ROBUST CONTROL OF A MAGNETIC BEARING SYSTEM USING CONSTANTLY SCALED H_∞ CONTROL

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ABSTRACT

In this paper, we propose a design method for a robust controller of a magnetic bearing system with structured time-varying uncertainties. We apply the method of constantly scaled H_∞ control to this system. The constantly scaled H_∞ controller is solved by using a dual iterative algorithm based on LMI constraints. Simulation results are presented to demonstrate the control effects.

INTRODUCTION

Active magnetic bearings (AMBs) have been increasingly interesting for indutrial applications such as turbo-molecular pump and machining spindle. Because it offers unique advantages of non-contact, elimination of lubrication, low power loss and controllability of the bearing dynamics characteristics. Nowadays the research works on magnetic bearing have been aggressively carried out, and the importance of the robust control has been increasing. For this reason, various attempts have been paid to the robust control for AMB systems. Especially, H_∞ control and μ-Synthesis are well understood. In H_∞ control method, we can get robust stability for the multiplicative or additive uncertainties only solving two Riccati equations(Zhou, Doyle & Glover 1996). But if the system has structural uncertainties such as physical parameter uncertainties, it yields conservative results. On the other hand, μ-Synthesis method can treat the structured uncertainties, and yield less conservative performance results(Packard & J.Doyle 1993).

In the μ-Synthesis, iterative methods such as D-K iteration or D,G-K iteration are used for the calculation of the controller(Balas, Doyle, Glover, Packard & Smith 1995, Young 1994). But in the process of the iteration, various parameter, e.g., the degree of D-scaling, the range and the number of the frequency points for fitting, must be given by the designer, and the results depend deeply on these parameters. Then it require much effort, much time and much trial and error. Furthermore, the order of the controller tend to be high, and it increases the cost of the implementation of the controller.

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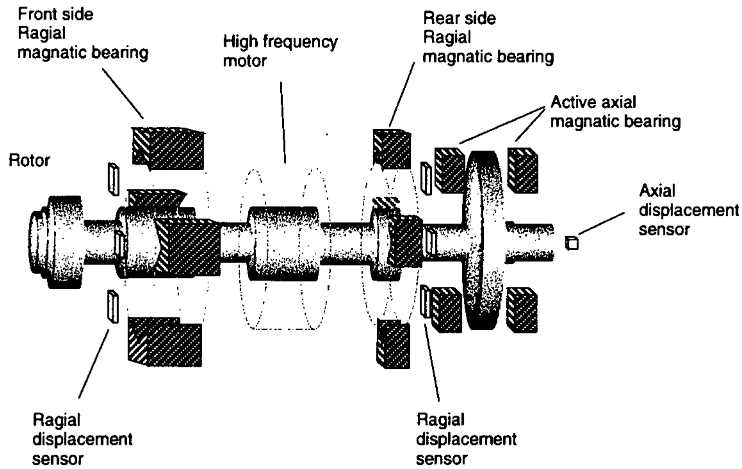

Figure 1: Flexible rotor-magnetic bearing system

In recent years, a constantly scaled H_∞ method has been developed. It can deal with structured time-varying uncertainties, and the order of the controller is same as that of the generalized plant and usually less than that of the μ controller. The constantly scaled H_∞ norm is an upper-bound of μ. Then it can also be used as the optimization method of the upper bound of μ. The optimization problem of the scaled H_∞ norm is not convex, so it is difficult to compute the global solution. However some iterative methods have been developed to find a sub-optimal solution, and we can get a good solution up to the medium size problems(Yamashiro, Iwasaki & Hara 1996).

In this paper, we propose a design method for the robust controller of magnetic bearing systems with parameter uncertainties by using constantly scaled H_∞ method. The nonlinearity of the magnetic bearings are modeled as structured time-varying uncertainties. The unmodeled dynamics and the sensitivity performance are also take into consideration. The controller is calculated by using a dual iterative algorithm based on LMI(Linear Matrix Inequality) constraints. We also design μ controller to compare the control performances with the constantly scaled H_∞ method. Finally, simulation results are presented to demonstrate the effectiveness of the proposed method.

