

UNBALANCE CANCELLATION WITH ROTATING REFERENCE CONTROL FOR A HORIZONTAL SHAFT

Jérôme De Miras,¹ Ali Charara¹

ABSTRACT

Considering the nonlinear model of Active Magnetic Bearings, a new means of nonlinear control is presented which treats variables as rotating vectors in the control plane, specially the unbalance. It becomes easy to cancel out the effect of this perturbation in a rotating reference. Therefore, the rotor can rotate around any desired axis, particularly the geometric axis or the axis of inertia. Simulations illustrate the efficiency of this control law.

INTRODUCTION

Non-contacting active magnetic bearings (AMB) are used to control one or two degrees of freedom of a free rotating body in space. In the last few years, active suspensions have become an important method for increasing the performance of industrial applications (Schweitzer et al., 1994). This kind of suspension allows active control of the vibrations created by mechanical imperfections (vibrations due to eccentricity). In the area of active elimination of unbalance vibration, many studies have been done on the linear AMB model obtained when a constant premagnetization bias current I_0 is used in each coil of the AMB, and a linearization is carried out about (I_0, ϵ_0) , where ϵ_0 is the nominal air-gap (linear functioning) (Boldea et al., 1990; Knospe, 1991; Nonami and Yamagushi, 1991; Cho et al., 1993; Herzog et al., 1996; Lum et al., 1996; Rundell et al., 1996). But a more interesting use of nonlinear functioning is in the reduction of the energy consumption, even if the model is nonlinear with a fast open loop instability (Charara et al., 1996)

In this paper, a nonlinear modeling of a horizontal shaft is presented. A nonlinear control voltage based on passivity and rotating references is proposed to obtain an active elimination of unbalance. Some simulations are given to show the performances of this method.

MODELING OF THE PROCESS

The process used for our studie is a horizontal shaft (Figure 1) with two control planes, an axial bearing, and a motor drive to control the rotor's rotation speed. To establish the control law and the simulations, three different models will be used. First, the equations of one control plane only will be taken into consideration in order to build the command (Figure 2). The second model is used to build desired currents and voltage inputs. The third model is a complete modeling of the rotor and actuators dynamics to obtain realistic behaviors in simulations.

¹UTC-HEUDIASYC UMR CNRS 6599, B.P. 20529, 60205 Compiègne Cedex, France, e-mail: jerome.demiras@utc.fr, ali.charara@utc.fr.

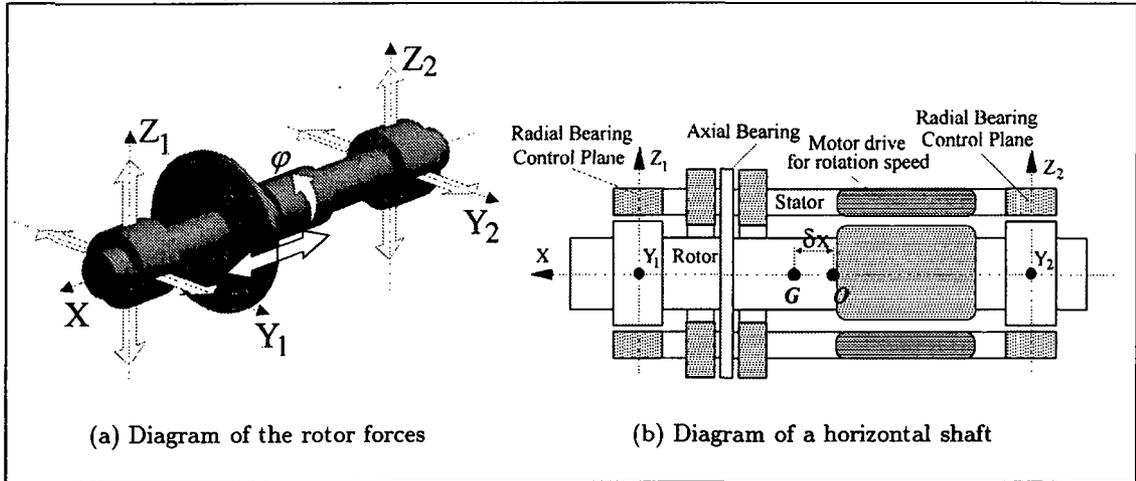


Figure 1: Experimental device

MODEL OF A CONTROL PLANE $y - z$

Consider the Lagrangian equation

$$\frac{d}{dt} \left(\frac{\partial \mathcal{L}}{\partial \dot{q}_i} \right) - \frac{\partial \mathcal{L}}{\partial q_i} = b_i F_i + \frac{\partial \mathfrak{R}}{\partial \dot{q}_i} \quad (1)$$

