SYNTHESIS TECHNIQUES FOR ROBUST ADAPTIVE VIBRATION CONTROL

Carl R. Knospe,[1] Jeffrey D. Lindlau,[1] R. Winston Hope[2]

ABSTRACT

The problem of synthesizing robustly stable gain matrices for the adaptive vibration control of unbalanced rotors is examined and several synthesis techniques are developed. The quite general case of uncertainties entering into the magnetic bearing system model in a linear-fractional form are considered. The uncertainties may be highly structured and either parametric or dynamic. It is shown that the resulting robust synthesis problem may be written as a nonlinear matrix inequality in both the gain matrix and scaling matrices. Three synthesis algorithms are then developed. In each, an iteration is conducted between the two problems of gain matrix optimization and scale optimization. It is also shown that the optimization with respect to the gain matrix can be eliminated by the use of a projection allowing direct construction of a satisfactory gain matrix. In each case, the optimization with respect to the scales can be reformulated as either a linear matrix inequality or a structured singular value computation. Furthermore, it is demonstrated that a family of robust gain matrices may be constructed from any satisfactory solution.

1. INTRODUCTION

Researchers worldwide have demonstrated that active magnetic bearings may be used to greatly attenuate the synchronous vibration of rotors due to unbalance [Haberman and Brunet, 1984; Matsumura et al., 1990; Burrows and Sahinkaya, 1983; Burrows et al., 1989; Higuchi et al., 1990; Larsonneur and Herzog, 1994; Shafai et al., 1994; Knospe et al., 1994; Hope, 1994; Knospe et al., 1997a]. Much investigation has focused on the use of Adaptive Vibration Control (AVC) algorithms (previously referred to as adaptive open loop). Recently, this technique has been successfully applied to a number of important industrial applications [Hope, 1997].

Previous efforts have established that the stability and performance robustness of this algorithm with respect to structured uncertainty can be analyzed through use of the structured singular value (μ) and that the synthesis of robust gain matrices can be achieved through numerical optimization [Knospe et al., 1997a, 1997b]. Experimental results have also shown both the analysis and synthesis tools are highly effective in practice [Knospe et al., 1997d].

---

[1]Associate Professor, Research Assistant, respectively. Department of Mechanical, Aerospace, and Nuclear Engineering, University of Virginia, Charlottesville, VA 22903, USA.
[2]Development Engineer, Revolve Magnetic Bearings, Calgary, Alberta, Canada.

In this paper, new more effective algorithms are presented for the synthesis of robustly stable gain matrices for AVC. Previous efforts have used computationally intensive gradient minimization methods in conjunction with μ-analysis for solution [Knospe et al., 1997b, 1997d]. Indeed, one of the new algorithms requires no direct optimization with respect to the gain matrix parameters as it involves only iterative steps of μ-analysis and algebraic construction.

In Section 2, the AVC algorithm is reviewed and its robustness is discussed. In Section 3, three new algorithms for synthesis of robustly stable AVC gain matrices are developed and discussed. Then, the most efficient numerical algorithm is tested in the synthesis of gain matrices for twenty example problems in Section 4.

MATHEMATICAL NOTATION

The two-norm of a vector v is indicated by the notation $\|v\|$. The maximum singular value of a matrix P is denoted by $\bar{\sigma}(P)$ and the spectral norm by $\rho(P)$. The superscript *, +, and $\perp$ indicate the complex-conjugate transpose, pseudo-inverse, and null space basis, respectively, of a matrix. The lower and upper linear fractional transformations [Doyle et al., 1991] of P are given the notations $F_l(P,Q)$ and $F_u(P,R)$ respectively where the matrices Q and R are assumed to be appropriately dimensioned. The Redheffer star-product of appropriately dimensioned matrices P and Q will be denoted by $S(P,Q)$. The structured singular value [Doyle et al., 1991] of a matrix P is indicated by the notation $\mu_\Delta(P)$ and its upper bound by the notation $\bar{\mu}_\Delta(P)$. The symbol S_Δ is used to denote the set of all matrices of a defined block structure.

2. REVIEW OF ADAPTIVE VIBRATION CONTROL

The concept of adaptive open vibration control is quite simple. Synchronous perturbation control signals are generated and added to the feedback control signals. These synchronous signals consist of sinusoids that are tied to the shaft angular position via a keyphasor signal. The magnitudes and phases of these sinusoids are periodically adjusted so as to minimize the rotor unbalance response. These updates occur slowly in comparison to the decay of the rotor's transient response. For this reason, this method is considered as the adaptation of a set of open loop synchronous signals.