MODELING

Fig.1 shows the AMB system which has five degrees of freedom of a flexible rotor. The position of axial direction is controlled by the conventional PID controller, we only model the dynamics of the rotor in radial directions. For simplicity, we assume the following assumptions: (a1) the force of attraction is proportional to the square power of the electric coil current and is inversely proportional to the square power of the gap length. (a2) the back EMF voltage which is proportional to the velocity is negligible. (a3) the inductance of the coil is consistent regardless of frequency or the gap length. (a4) there is no interaction such as gyroscopic effect between x direction and y direction in radial direction. (a5) we only treat the small deviation from the nominal position.

Based on the conditions above, we analyze the model of x direction of the AMB system which is depicted in Fig.2. The derivation of the model of y direction is similar to the x

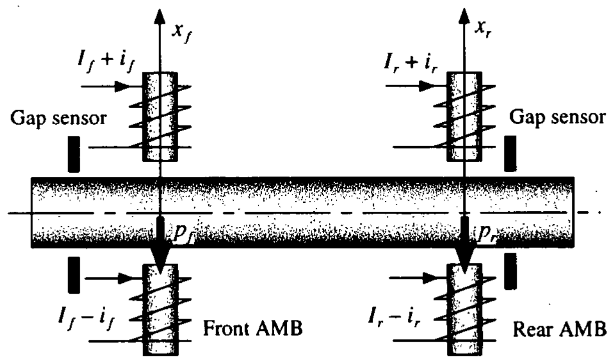

Figure 2: Model of x direction

direction.

DYNAMICAL MODEL OF FLEXIBLE ROTOR-MAGNETIC BEARING SYSTEM

The dynamical model of the flexible rotor is derived by the finite element method. The rotor can be taken simply into account in 27 parts, and the discrete model of 56-order is obtained as follows:

$$M\ddot{q} + Kq = 0, \tag{1}$$

where $q \in \mathcal{R}^{56}$ represents the displacement and the angle of the rotor, $M \in \mathcal{R}^{56\times 56}$ is the mass matrix and $K \in \mathcal{R}^{56\times 56}$ is the stiffness matrix. The rotor is supported by the attractive forces which is described by p_f and p_r as shown in Fig.2. This gives:

$$M\ddot{q} + Kq = \tilde{F}p, \quad p := [p_f,\ p_r]^T, \tag{2}$$

where the scripts f and r represent the front side and the rear side of the roter respectively, and $\tilde{F} \in \mathcal{R}^{56\times 2}$ is a matrix which indicates the acting position of electro-magnet force.

Using Taylor series expansion, the attractive force of electro-magnets at the nominal position can be described by the following equations.

$$p_f \;:=\; k_f \cdot x_f - g_f \cdot i_f + \delta_f(x_f, i_f), \tag{3a}$$
$$p_r \;:=\; k_r \cdot x_r - g_r \cdot i_r + \delta_r(x_r, i_r), \tag{3b}$$

where i_f and i_r are the control current which are added to the bias current, x_f and x_r are the displacements from the nominal positions. Furthermore, k_f and k_r are the force-current factor, g_f and g_r are the force-displacement factor, δ_f, δ_r are the high-order term in Taylor series expansion which are taken into account in the controller design.

STATE-SPACE MODEL WITH PARAMETRIC UNCERTAINTIES

MODEL REDUCTION

As is well known, it is difficult to use the model of 56-order as the design model, we reduce the order of the model of the flexible rotor. Using the modal axis transformation and the extraction of only n lower modes, (2) can be reduced to the following equation.

$$\ddot{\xi} + \Lambda\dot{\xi} + \Omega^2\xi = Fp, \quad \xi := [\xi_1, \ldots, \xi_n]^T, \tag{4}$$

where

$$\begin{aligned} \Omega &:= \text{diag}[\omega_1, \ldots, \omega_n] \quad (\omega_i \leq \omega_{i+1}), \\ \Lambda &:= \text{diag}[2\zeta_1\omega_1, \ldots, 2\zeta_n\omega_n]. \end{aligned}$$

