NONLINEAR REGULATOR DESIGN FOR GRINDING AMB SPINDLE BASED ON OPTIMAL CONTROL*

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ABSTRACT

For AMB system with high stiffness, such as applied in grinding, the system can be treated as linearity, only if the rotor operates among a small range of linearity. At this condition, linear control system designing is satisfaction. However, once the rotor operates out of the linear range, its real performance will gradually become worse. To be aimed at this question, this paper put forward a linear optimal control at first, and then, a nonlinear variable gain control for high stiffness and good stabilization, by using 3-D frequency domain analyzing method. The method especially suits for such a system whose regulator is made of analogue circuit.

Keywords: Nonlinear control; 3-D frequency domain analysis; Active Magnetic Bearings

1. INTRODUCTION

When AMB-electrospindle is applied to machining such as inner circular grinding, the performance are required to be high bearing stiffness, high rotation speed, high rotation accuracy. To realize such high performance, the key point is how to design a controller. Under normal conditions, when the floating rotor is restricted to a small range, the AMB system is regarded as a linear system, i.e. the designing of a linear regulator is satisfactory. However, for a settled mechanism of AMB, higher and higher the feedback gain becomes, narrower and narrower the range of electromagnetic force relative to the displacement of rotor will be. The magnet force will abide by the linear model, only when the operating range of the rotor is working among the small range. Once the rotor operates over the range, its

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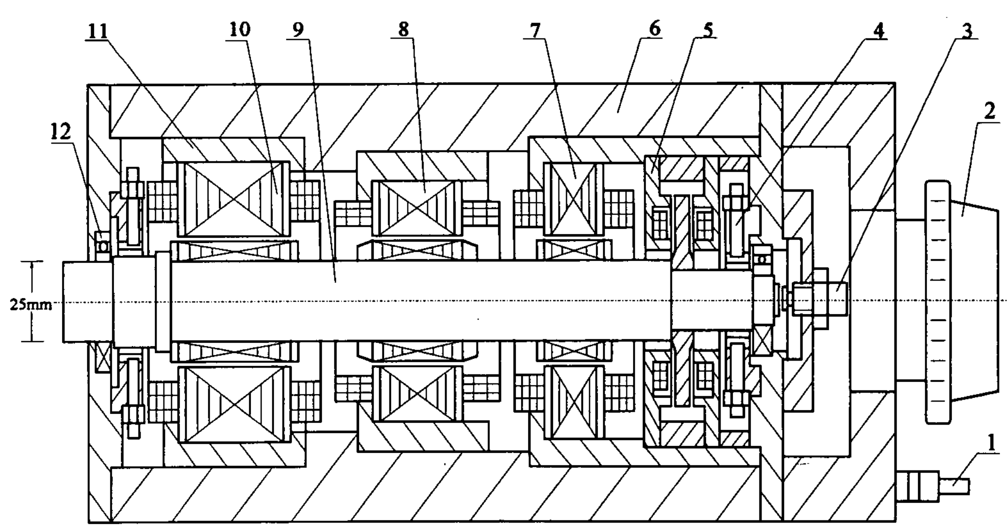
performance will drop down, but it is unavoidable that the rotor may exceed the linear range, for instance, during the starting up of the rotor etc., Theoretical analysis proves that some unstable factors exist in the system at this conditions(Zhao, 1996) . Therefore, such a AMB system that requires high stiffness has linear characteristic in the small range and nonlinear out of that range, which is different from the AMB system without bias current(Lottin, Mouille, Ponsart, 1994; Ponsart, Lottin, Mouille, 1996). This kind of problem is mentioned by Schmied(Schmied 1990), T. Inoue(Inoue et al., 1990) , but the analyses and solution methods are not provided in details.

In this paper, expecting to obtain high stiffness and good stability, a basic analogue regulator based on decentralized optimal control is put forward firstly, whose weighting matrix of cost function are determined according to the principle of minimizing deviation(Schweitzer, Bleuler, Traxler, 1994). In fact, there are many nonlinear relationship in AMB system yet, such as the nonlinearity between coil current and electromagnetic force etc.. Now taking into consideration these factors, the regulator is improved by nonlinear variable gain control method further more, which especially can be adopted to such a system whose regulator is made of analogue circuit. The improved method is presented also, according to the conclusion of three dimensional frequency domain method described in paper(Qi, 1997).