A model of the rotor system may be formulated relating the vibration to the applied open loop signals via

$$X = TU + X_0$$

where X is a n-vector of the complex synchronous Fourier coefficients of the n (generalized) vibration measurements, U is a m-vector of the complex synchronous Fourier coefficients of the m applied open loop signals, X_0 is a n-vector of the complex synchronous Fourier coefficients of the (generalized) uncontrolled vibration, and T is a $n \times m$ matrix of complex influence coefficients relating the open loop signals to the vibration measurements. The influence coefficient matrix is the transfer function matrix of the supported rotor (with feedback control) from perturbation forces at the bearings to the displacements at the sensors, evaluated at the rotor operating speed Ω.

Since the convergent control algorithm updates the control vector U periodically, the subscript i will be used to denote the i'th update of control and measurement. The AVC algorithm uses a simple control update or *adaptation* law

$$U_{i+1} = U_i + AX_i \tag{1}$$

to determine the next set of harmonic forces to apply. Here the matrix A is referred to as the *gain* matrix and ultimately determines the efficacy of the algorithm. Determination of the proper gain matrix is hindered by the fact that the influence coefficients may be poorly known or may change. Thus, only an estimate of the T matrix, $\hat{T}$, is available. This estimate can be considered to correspond to a nominal state space model of the system given as follows

$$\overline{P}_{f \rightarrow d}(s) \equiv \left[\begin{array}{c|c} \overline{A} & \overline{B} \\ \hline \overline{C} & \overline{D} \end{array} \right] \tag{2}$$

and the estimate is related to this nominal model via the equation

$$\hat{T} = \overline{P}_{f \rightarrow d}(j\Omega) \tag{3}$$

We will consider in this paper only the case where the matrix T is square (number of sensors is equal to the number of actuators - see [Knospe, 1997b] for an examination of the non-square case). When the estimate used in the determination of A is in error, the adaptation process results in the control vector either growing unbounded or converging to the optimal control vector. If the control vector converges to its optimal value, the adaptation process is said to be stable. A necessary and sufficient condition for stability is $\rho(I + AT) < 1$. A sufficient condition for adaptation process stability is given by the following condition [Hope, 1994]:

$$\overline{\sigma}(I + AT) < 1 \tag{4}$$

This stability condition requires that the distance of the control vector from the optimal control vector, $\left\| U_i - U_{opt} \right\|$, decrease with each update.

Uncertainties in the machine's dynamics usually can be represented by a structured uncertainty representation. That is, several parameters $\theta_1, \theta_2, \ldots \theta_i, \cdots \theta_p$ of the dynamic model (e.g., the effective stiffness or damping of a seal) are different from those of the nominal model (corresponding to $\hat{T}$). The structured representation indicates how each of these uncertainties affects the elements of the influence coefficient matrix T.

Since the system's state space representation (A, B, C, D) may be considered to be affinely dependent on these uncertainties, the influence coefficient matrix can be represented by a linear fractional transformation (LFT) of the following form [Knospe et al., 1997a]

$$T = F_u\left(\tilde{T}, \Delta_s\right) \equiv \tilde{T}_{22} + \tilde{T}_{21}\Delta_s\left[I - \tilde{T}_{11}\Delta_s\right]^{-1}\tilde{T}_{12} \tag{5}$$

where Δ_s is a block diagonal matrix of the uncertainties, its structure dictated by the particular problem considered.

Here
$$\tilde{T} = \begin{bmatrix} \tilde{T}_{11} & \tilde{T}_{12} \\ \tilde{T}_{21} & \tilde{T}_{22} \end{bmatrix}$$

and the nominal influence coefficient matrix is given by $\hat{T} = \tilde{T}_{22}$. The set of all matrices of a given block structure is denoted by S_Δ. Note that through appropriate scaling of the matrices $\tilde{T}_{11}$ and $\tilde{T}_{12}$, the uncertainty block matrix can be considered to satisfy

$$\overline{\sigma}(\Delta_s) \leq 1$$

Theorem 1: Stability Robustness [Knospe et al, 1997a]