The displacements x_f and x_r can be represented by the following equation

$$\begin{bmatrix} x_f \\ x_r \end{bmatrix} = F^T\xi. \tag{5}$$

Substituting (3) and (5) into (4), the reduced model of the flexible rotor-magnetic bearing system can be represented as follows:

$$\ddot{\xi} + \Lambda\dot{\xi} + (\Omega^2 - F\,G_x\,F^T)\xi = -F\,G_u\,u_i, \tag{6}$$

where

$$G_x := \text{diag}[k_f,\ k_r], \quad G_u := \text{diag}[g_f,\ g_r], \tag{7}$$

$$u_i := [i_f,\ i_r]^T. \tag{8}$$

Using (6), we can get the state-space model of the nominal case as follows:

$$\dot{x} = Ax + Bu_i, \tag{9a}$$

$$y = Cx, \tag{9b}$$

where

$$A := \begin{bmatrix} 0 & I \\ \Lambda & -(\Omega^2 - FG_xF^T) \end{bmatrix}, \tag{10}$$

$$B := \begin{bmatrix} 0 \\ -FG_u \end{bmatrix}, \ C := \begin{bmatrix} C_s & O \end{bmatrix}, \tag{11}$$

$$x := \begin{bmatrix} \xi \\ \dot{\xi} \end{bmatrix}, \ y := \begin{bmatrix} y_f \\ y_r, \end{bmatrix}. \tag{12}$$

Here, y_f and y_r denote the displacement observed by the position sensor, C_s is the matrix which represents the relation between y and ξ.

LFR OF PARAMETRIC UNCERTAINTIES

In this section, the LFR (Linear Fractional Representation) of the parametric uncertainties is discussed. In general, magnetic force is a nonlinear function of the gap length and the coil current. Now, we represent the nonlinearity which is described by δ_f or δ_r in (3) as the LFR model.

As the first step, we put these nonlinearity into the force-current factors, and suppose that k_f and k_r have time-varying uncertainties. Introducing the normalized time-varying parameter $\delta_i \in \mathcal{R}$, k_f and k_r can be represented as follows:

$$k_f = k_{f0} + \Delta_1\delta_1, \quad k_r = k_{r0} + \Delta_2\delta_2, \tag{13}$$

where Δ_1 and Δ_2 give the bound of the uncertainties.

Using the method described in (Hirata, Liu & Mita 1996), the magnetic bearing system with parametric uncertainties can be represented using LFR as follows:

$$y = \mathcal{F}_u(P_0, \Delta_\delta)u_i \tag{14}$$

where P_0 is a generalized plant, and Δ_δ is diagonal matrix which is defined as

$$\Delta_\delta := \mathrm{diag}[\delta_1,\ \delta_2],$$

$\mathcal{F}_u$ denotes the upper linear fractional transformation(Zhou et al. 1996).

CONSTANTLY SCALED H_∞ PROBLEM

Fig.3 shows the closed-loop system with a diagonalized time-varying uncertainty Δ.

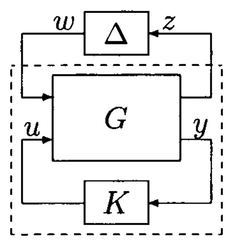

Figure 3: Closed-loop system

In this figure, G and K denote the generalized plant and the controller respectively. $\mathcal{S}$ is defined as the set of constant scaling matrices which have commutative structure with Δ. Under this setup, the constantly scaled H_∞ problem can be defined as follows.

Problem 1 *For given G, find a constant scaling matrix $S \in \mathcal{S}$ and internally stabilizing controller K minimizing γ which satisfy the following inequality:*

$$\| S^{\frac{1}{2}}\mathcal{F}_l(G,K)S^{-\frac{1}{2}} \|_\infty < \gamma \tag{15}$$

□

In the μ-Synthesis with D-K iteration, the scaling matrix S can be chosen as a real-rational, stable, minimum-phase transfer function. Hence we see that the γ satisfying (15) is a upper bound of μ.