The experimental apparatus is an inner circular grinding test machine equipped with AMBs, the operating speed of the rotor is 50,000rpm, the bearing stiffness in linear range is 34N/μm. Field experimental results demonstrated that the above regulator can meet the demand of stiffness and stability during the grinding process.

2. EXPERIMENTAL APPARATUS DESCRIPTION



1. Chill-down duct; 2. Bus joint; 3. Axial Sensor; 4. Radial Sensor; 5. MB3; 6. Pedestal; 7. MB1; 8. Motor; 9. Rotor; 10. MB2; 11. Barrel; 12. Retained Bearing

Figure 1. AMB electrospindle experimental apparatus

The structure of a AMB-electrospindle, designed operating speed is 50,000rpm, applied in inner circular machining is shown in Fig. 1. The parameters are listed in table 1.

TABLE 1 — MB PARAMETERS

parameter	MB1	MB2	MB3
bias current I_0(A)	4.0	4.0	4.0
ampere-turn NI_0(A)	544	544	504
air gap h_0(mm)	0.3	0.3	0.4
mass (see Equ.(5)) (kg)	m_a=1.242	m_b=1.171	m=2.176

3. OPTIMAL CONTROL OF AMB SYSTEM

3.1 MODELING

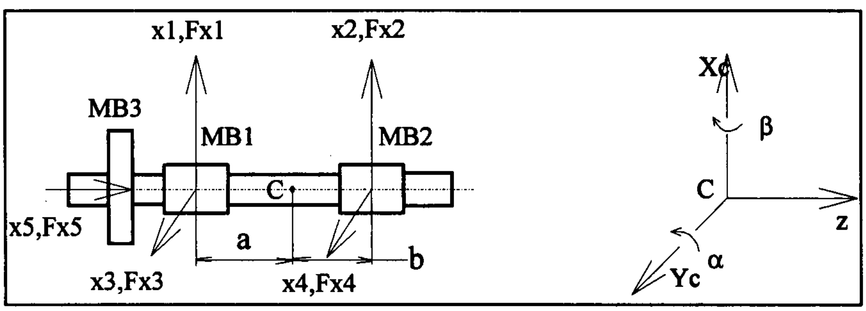


Figure 2. Force exerted distribution of a rotor

The force exerted distribution of a rotor suspended by AMBs are shown in Fig. 2, in which include two radial AMB, MB1, MB2, and a axial AMB, MB3.

According to variables denoted in Fig.2, the equation of motion of the rotor is

$$\begin{cases} m x_c'' = F_{x1} + F_{x2} \\ m y_c'' = F_{x3} + F_{x4} \\ J_d \alpha'' = -aF_{x1} + bF_{x2} - \omega J_p \beta' \\ J_d \beta'' = -aF_{x3} + bF_{x4} + \omega J_p \alpha' \\ m x_5'' = F_{x5} \end{cases} \tag{1}$$

where ( )′ and ( )″ present the first derivative and the second derivative of those variable respectively,

Supposing, $|X_i| \ll h_{oi}$; $|I_{ci}| \ll I_{oi}$, and not taking into consideration of the coupling of sensors location and magnetic circuits etc., let

$Z_c=[X_c, \alpha, Y_c, \beta, X_5]^T$; $Z_B=[X_1, X_2, X_3, X_4, X_5]^T$; $U_c=[I_{c1}, I_{c2}, I_{c3}, I_{c4}, I_{c5}]^T$,

taking account to the electromagnetic forces exerted on the rotor expressions as

$$F_{xi} = k_{ci} * I_{ci} + k_{di} * x_i \tag{2}$$

where x_i and I_i are the displacements and control currents in each degree-of-freedom respectively, k_{ci} and k_{di} are respectively force-current factor and force-displacement factor. i=1,2,3,4,5.

yield:

$$M_1 Z_C'' + G_1 Z_C' + K_1 Z_B + K_{I1} U_C = 0 \tag{3}$$

where M_1, G_1, K_1, K_{I1} are coefficient matrix , $Z_C = T_B Z_B$ (T_B is transition matrix), and then,

$$T_B{}^T M_1 T_B Z_B'' + T_B{}^T G_1 T_B Z_B' + T_B{}^T K_1 Z_B + T_B{}^T K_{I1} U_C = 0$$

changing above equation to:

$$M Z_B'' + G Z_B' + K Z_B + K_I U_C = 0 \tag{4}$$

expanding the Equ. (4) , then yield

$$\begin{bmatrix} m_a & m_0 & 0 & 0 & 0 \\ m_0 & m_b & 0 & 0 & 0 \\ 0 & 0 & m_a & m_0 & 0 \\ 0 & 0 & m_0 & m_b & 0 \\ 0 & 0 & 0 & 0 & m \end{bmatrix} Z_B'' + \begin{bmatrix} 0 & 0 & 1 & -1 & 0 \\ 0 & 0 & -1 & 1 & 0 \\ -1 & 1 & 0 & 0 & 0 \\ 1 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \frac{J_p \omega}{l^2} Z_B' -$$
$$\begin{bmatrix} k_{d1} & & & & \\ & k_{d2} & & & \\ & & k_{d3} & & \\ & & & k_{d4} & \\ & & & & k_{d5} \end{bmatrix} Z_B - \begin{bmatrix} k_{c1} & & & & \\ & k_{c2} & & & \\ & & k_{c3} & & \\ & & & k_{c4} & \\ & & & & k_{c5} \end{bmatrix} U_C = 0 \tag{5}$$

where $m_a=(mb^2+J_d)/l^2$, $m_b=(ma^2+J_d)/l^2$, $m_0=(mab+J_d)/l^2$,

3.2 LINEAR QUADRATIC OPTIMAL CONTROL FOR AMB

For a linear system, the linear quadratic optimal controller will be obtained from the quadratic performance index which determined by taking the integration of quadratic function of state vector and control vector as performance index function. The controller lay the foundation for the linear regulator of PID. Supposing that the system operate around in the small, the state vector and control vector and output vector are respectively defined as

$$X(t)=\left[x_1,x_2,x_3,x_4,x_5,x_1',x_2',x_3',x_4',x_5'\right]^T;$$

$$U(t)=\left[I_{c1},I_{c2},I_{c3},I_{c4},I_{c5}\right]^T;\quad Y(t)=\left[y_1,y_2,y_3,y_4,y_5\right]^T,$$

where y_i (i=1,2,3,4,5) are the displacement signals of rotor at each direction, then, the state equation of the AMB-rotor system is

$$\begin{cases} X(t)' = AX(t)+BU(t) \\ Y(t)=CX(t) \end{cases} \tag{6}$$

where

$$K_{ij}=\frac{1}{m}\begin{bmatrix}(1+a^2/\rho^2)k_{di} & (1-ab/\rho^2)k_{dj} \\ (1-ab/\rho^2)k_{di} & (1+b^2/\rho^2)k_{dj}\end{bmatrix},\qquad G=\frac{\omega J_p}{lJ_d}\begin{bmatrix}a & -a \\ -b & b\end{bmatrix},$$

$$C=\begin{bmatrix}I_5 & 0\end{bmatrix},$$

$$A=\begin{bmatrix}0 & 0 & 0 & I_2 & 0 & 0 \\ 0 & 0 & 0 & 0 & I_2 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ K_{12} & 0 & 0 & 0 & -G & 0 \\ 0 & K_{34} & 0 & G & 0 & 0 \\ 0 & 0 & k_{d5}/m & 0 & 0 & 0\end{bmatrix},\qquad B=\begin{bmatrix}0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ H_{12} & 0 & 0 \\ 0 & H_{34} & 0 \\ 0 & 0 & k_{c5}/m\end{bmatrix},$$

$$H_{ij}=\frac{1}{m}\begin{bmatrix}(1+a^2/\rho^2)k_{ci} & (1-ab/\rho^2)k_{cj} \\ (1-ab/\rho^2)k_{ci} & (1+b^2/\rho^2)k_{cj}\end{bmatrix}$$

let $Z(t)'=Y(t)$, and then the equation (6) can be changed as

$$X_1' = A_1X_1 + B_1U, \tag{7}$$

where , $X_1 = \begin{bmatrix} X \\ Z \end{bmatrix}$, $A_1 = \begin{bmatrix} A & 0 \\ C & 0 \end{bmatrix}$, $B_1 = \begin{bmatrix} B \\ 0 \end{bmatrix}$。

now taking the quadratic performance index function as :

$$J = \int_0^\infty [X_1^T Q X_1 + U^T R U] dt = \int_0^\infty [X^T Q_1 X + Z^T Q_2 Z + U^T R U] dt \tag{8}$$