The uncertain rotor system described by

$$T = F_u(\tilde{T}, \Delta_s) \qquad\qquad \Delta_s \in S_{\Delta_s}: \quad \overline{\sigma}(\Delta_s) \leq 1 \qquad\qquad (6)$$

will robustly converge with rate ε_c, to the optimal control vector if

$$\mu_\Delta[M] < 1 \qquad\qquad M \equiv \begin{bmatrix} \tilde{T}_{11} & \tilde{T}_{12} \\ \dfrac{1}{\varepsilon_c} A\tilde{T}_{21} & \dfrac{1}{\varepsilon_c}(I + A\tilde{T}_{22}) \end{bmatrix} \qquad\qquad (7)$$

where the uncertainty block has the structure

$$\Delta = \begin{bmatrix} \Delta_s & \\ & \Delta_f \end{bmatrix} \qquad\qquad (8)$$

where Δ_s is a structured block representing the plant uncertainty, $\Delta_s \in S_{\Delta_s}$, and Δ_f is a full complex block, $\Delta_f \in C^{m \times m}$.

3. SYNTHESIS OF ROBUST GAIN MATRICES

Since calculation of the structured singular value μ is very difficult, we will make use of its well known upper bound for deriving equations for gain matrix synthesis. If the upper bound on $\mu_\Delta(M)$ is minimized through choice of A to be less than one, then the convergence will be achieved robustly.

Without loss of generality, we will define the structure of the block diagonal uncertainty matrix Δ by

$$\Delta = \text{block diag}\left(\delta_1^r I_{k_1}, \ldots, \delta_m^r I_{k_m}, \delta_1^c I_{k_{m+1}}, \ldots, \delta_m^c I_{k_{m+n}}, \Delta_1^C, \ldots, \Delta_q^C\right) \qquad (10)$$

where k_i are indices providing the dimensions of each block. The first $m+n$ blocks are the (repeated) real and complex uncertainties δ_i^r and δ_i^c. The complex block Δ_i^C has dimensions $k_{m+n+i} \times k_{m+n+i}$. Corresponding to this structure, define scaling matrix sets

$$D = \left\{ D : D = \text{block diag}\left(D_1,\ldots,D_{m+n},d_1 I_{k_{m+n+1}},\ldots,d_q I_{k_{m+n+q}}\right), \quad D_i = D_i^* > 0, D_i \in C^{k_i \times k_i}, d_i \in R \right\}$$

$$G = \left\{ G : G = \text{block diag}\left(G_1,\ldots,G_m,0_{k_{m+1}},\ldots,0_{k_{m+n+q}}\right), \quad G_i = G_i^* \in C^{k_i \times k_i} \right\} \tag{11}$$

An upper bound on the structured singular value may then be computed by solving the following optimization problem:

$$\overline{\mu}_\Delta(M) = \inf_{D \in D, G \in G} \left\{ \inf_{\beta \geq 0} \left\{ \beta : M^* DM + j\left(GM - M^*G\right) - \beta^2 D < 0 \right\} \right\} \tag{12}$$

An alternative optimization problem may also be used to compute the upper bound. First, define the sets of scaling matrices

$$\overline{D} = \left\{ \begin{array}{l} D : D = \text{block diag}\left(D_1,\ldots,D_{m+n},d_1 I_{k_{m+n+1}},\ldots,d_q I_{k_{m+n+q}}\right), \\ D_i = D_i^*, \det(D_i) \neq 0, D_i \in C^{k_i \times k_i}, d_i \neq 0, d_i \in R \end{array} \right\} \tag{13}$$

$$\overline{G} = \left\{ G : G = \text{block diag}\left(g_1,\ldots,g_{n_r},0_{n_c}\right), \quad g_i \in R, \quad n_r = \sum_{i=1}^m k_i, \quad n_r = \sum_{i=m+1}^{m+n+q} k_i \right\}$$

Then, the alternative upper bound problem is

$$\overline{\mu}_\Delta(M) = \inf_{D \in \overline{D}, G \in \overline{G}} \left\{ \inf_{\beta > 0} \left\{ \beta : \overline{\sigma}\left(\left(I+G^2\right)^{-\frac{1}{4}} \left(\frac{DMD^{-1}}{\beta} - jG\right)\left(I+G^2\right)^{-\frac{1}{4}} \right) < 1 \right\} \right\} \tag{14}$$