There are several methods for optimizing (15) (Yamashiro et al. 1996, Yamada & Hara 1998). In this paper, we use the dual iteration method described in (Yamashiro et al. 1996). In this method, the optimized variables which are fixed at each iteration are diminished to extend the optimizing space, and we could have good sub-optimum solution with a few iterations.

The state-space realization of G is defined as follows:

$$G(s) := \left[\begin{array}{c|cc} A & B_1 & B_2 \\ \hline C_1 & D_{11} & D_{12} \\ C_2 & D_{21} & O \end{array}\right] \tag{16}$$

Using the matrices in (16), define the following matrices:

$$N := \left[\begin{array}{cc} B_2^T & D_{12}^T \end{array}\right]^{\perp}, \quad M := \left[\begin{array}{cc} C_2 & D_{21} \end{array}\right]^{\perp} \tag{17}$$

$$\mathcal{N} := \mathrm{diag}[N,\ I], \quad \mathcal{M} := \mathrm{diag}[M,\ I] \tag{18}$$

where $A^{\perp}$ denotes the column full-rank matrix satisfying $AA^{\perp} = 0$.

Under this setup, the following theorem can be shown(Yamashiro et al. 1996).

Theorem 1 *For a given γ, Problem 1 is solvable if and only if the following two equivalent conditions* **C1)** *and* **C2)** *are satisfied.*

C1) *There exist a real matrix F, symmetric matrices P, Y and $S \in \mathcal{S}$ satisfying the following three LMIs:*

$$\begin{bmatrix} PA_F + A_F^T P & PB_F & {C_F}^T S \\ {B_F}^T P & -\gamma S & {D_F}^T S \\ SC_F & SD_F & -\gamma S \end{bmatrix} < 0, \tag{19a}$$

$$\mathcal{M}^T \begin{bmatrix} YA + A^T Y & YB_1 & {C_1}^T S \\ {B_1}^T Y & -\gamma S & {D_{11}}^T S \\ SC_1 & SD_{11} & -\gamma S \end{bmatrix} \mathcal{M} < 0, \tag{19b}$$

$$Y \geq P > 0, \tag{19c}$$

where

$$\begin{pmatrix} A_F & B_F \\ C_F & D_F \end{pmatrix} := \begin{pmatrix} A & B_1 \\ C_1 & D_{11} \end{pmatrix} + \begin{pmatrix} B_2 \\ D_{21} \end{pmatrix} F. \tag{20}$$

C2) *There exist a real matrix L, symmetric matrices Q, X and $R \in \mathcal{S}$ satisfying the following three LMIs:*

$$\mathcal{N}^T \begin{bmatrix} AX + XA^T & X{C_1}^T & B_1 R \\ C_1 X & -\gamma R & D_{11} R \\ R{B_1}^T & R{D_{11}}^T & -\gamma R \end{bmatrix} \mathcal{N} < 0, \tag{21a}$$

$$\begin{bmatrix} A_L Q + Q{A_L}^T & Q{C_L}^T & B_L R \\ C_L Q & -\gamma R & D_L R \\ R{B_L}^T & R{D_L}^T & -\gamma R \end{bmatrix} < 0, \tag{21b}$$

$$X \geq Q > 0, \tag{21c}$$

where

$$\begin{pmatrix} A_L & B_L \\ C_L & D_L \end{pmatrix} := \begin{pmatrix} A & B_1 \\ C_1 & D_{11} \end{pmatrix} + L \begin{pmatrix} C_2 & D_{21} \end{pmatrix}. \tag{22}$$

□

The following relations are satisfied between the condition **C1)** and **C2)**.

$$P = X^{-1}, \quad Y = Q^{-1}, \quad S = R^{-1} \tag{23}$$

Note that the condition **C1)** will be LMI conditions with variables (P, Y, S) if F is fixed. Similar to this, the condition **C2)** will be LMI conditions with variables (Q, X, R)

