_{rv} + g \\ a\dot{g}_{lr1} + (-\frac{1}{m} - \frac{l_l^2}{J_y})f_{lh} + (-\frac{1}{m} + \frac{l_l l_r}{J_y})f_{rh} \\ -b\dot{g}_{lr1} + (-\frac{1}{m} + \frac{l_l l_r}{J_y})f_{lh} + (-\frac{1}{m} - \frac{l_r^2}{J_y})f_{rh} \\ -\frac{R}{L}\, i_{l1} \\ -\frac{R}{L}\, i_{r1} \\ -\frac{R}{L}\, i_{l3} \\ -\frac{R}{L}\, i_{r3} \end{bmatrix}$$

$$g_{lr1}(t) = g_{l1} - g_{r1}, \quad g_{lr3}(t) = g_{l3} - g_{r3}, \quad a = \frac{J_x}{J_y}\frac{l_l}{l_l + l_r}p(t), \quad b = \frac{J_x}{J_y}\frac{l_r}{l_l + l_r}p(t)$$

Model parameters of this system are given by Table 1.

Table 1: Parameters of Experimental Machine

Parameter	Symbol	Value	Unit
Mass of the Rotor	m	13.9	kg
Moment of Inertia about X	J_x	0.0135	$kg \cdot m^2$
Moment of Inertia about Y	J_y	0.233	$kg \cdot m^2$
Distance between Center of Mass and Left EM	l_l	0.130	m
Distance between Center of Mass and Right EM	l_r	0.130	m
Distance between Center of Mass and Motor	l_m	0	m
Steady Attractive Force	$F_{\{l1,\ r1\}}$	90.9	N
	$F_{\{l2\sim l4,\ r2\sim r4\}}$	22.0	N
Steady Current	$I_{\{l1,\ r1\}}$	0.630	A
	$I_{\{l2\sim l4,\ r2\sim r4\}}$	0.310	A
Steady Gap	W	5.50×10^{-4}	m
Resistance	R	10.7	Ω
Inductance	L	0.285	H

EXACT LINEARIZATION VIA FEEDBACK

We derive an exactly linearized system of the nonlinear plant(5) according to state transformation and input transformation which does not lose any nonlinear information. Using definitions of Lie derivative, Lie bracket and involutive [Ishidori, 1989], the solvable condition of the state space exact linearization problem is given by the following theorem.

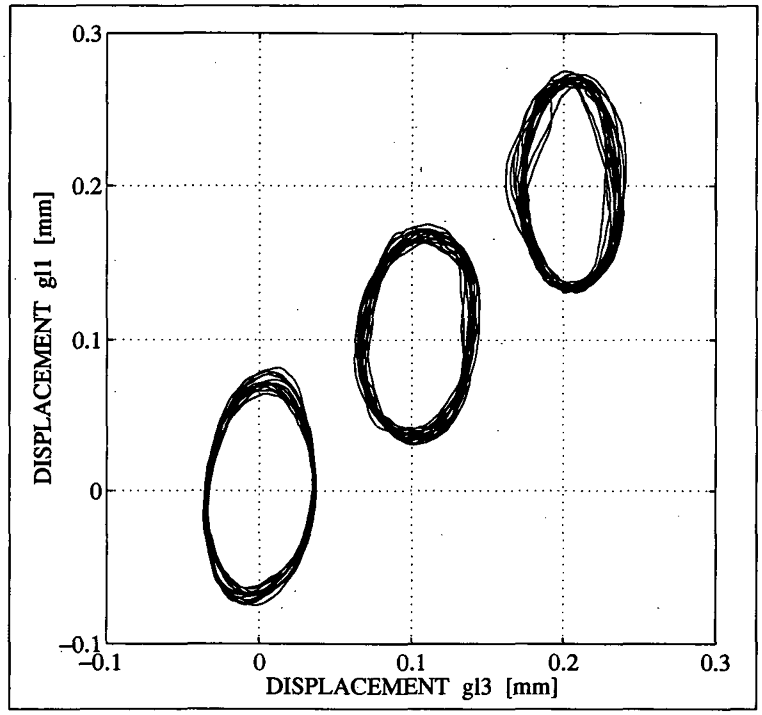

Figure 3: Displacement of Rotor at 1000rpm (Conventional Method)