Therefore, the robust gain matrix synthesis problem may be put in either of two forms

$$A_{syn} = \left\{ A : \overline{\mu}_\Delta(M) < 1 \right\}$$

$$= \arg\min_{A \in C^{m \times m}} \left\{ \min_{\substack{D \in D, G \in G \\ \beta \geq 0}} \left\{ \beta : M^* DM + j\left(GM - M^*G\right) - \beta^2 D < 0 \right\} \right\} \tag{15}$$

OR

$$= \arg\min_{A \in C^{m \times m}} \left\{ \min_{\substack{D \in \overline{D}, G \in \overline{G} \\ \beta > 0}} \left\{ \beta : \overline{\sigma}\left(\left(I+G^2\right)^{-\frac{1}{4}} \left(\frac{DMD^{-1}}{\beta} - jG\right)\left(I+G^2\right)^{-\frac{1}{4}} \right) < 1 \right\} \right\} \tag{16}$$

Note that in first form the constraint is linear in the scales but quadratic in the gain matrix A (since M is linear in A). Thus, optimization with respect to D and G is a convex optimization problem which may be solved straightforwardly with available linear matrix inequality (LMI) software packages. However, minimizing β via the gain matrix A is much more difficult due to the constraint's quadratic dependence on A . The constraint may be reformulated so as to be linear in A through use of Schur's complement

$$M^* DM + j\left(GM - M^*G\right) - \beta^2 D < 0, \qquad D > 0 \tag{17}$$

$$\Updownarrow$$

$$M^*DM + j(GM - M^*G) + \Phi_2 - (\beta^2 D + \Phi_1) < 0, \qquad D > 0 \qquad \Phi_2 > \Phi_1 > 0$$

$$\Updownarrow$$

$$(\beta^2 D + \Phi_1) - \{M^*DM + j(GM - M^*G) + \Phi_2\} > 0, \qquad D > 0 \qquad \Phi_2 > \Phi_1 > 0$$

$$\Updownarrow$$

$$(\beta^2 D + \Phi_1) - \begin{bmatrix} M^* & I \end{bmatrix} \begin{bmatrix} D & -jG \\ jG & \Phi_2 \end{bmatrix} \begin{bmatrix} M \\ I \end{bmatrix} > 0, \qquad D > 0 \qquad \Phi_2 > \Phi_1 > 0$$

$$\Updownarrow$$

$$\begin{bmatrix} (\beta^2 D + \Phi_1) & \begin{bmatrix} M^* & I \end{bmatrix} \\ \begin{bmatrix} M \\ I \end{bmatrix} & \begin{bmatrix} D & -jG \\ jG & \Phi_2 \end{bmatrix}^{-1} \end{bmatrix} > 0, \qquad \begin{bmatrix} D & -jG \\ jG & \Phi_2 \end{bmatrix} > 0, \quad D > 0, \quad \Phi_2 > \Phi_1 > 0 \qquad (18)$$

at the expense of introducing the two slack variables Φ_1 and Φ_2. However, this form is now nonlinear in the scale matrices D and G due to the inverse involving these terms in the lower right-hand corner of the first constraint. An iterative algorithm to minimize beta may be formulated using both constraints (17) and (18):

Synthesis Algorithm #1: Dual LMI Iteration
 1) initialize A
 2) with A fixed, minimize β over D and G subject to the LMI constraint
$$M^*DM + j(GM - M^*G) - \beta^2 D < 0, \qquad D > 0$$
 3) pick matrix $\Phi_2 > GD^{-1}G$ and set $\Phi_1 = \Phi_2$
 4) with D, G, Φ_1, Φ_2 fixed, minimize β over A subject to the LMI constraint
$$\begin{bmatrix} (\beta^2 D + \Phi_1) & \begin{bmatrix} M^* & I \end{bmatrix} \\ \begin{bmatrix} M \\ I \end{bmatrix} & \begin{bmatrix} D & -jG \\ jG & \Phi_2 \end{bmatrix}^{-1} \end{bmatrix} > 0$$
 5) if $\beta < 1$ quit, else return to step 2.