where q_i are the generalized coordinates, and \mathcal{L} is the difference between the kinetic energy $T(q, \dot{q})$ and the potential energy $U(q)$. T can be defined as $T \triangleq T_m(q, \dot{q}) + T_e(q, \dot{q})$. (The notations are detailed at the end of the paper in Table I). In the first part of this analysis, the potential energy (gravity) is not considered. We shall show later how to compensate for gravity. We have

$$T_m = \frac{1}{2} (m_p V_g^2 + I_x \dot{\phi}^2); \quad T_e = \frac{1}{2} \left(\frac{\lambda i_{y1p}^2}{\epsilon_0 - y} + \frac{\lambda i_{y1m}^2}{\epsilon_0 + y} + \frac{\lambda i_{z1p}^2}{\epsilon_0 - z} + \frac{\lambda i_{z1m}^2}{\epsilon_0 + z} \right) \quad (2)$$

$$q_g = \left[x_g, y_g, z_g, \phi, \int i_{y1p} dt, \int i_{y1m} dt, \int i_{z1p} dt, \int i_{z1m} dt \right].$$

V_g is the translatory motion speed of the center of gravity G . But the location of G is unknown (Figure 2). The only known point is the *geometrical center* O which is given by measurements. So, the following transformation will be used

$$V_g = V_o + \Omega_{C'/C} \wedge [T_{C'/C} P_{G/C'}]$$

$$P_{G/C'} = [0, \delta y, \delta z]^T; \quad \Omega_{C'/C} = (\dot{\phi} \ 0 \ 0)^T; \quad T_{C'/C} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos \phi & -\sin \phi \\ 0 & \sin \phi & \cos \phi \end{pmatrix} \quad (3)$$

$$\Omega_{C'/C} \wedge [T_{C'/C} P_{G/C'}] = \bar{\Omega}_{C'/C} T_{C'/C} P_{G/C'}; \quad \bar{\Omega}_{C'/C} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -\dot{\phi} \\ 0 & \dot{\phi} & 0 \end{pmatrix}$$

where $\Omega_{C'/C}$ is the rotational speed between references C and C' , $T_{C'/C}$ the transformation to take us from C to C' , and $P_{G/C'}$ the position of G in C' . Coordinates of O will replace G .

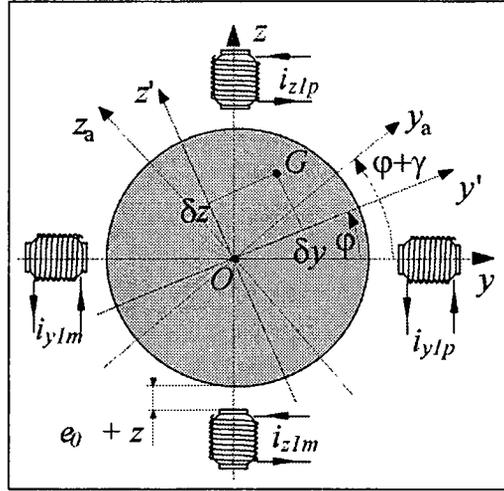


Figure 2: Diagram of a control plane

We can use Equation (3) in Equation (2). It is assumed that $\delta z = 0$ without loss of generality. The following calculation is performed

$$V_g^2 = V_o^2 + P_{G/C'}^T T_{C'/C}^T \bar{\Omega}_{C'/C}^T \bar{\Omega}_{C'/C} T_{C'/C} P_{G/C'} + 2V_o^T \bar{\Omega}_{C'/C} T_{C'/C} P_{G/C'}. \quad (4)$$