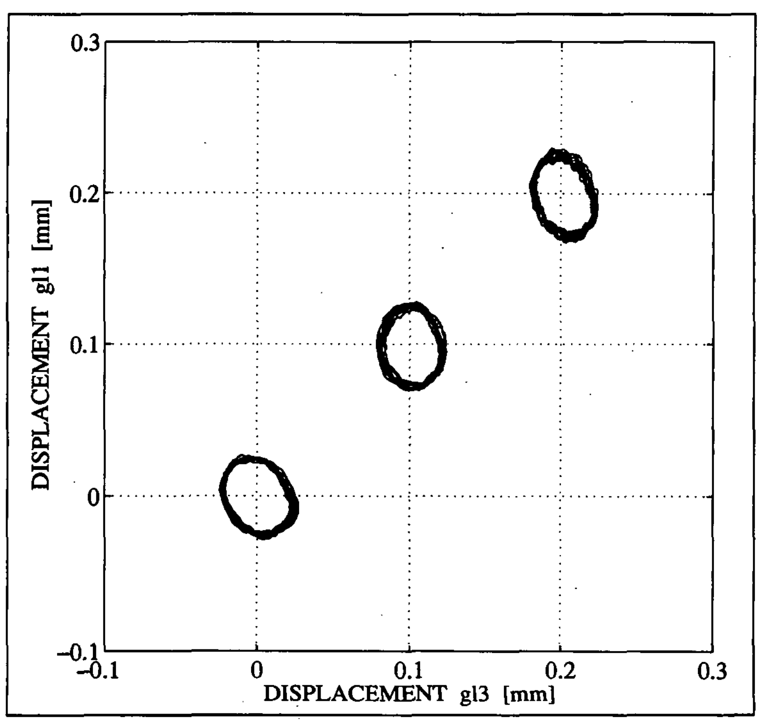

Figure 4: Displacement of Rotor at 1000rpm (Exact Linearization)

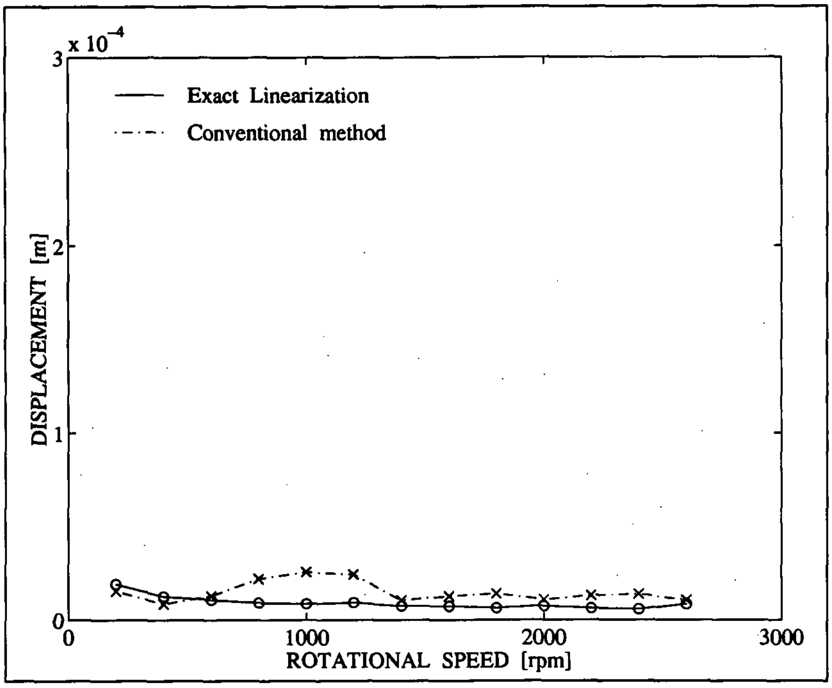

Figure 5: Max. Displacement of Rotor against Sinusoidal Disturbance (without Weight)