An alternative algorithm may be formulated using Eqn. 16. First, note that

$$\overline{\sigma}\left((I + G^2)^{-\frac{1}{4}} \left(\frac{DMD^{-1}}{\beta} - jG \right) (I + G^2)^{-\frac{1}{4}} \right) < 1$$

$$\Updownarrow$$

$$\overline{\sigma}\left((I + G^2)^{-\frac{1}{4}} (DMD^{-1} - j\beta G)(I + G^2)^{-\frac{1}{4}} \right) < \beta$$

$$\Updownarrow$$

$$\overline{\sigma}\left((D_L M D_R - \beta \Gamma) \right) < \beta$$

where $D_L \equiv (I + G^2)^{-\frac{1}{4}} D$, $D_R \equiv D^{-1} (I + G^2)^{-\frac{1}{4}}$, and $\Gamma \equiv j(I + G^2)^{-\frac{1}{4}} G(I + G^2)^{-\frac{1}{4}}$. This last constraint may be rewritten as the linear matrix inequality

$$\begin{bmatrix} \beta I & D_L M D_R - \beta \Gamma \\ (D_L M D_R - \beta \Gamma)^* & \beta I \end{bmatrix} > 0 \qquad (19)$$

Based on this formulation, a second synthesis algorithm with fewer LMI variables may be developed:

Synthesis Algorithm #2: μ-LMI Iteration

1) initialize A
2) With A fixed, minimize β over D and G as specified in Eqn. (14)
3) calculate D_L, D_R, Γ
4) with D_L, D_R, Γ fixed, minimize β over A subject to the constraint

$$\begin{bmatrix} \beta I & D_L M D_R - \beta \Gamma \\ (D_L M D_R - \beta \Gamma)^* & \beta I \end{bmatrix} > 0$$

5) if $\beta < 1$ quit, else return to step 2.

Step #2 may be accomplished in an efficient fashion by using the **mu** command in the MATLAB μ-Toolbox which is highly optimized for this calculation.

While this algorithm is considerably faster than Synthesis Algorithm #1, it can be further accelerated by eliminating the LMI optimization in step (3). As will be shown, *the gain matrix which minimizes β subject to the constraint (19) may be directly calculated.*

To derive the necessary equations, we will first rewrite constraint (19) explicitly indicating its dependence on A

$$\begin{bmatrix} \beta I & D_L \{M_0 + M_1 A M_2\} D_R - \beta \Gamma \\ (D_L \{M_0 + M_1 A M_2\} D_R - \beta \Gamma)^* & \beta I \end{bmatrix} > 0$$

This, in turn, may be rewritten as

$$Q + BAC + (BAC)^* > 0 \qquad (20)$$

where

$$Q \equiv \begin{bmatrix} \beta I & D_L M_0 D_R - \beta \Gamma \\ (D_L M_0 D_R - \beta \Gamma)^* & \beta I \end{bmatrix} \qquad B \equiv \begin{bmatrix} D_L M_1 \\ 0 \end{bmatrix} \qquad C \equiv \begin{bmatrix} 0 & M_2 D_R \end{bmatrix}$$

Constraint (20) will have a solution A if and only if the following projected feasibility conditions are satisfied [Skelton et al, 97]:

$$B^\perp Q B^{\perp^*} > 0 \qquad C^{*\perp} Q C^{*\perp^*} > 0$$

First we will determine the smallest β such that there is a feasible solution. Since Q is an affine function of β

$$Q = Q_0 + \beta Q_1 \qquad Q_0 \equiv \begin{bmatrix} 0 & D_L M D_R \\ (D_L M D_R)^* & 0 \end{bmatrix} \qquad Q_1 \equiv \begin{bmatrix} I & -\Gamma \\ -\Gamma^* & I \end{bmatrix} \quad (21)$$

these projected constraints can be written as

$$B^\perp Q_0 B^{\perp^*} + \beta \left\{ B^\perp Q_1 B^\perp \right\} > 0 \qquad C^{*\perp} Q_0 C^{*\perp^*} + \beta \left\{ C^{*\perp} Q_1 C^{*\perp^*} \right\} > 0$$

Applying the congruence transforms

$$F_B = F_B^* \equiv \left(B^\perp Q_1 B^{\perp^*} \right)^{-\frac{1}{2}} \qquad F_C = F_C^* \equiv \left(C^{*\perp} Q_1 C^{*\perp^*} \right)^{-\frac{1}{2}} \quad (22)$$

to the two constraints yields

$$\beta I + F_B B^\perp Q_0 B^{\perp^*} F_B > 0 \qquad \beta I + F_C C^{*\perp} Q_0 C^{*\perp^*} F_c > 0$$