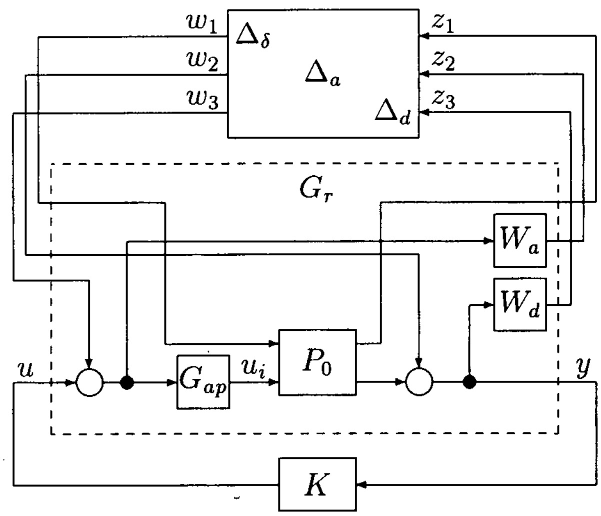

Figure 4: Block diagram of generalized plant

if L is fixed. Using this fact and the relation (23), the following algorithm which is called "dual iterative algorithm" can be available.

[Dual iterative algorithm]

Step 0 Set $S = I$ and $i = 0$, compute (P, Y, F) minimizing γ_0 under the condition **C1)**. Let $i \leftarrow i + 1$ and go to **Step 1**.

Step 1 Fix F and compute (P, Y, S) minimizing γ_i under the condition **C1)**. Using this (Y, S, γ_i), compute (Q, R) by using (23) and L satisfying (21b). Let $i \leftarrow i + 1$.

Step 2 Fix L and compute (Q, X, R) minimizing γ_i under the condition **C2)**. Using this (X, R, γ_i), compute (P, S) by using (23) and L satisfying (19a).

Step 3 If the relative error $|\gamma_i - \gamma_{i-1}|$ is less than the tolerance, quit the iteration. Otherwise, let $i \leftarrow i + 1$ and go to **Step 1**.

CONTROLLER DESIGN

The nominal model which has only two rigid modes is obtained by setting $n = 2$ in (4). It is assumed that the bounds of the time-varying parameters k_f, k_r are $\pm 0.5\%$ respectively. Under this condition, P_0 in (14) is obtained. Fig.4 shows the generalized plant for controller design. In this figure, W_a is the weighting function for the additive perturbation which is neglected in the nominal model, W_d is the weighting function for the robust performance against to the various force disturbances, G_{ap} is the model of the current amplifier which is a 2×2 transfer function from the input voltage to the coil current. Δ_a and Δ_d are 2×2 normalized full blocks. The frequency responses of the weighting functions are depicted in Fig.5 in which the additive error is plotted as the dotted lines.

Using this G_r, we apply the dual iteration algorithm for the constantly scaled H_∞ synthesis and obtain the controller K_{cs}. The history of γ_i over 10 iteration is shown in Fig.6. We also compute two more controllers for comparison. One is the controller K_{dk1}