The square matrices Q and R are the weights of the variables respectively, and $Q=[Q_1, Q_2]$. In the right hand of the equation (8), the first term express the requirement that the controlled variable should attenuate as fast as possible, the second term express the requirement that the area around the response curve should minimize as possible as it can, the third term express the limitation of control energy. The optimum feedback control of satisfying the most minimizes performance index function is

$$U^*(t) = -FX_1(t) = -R^{-1}B_1^T P X_1(t) \tag{9}$$

where F is defined as optimum feedback matrix, and P is the solution of Riccati equation as follows

$$PA_1 + A_1^T P - PB_1R^{-1}B_1^T P + Q = 0 \tag{10}$$

treating the Q from the equation (8) as

$$Q = \begin{bmatrix} Q_P I_5 & & \\ & Q_D I_5 & \\ & & Q_I I_5 \end{bmatrix}$$ and $Q_p=1.5E+9$, $Q_D=1$, $Q_I=1.0E+11$, $R=I_5$

then the optimum feedback matrix F at the speed of 50,000rpm for the AMBs-rotor system can be yield

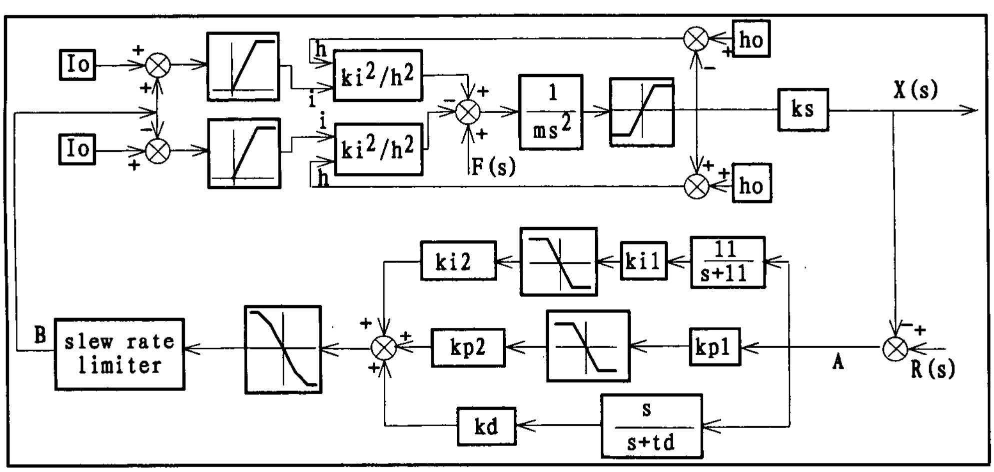
$$F = \begin{bmatrix} 52269.4 & -1292.5 & 926.1 & -918.1 & 0.00 & 24.99 & -1.71 \\ 1311.7 & 52214.5 & -866.5 & 683.6 & 0.00 & -0.36 & 17.33 \\ -926.1 & 918.1 & 52269.4 & -1292.5 & 0.00 & 0.01 & -0.07 \\ 866.5 & -683.6 & 1311.7 & 52214.5 & 0.00 & 0.14 & -0.01 \\ 0.00 & 0.00 & 0.00 & 0.00 & 47238.4 & 0.00 & 0.00 \end{bmatrix}$$

$$\left.\begin{matrix} -0.01 & 0.07 & 0.00 & 315907 & -10000 & 7204 & -7148 & 0.00 \\ -0.14 & 0.01 & 0.00 & 10276 & 315944 & -6746 & 5325 & 0.00 \\ 24.99 & -1.71 & 0.00 & -7204 & 7148 & 315907 & -10000 & 0.00 \\ -0.36 & 17.33 & 0.00 & 6746 & -5325 & 10276 & 315944 & 0.00 \\ 0.00 & 0.00 & 36.19 & 0.00 & 0.00 & 0.00 & 0.00 & 316228 \end{matrix}\right]$$

Dividing the matrix F into three square matrices of five by five, it is evident that all the non-diagonal elements are smaller than the diagonal elements in the three square matrices. Therefore, a decentralized control scheme is available. The simplification has been achieved by dividing the large matrix F into 5 subsystems and by controlling each subsystem locally. For each subsystem, the PID regulator is available.

4. DESIGN OF NONLINEAR REGULATOR

When the designed linear PID controller is tested on the apparatus, the rotor often vibrates seriously or has some difficulty to start up sometimes. For this reason, a nonlinear control method will be intented to settle the contradictory between high stiffness and rotor stability. Figure 3 shows a AMB system model with the main nonlinear factor considered, and the power amplifier are modeled as slew rate limiter.