The minimum value of β subject to this constraint therefore is

$$\beta_{\min} = \max \left\{ \max \operatorname{Re}\left(\lambda_i \left(-F_B B^\perp Q_0 B^{\perp^*} F_B \right) \right), \max \operatorname{Re}\left(\lambda_i \left(-F_c C^{*\perp} Q_0 C^{*\perp^*} F_c \right) \right) \right\} \quad (23)$$

From this, the minimizing Q can be constructed via $Q = Q_0 + \beta Q_1$. Then, a solution corresponding to this value of β can be reconstructed via {Skelton et al, 1998]

$$A = RB^* \Psi C^* \left(C \Psi C^* \right)^{-1} + S^{\frac{1}{2}} L \left(C \Psi C^* \right)^{-\frac{1}{2}} \quad (24)$$

where

$$S = R - RB^* \left[\Psi - \Psi C^* \left(C \Psi C^* \right)^{-1} C \Psi \right] BR$$

$$\Psi = \left[BRB^* + Q \right]^{-1} > 0 \quad (25)$$

$$R = B^+ \left[QB^{\perp^*} \left(B^\perp Q B^{\perp^*} \right)^{-1} B^\perp Q - Q \right] B^{+*} + W^* W$$

and L and W are arbitrary matrices with L satisfying $\overline{\sigma}(L) < 1$.

<u>Synthesis Algorithm #3: μ / Construction Iteration</u>
    1) initialize A
    2) with A fixed, minimize β over D and G as specified in Eqn. (14)
    3) calculate D_L, D_R, Γ
    4) with D_L, D_R, Γ fixed, find the minimum feasible β via Eqn. (23).
    5) Compute a gain matrix A which achieves this β via Eqns. (24) and (25)
    6) if $\beta < 1$ quit, else return to step 2.

With the last value of β determined, a family of robustly stabilizing gain matrices may be generated via Eqns (24) and (25) using the freedom available in the choice of L and W. Further note that this family is affine in the matrix L. It is the hope of the authors that these

degrees of freedom may be exploited to guarantee robust performance for the non-square problem (where the number of sensors exceeds the number of actuators) in which the optimal performance is non-zero.

While Synthesis Algorithm #3 is typically much faster than #2, the second algorithm allows the designer to place constraints upon the gain matrix elements. Such constraints, for example, could be used to develop decentralized adaptive vibration controllers.

It should also be noted that the projection used above may also be applied to Constraint (18) to eliminate the gain matrix A. This yields a nonlinear matrix inequality in D, G, Φ_1, and Φ_2 and an alternative optimization problem.

4. EXAMPLE PROBLEMS

Twenty example problems (i.e., $\tilde{T}$) were generated in such a fashion as to guaranteed that a robustly stabilizing gain matrix existed. These examples had 6 control inputs and 6 sensor outputs. In each case, the uncertainty considered had the following structure:

$$\Delta = \begin{bmatrix} \delta_1 I_{12} & & & \\ & \delta_2 I_2 & & \\ & & \delta_3 I_2 & \\ & & & \delta_4 I_6 \end{bmatrix} \qquad \begin{matrix} \delta_1 \in R \\ \delta_2 \in R \\ \delta_3 \in R \\ \delta_4 \in C \end{matrix}$$

The first parametric uncertainty (δ_1) may be considered to correspond to an "uncertainty" in the operating speed [Knospe,1997c]. Inclusion of this in the structure results in a gain matrix which will be satisfactory over a wide range of operating speeds, reducing the number of elements needed in the gain matrix look-up table. The second two parametric uncertainties (δ_2, δ_3) may be considered to represent uncertainties in the properties of a seal or coupling. The last complex uncertainty (δ_4) may be considered to represent dynamic uncertainty in the amplifiers or actuators.

For each of the 20 example problems, Synthesis Algorithm #3 successfully found a robustly stabilizing gain matrix. In each case, the algorithm was started with a random guess for A. The average time for solution of each problem on a 300 Mhz Pentium II PC was 565 seconds. The maximum and average number of iterations for solution were 3 and 2.05 respectively.

CONCLUSIONS

Three new algorithms were presented for the synthesis of robustly stable AVC gain matrices. All three algorithms involve the iteration between two convex problems, and are therefore monotonically decreasing. None, however, are guaranteed to find a solution if one exists. The last algorithm takes advantage of a projection to eliminate the minimization with respect to elements of the gain matrix. Instead, a direct method of construction of the solution is used. Thus, synthesis is little more than repeated μ calculations. This formulation also allows characterization of a family of robustly stabilizing gain matrices. A number of random example problems demonstrated the synthesis algorithms effectiveness.