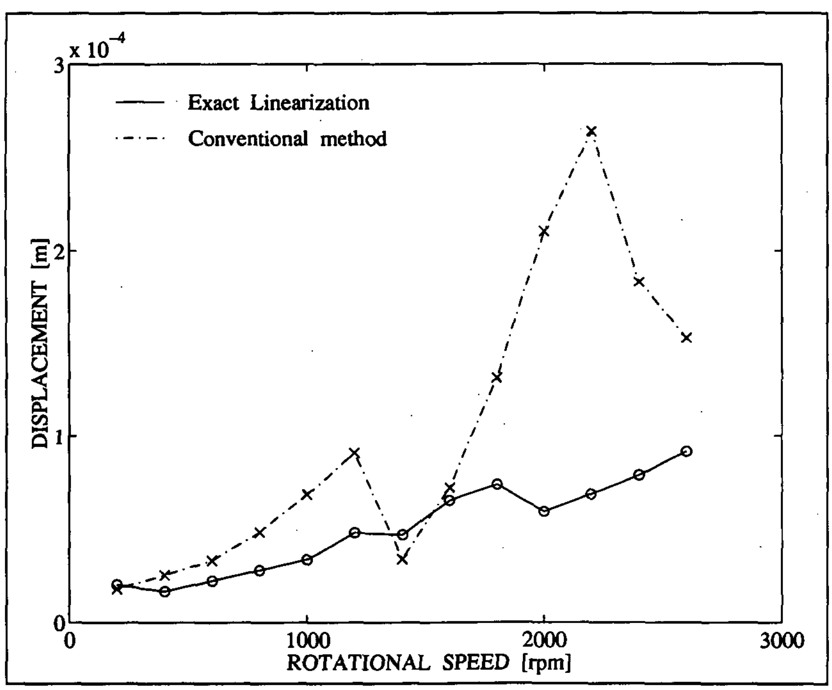

Figure 6: Max. Displacement of Rotor against Sinusoidal Disturbance (with Weight)

ROTATIONAL SPEED AND UNBALANCE VIBRATION

Next we fix the operating point at the origin $[g_{l1}, g_{l3}]=[0, 0][\mu\mathrm{m}]$, and measure the maximum displacement caused by unbalance vibration against the change of the rotational speed $p(t)$ of the rotor. Suppose the following two experiments.

- **[Exp4]: Without Weight** (Fig. 5) • **[Exp5]: With Weight (30[g])** (Fig. 6)

Rotational speed is changed in the range from 200[rpm] to 2600[rpm] by 200[rpm] discretely, this rotating range is decided by the precision of the tachometer. In Figs.5 and 6, maximum radiuses of the unbalance vibration [m] against the change of the rotational speed [rpm]. The vertical axis shows maximum radiuses and the horizontal axis shows rotational speed, and the solid lines show the results with [EL] and dash-dot lines show with [LL]. Fig. 5 shows results without any weights ([Exp4]), and Fig. 6 shows results with unbalance weight(30[g]) ([Exp5]). In Fig. 5, we can see that there are not so remarkable difference between two methods because there are little unbalance in the original rotor. In Fig. 6, however, the difference between [EL] and [LL] becomes clear. Exact linearization method:[EL] preserves the nonlinear informations s.t. electromagnetic force:$f(x(t))$ and rotational speed:$p(t)$, then it has also shown better responses than [LL] concerned with variation of the rotational speed.

CONCLUSION

In this paper, we applied the exact linearization method to a magnetic bearing system to achieve wide area stabilization. First, we derived a nonlinear model of the magnetic bearing system, then we proved that the model of the magnetic bearing was exactly linearizable. Next we derived an exactly linearized system according to state transformation and input transformation which does not lose any nonlinear information. For the obtained linear model, we designed a state feedback control system based on LQ optimal regulator. Finally we performed several control experiments which examine stability and performance of the magnetic bearing controlled system against the change of the operating point and the variation of the rotational speed, then we indicated the effectiveness of this proposed method. This paper does not considered the robustness against changes of the model parameters, hence the next challenging issue is a robust stability/performance problem. Further we only focused on the nonlinearity in the electromagnetic force $f(x(t))$, but there are other unmodeled nonlinearities e.g., inductance $L(x(t))$ and rotor nonlinear dynamics, hysterics, saturation, and so on. We have to consider these nonlinearities and investigate their behavior, and they should be expressed in detail.

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