Io - bias current; ho - air gap; ks - sensor gain; ki1, ki2 - integrator gains; kp1,kp2- preparation gains; kd - differentiator gain; F(s) - disturbance force; m - rotor mass; h - rotor displacement

Figure 3. Nonlinear control system of AMB

For a nonlinear system, the ratio of output vs. input relates not only to the frequency of input, but to the amplitude of input also. How to analysis and design this kind of nonlinear system is a challenge matter. In classical control theory, the Bode plots is very successful for linear system, but is limited for nonlinear system. It will be very helpful to analyze the ratio characteristic for a nonlinear system, if a coordinate axis of amplitude of input is added to Bode plots to construct three dimensional "Bode plots". Therefore, according to describing function and taking advantage of micro computer technique, a 3-D frequency domain analysis method is put forward to determine the nonlinear parameters of the system based on linear system design results. This method is implemented by MATLAB program, and can be applied to high order and complicated systems.

Fig. 4 shows the three dimensional response of the regulator from A to B shown in Fig. 3 The regulator is linear PID without variable gain control but including some inherent

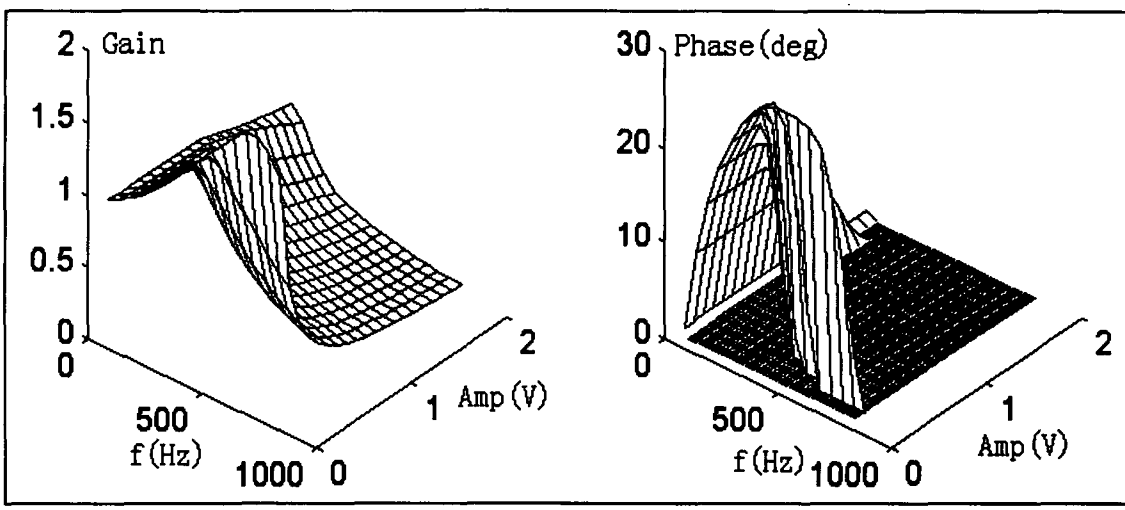


Figure 4. 3-dimsion Bode plot of the controller(from A to B)

nonlinearity, such as voltage saturation.

From this figure, it is obvious that the damping is not enough in large displacement and high frequency due to a large phase lag . Therefore, when rotor drops into a large displacement area, the system will be unstable. Fig. 5 shows the simulation results of rotor displacement in starting up.

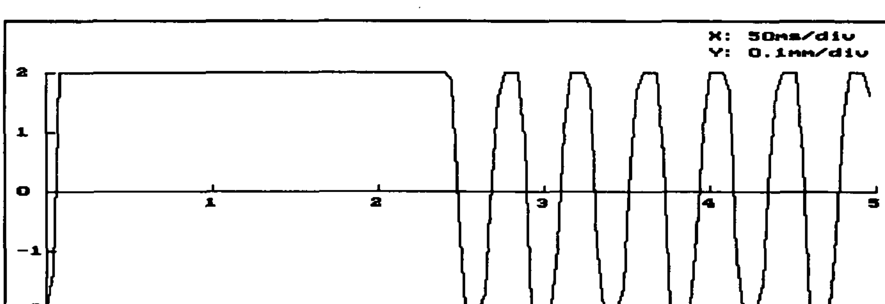


Figure 5. Simulation result of the rotor displacement in startup

In Fig. 6 the simulation results of a variable gain regulator is shown. The gain parameter