ACKNOWLEDGMENTS

REFERENCES

Burrows, C.R. and Sahinkaya, M.N., 1983, "Vibration Control of Multi-Mode Rotor-Bearing Systems," *Proceedings of the Royal Society of London*, Vol. 386, pp. 77-94.

Burrows, C.R., Sahinkaya, M.N., and Clements, S., 1989, "Active Vibration Control of Flexible Rotors: an Experimental and Theoretical Study", in *Proceedings of the Royal Society of London*, Vol. 422, pp. 123-146.

Doyle, J., Packard, A., and Zhou, K., 1991, "Review of LFTs, LMIs and μ," *Proceedings of the 30th IEEE Conference on Decision and Control*, England, pp 1227-1232.

Haberman, H. and Brunet, M., 1984, "The Active Magnetic Bearing Enables Optimum Damping of Flexible Rotors", ASME Paper 84-GT-117.

Higuchi, T., Otsuka, M., Mizuno, T., and Ide, T., 1990. "Application of Periodic Learning Control with Inverse Transfer Function Compensation in Totally Active Magnetic Bearings", in *Proceedings of the 2nd International Symposium on Magnetic Bearings*, Tokyo, Japan, July 12-14, 1990.

Hope, R., 1994, *Adaptive Open Loop Control of Synchronous Rotor Vibration using Magnetic Bearings*, M.S. Thesis, University of Virginia, August.

Hope, R., Miyaji, T., and Knospe, C., "Adaptive Vibration Control of Magnetic Bearing Equipped Industrial Turbomachinery," *Proceedings of MAG97*, Technomics Press, Aug. 21-27, Alexandria, VA.

Knospe, C., Hope, R., Fedigan, S., and Williams, R., 1994, "New Results in the Control of Rotor Synchronous Vibration", in *Proceedings of the Fourth International Symposium on Magnetic Bearings*, vdf Hochschulverlag AG, Zurich, Switzerland, August 23-26.

Knospe, C., Hope, R., Fedigan, S., and Williams, R., 1995a, "Experiments in the Control of Unbalance Response Using Magnetic Bearings," *Mechatronics*, Vol. 5, No. 4, pp 385-400.

Knospe, C., Tamer, S., and Fedigan, S., 1997a, "Robustness of Adaptive Rotor Vibration Control to Structured Uncertainty," *ASME Journal of Dynamic Systems, Measurement, and Control*. June, pp. 243-250.

Knospe, C., Tamer, S., and Fedigan, S., 1997b, "Synthesis of Robust Gain Matrices for Adaptive Rotor Vibration Control," *ASME Journal of Dynamic Systems, Measurement, and Control*. June, pp. 298-300.

Knospe, C., Tamer, S., and Fittro, R., 1997c, "Rotor Synchronous Response Control: Approaches for Addressing Speed Dependence," *Journal of Vibration and Control*, Vol. 3, No. 4, November, pp. 435-458.

Knospe, C., and Tamer, S., 1997d, "Experiments in Robust Control of Rotor Unbalance Response using Magnetic Bearings," *Mechatronics*, Vol. 7, No. 3, pp. 217-229.

Larsonneur, R. and Herzog, R., 1994, "Feedforward Compensation of Unbalance: New Results and Application Experiences", in *IUTAM Symposium on the Active Control of Vibration*, Bath, UK, Sept.

Mathworks, Inc., 1994, *μ-Analysis and Synthesis Toolbox*, Natick, Mass. 01760-1500, January.

Matsumura, F., Fujita, M., and Okawa, K., 1990, "Modeling and Control of Magnetic Bearing Systems Achieving a Rotation around the Axis of Inertia," in *Proceedings of the 2nd International Symposium on Magnetic Bearings*, Tokyo, Japan, July 12-14.

Shafai, B., Beale, S., LaRocca, P., and Cusson, E., 1994, "Magnetic Bearing Control Systems and Adaptive Forced Balancing", *IEEE Control Systems*, Volume 14, No. 2, pp. 4-13, April.

Skelton, R.E., Grigoriadis, K.M., and Iwasaki, T., 1997, *Control System Design: A Unified Algebraic Approach*, Taylor and Francis.