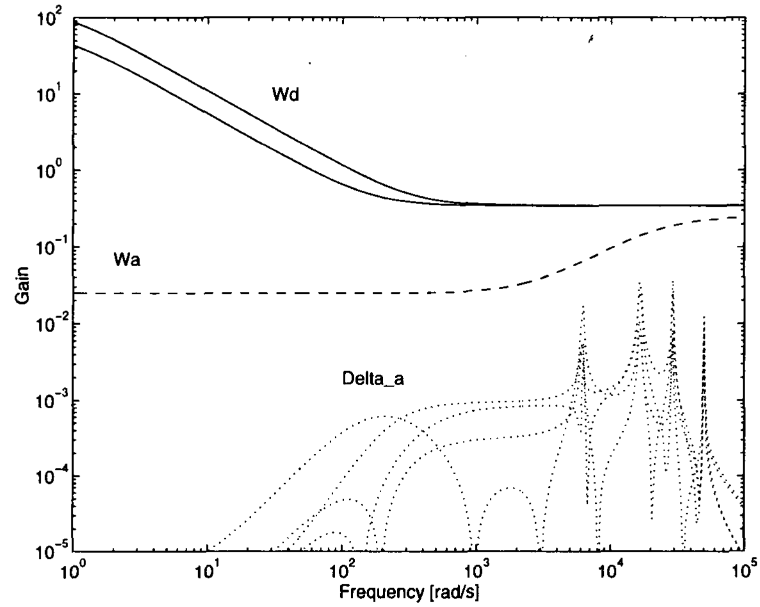

Figure 5: Frequency response of W_d and W_a

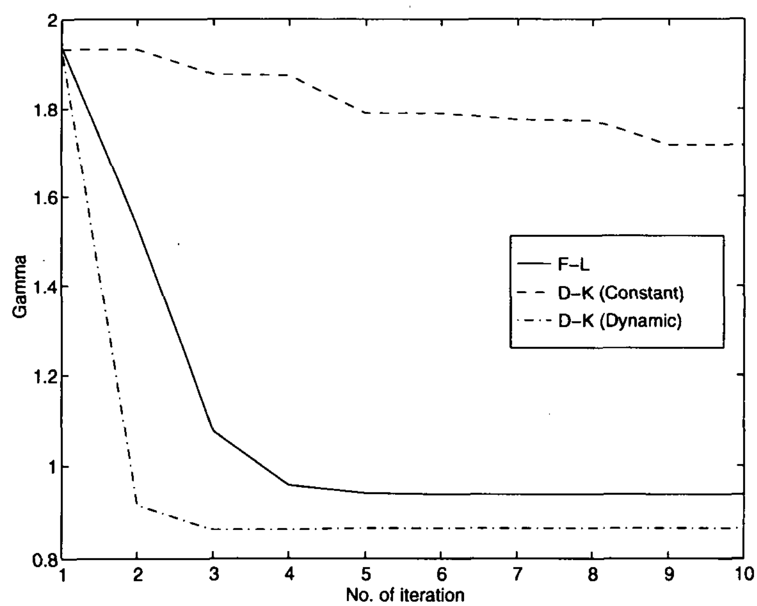

Figure 6: Transition of γ_i

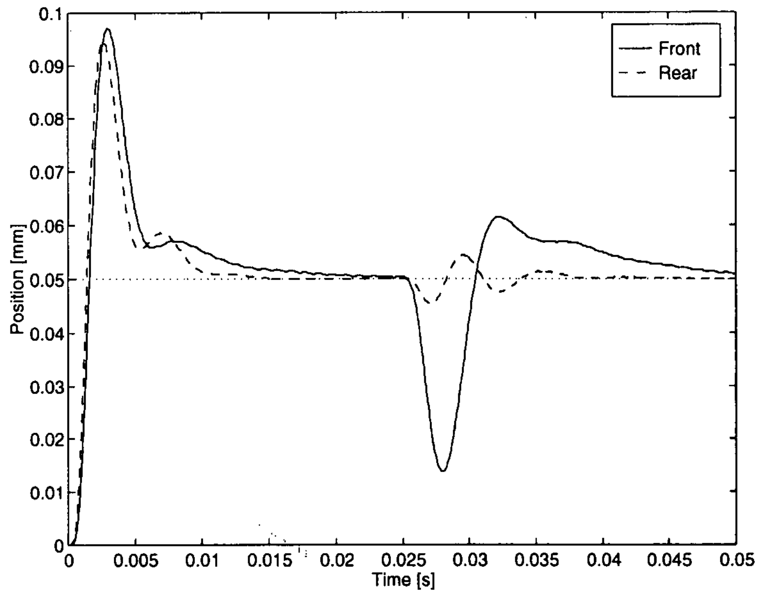

Figure 7: Time response via controller K_{cs}

obtained by using *D-K* iteration with constant scalings, and the other is the controller K_{dk2} obtained by using *D-K* iteration with dynamical scalings. The histories of γ_i of these two cases are also plotted in Fig.6.

In the dual iteration, the optimization is converged to the sub-optimal solution within 4 steps. On the other hand, the *D-K* iteration with constant scalings converges very slow, and the value of γ at 10th iteration is larger than that of the other two methods.