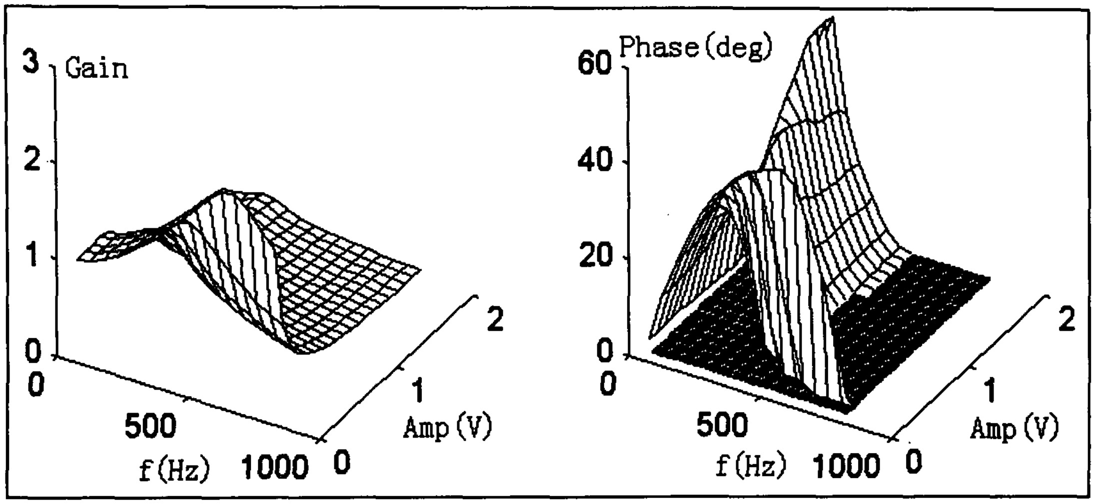


Figure 6. 3-dimsion Bode plot of variable gain regulator (from A to B)

in Fig. 3 is K_{i1}=63, K_{i2}=0.037, K_{p1}=30, K_{p2}=0.032, K_d=11. The result shows that damping is satisfactory in resonance area with high oscillating amplitude. So the rotor will be stable in a large displacement area. Fig. 7 displays the simulation results of rotor start up, and Fig. 8 shows the correspond experimental results, the additional jump is caused by power start rewind, the transient time about 130ms is caused by integration coefficient of regulator.

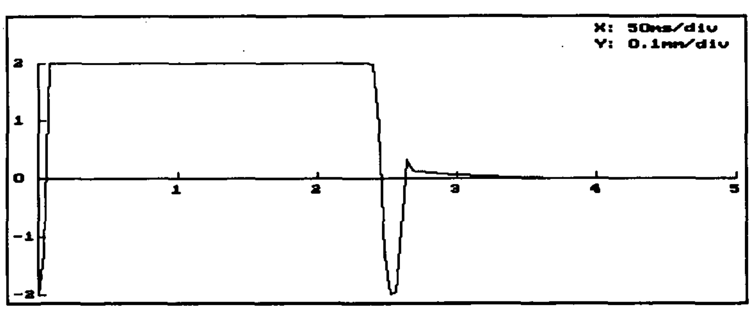


Figure 7. Simulation results of rotor displacement in start up.

5. CONCLUSION

Theoretical analyses and experimental results show that the optimal LQR is valid only for small movement and the variable gain regulator based on optimal LQR can improve the nonlinear AMB system performance in the following aspects:

1) The optimal method LQR is effect in linear control system. For the AMB system presented a decentralized control can be used.
2) For the decentralized control, the 3-D frequency domain analyses method is effective

for the nonlinear system.

3) Both high stiffness and the stability in the large can be obtained in AMB system.

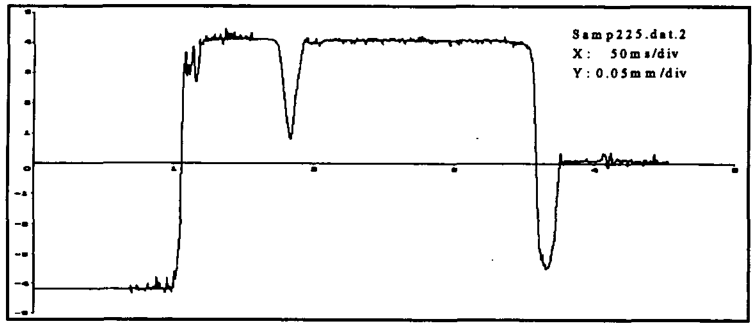


Figure 8. Experimental results of the rotor displacement (x-coordinate: 50ms/div)

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