SIMULATION

The obtained continuous-time controllers K_{cs} and K_{dk2} are discretized by the Tustin transformation with a sampling frequency of 8kHz. The simulation model which is constructed in SIMULINK can simulate the nonlinearity of the magnetic bearings, the saturation of the circuit amplifier, the computational delay of the controller and the vibration mode of the flexible rotor up to 4th mode.

To evaluate the performances achieved by K_{cs} and K_{dk2}, we show the step response of x_f and x_r in Fig.7 and Fig.8 in which the impulse disturbance acts on the the front side of the rotor at $t = 0.025[s]$. From these figures, it is confirmed that that the controller K_{cs} with 10th degrees of order can achieve a good performance as same as the controller K_{dk2} with 22nd order.

CONCLUSION

This paper has proposed the design method of the controller for magnetic bearing systems using the constantly scaled H_∞ method. It has been shown that the dual iteration algorithm is quite well for the optimization compared with the constantly scaled *D-K* iteration. The simulation results have been shown that the resulting performance of

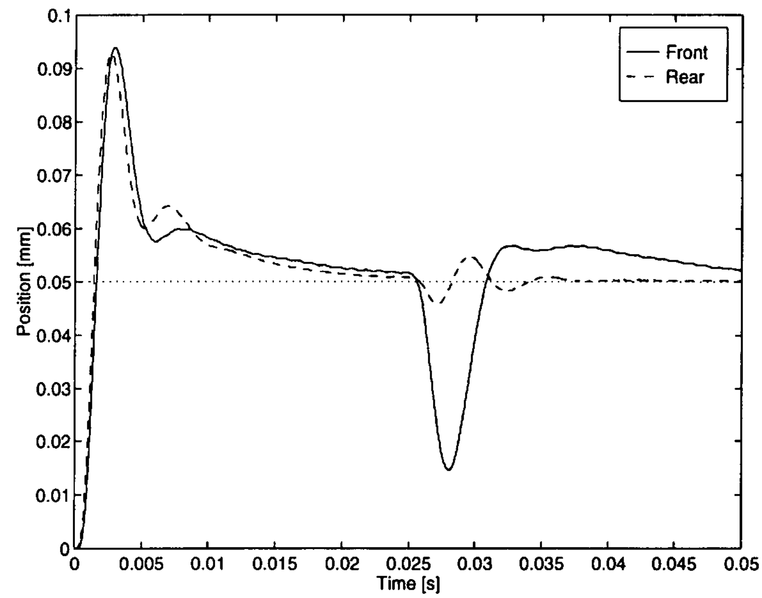

Figure 8: Time response via controller K_{dk2}

the constantly scaled H_∞ controller is satisfactory in the sense that it can achieve same performances as μ controller by the low order controller. This means that it is useful if the trade off between the performances and the realization cost of the controller must be considered. Furthermore, in the dual iteration method, there is few optimization parameters which must be given by the designer compared with the D-K iteration with dynamical scalings. So we could also use this method as the alternatives for μ-Synthesis.

REFERENCES

Balas, G., Doyle, J., Glover, K., Packard, A. & Smith, R. (1995), *μ-Analysys & Synthesis Toolbox User's Guide*, The MathWorks.

Hirata, M., Liu, K. & Mita, T. (1996), 'Active Vibration Control of A 2-Mass System Using μ-Synthesis with a Descriptor Form Representation', *Control Engineering Practice* **4**(4), 545–552.

Packard, A. & J.Doyle (1993), 'The Complex Structured Singular Value', *Automatica* **29**(1), 71–109.

Yamada, Y. & Hara, S. (1998), 'Global Optimization for H_∞ Control with Constant Diagonal Scaling — Triangle Convering Method and its Computational Complexity —', *IEEE Trans. on A.C.* **36**(2), 653–667.

Yamashiro, K., Iwasaki, T. & Hara, S. (1996), Scaled H_∞ Synthesis by the Dual Iteration, *in* 'Proc. of SICE'96', pp. 83–84.

Young, P. (1994), Controller Design with Mixed Uncertainties, *in* 'Proc. of the 1994 ACC', pp. 2333–2337.

Zhou, K., Doyle, J. & Glover, K. (1996), *Robust Optimal Control*, Prentice-Hall.