Then T_m and T_e can be rewritten as

$$\begin{aligned} T_m &= \frac{1}{2} q_m^T D_m q_m; \\ \dot{q}_m &= [\dot{x}_o, \dot{y}_o, \dot{z}_o, \dot{\phi}]^T; \end{aligned} \quad D_m = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & m_p & 0 & -m_p \delta y \sin(\phi) \\ 0 & 0 & m_p & m_p \delta y \cos(\phi) \\ 0 & -m_p \delta y \sin(\phi) & m_p \delta y \cos(\phi) & m_p \delta y^2 + I_x \end{bmatrix} \quad (5)$$

$$\begin{aligned} T_e &= \frac{1}{2} q_e^T D_e q_e; \\ \dot{q}_e &= [i_{y1}, i_{y2}, i_{z1}, i_{z2}]^T; \end{aligned} \quad D_e = \begin{bmatrix} \frac{\lambda}{e_0 - y} & 0 & 0 & 0 \\ 0 & \frac{\lambda}{e_0 + y} & 0 & 0 \\ 0 & 0 & \frac{\lambda}{e_0 - z} & 0 \\ 0 & 0 & 0 & \frac{\lambda}{e_0 + z} \end{bmatrix}. \quad (6)$$

The complete energy is then

$$T = \frac{1}{2} q^T D q; \quad \dot{q} = [\dot{q}_m, \dot{q}_e]^T; \quad D = \begin{bmatrix} D_m & 0 \\ 0 & D_e \end{bmatrix}. \quad (7)$$

Now, Equation (1) can be used to calculate the dynamic model. It can be written as

$$D(q) \ddot{q} + C(q, \dot{q}) \dot{q} - \frac{\partial \mathcal{R}}{\partial \dot{q}_i} = B F = E \quad (8)$$

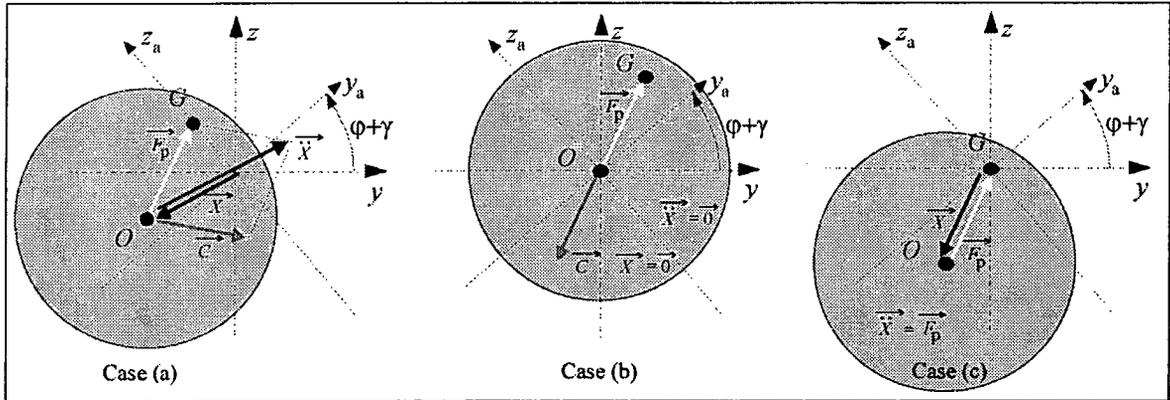


Figure 3: Relative position of different vector items in the control plane.

where $C(q, \dot{q})$ is the Christoffel symbols matrix (Spong and Vidyasagar, 1989). The system can be written as

$$\begin{aligned}
 m_p \ddot{y} &= m_p \delta y \sin(\phi) \ddot{\phi} + m_p \delta y \cos(\phi) (\dot{\phi})^2 + \frac{1}{2} \frac{\lambda i_{1yp}^2}{(\epsilon_0 - y)^2} - \frac{1}{2} \frac{\lambda i_{1ym}^2}{(\epsilon_0 + y)^2} + P_y \\
 m_p \ddot{z} &= -m_p \delta y \cos(\phi) \ddot{\phi} + m_p \delta y \sin(\phi) (\dot{\phi})^2 + \frac{1}{2} \frac{\lambda i_{1zp}^2}{(\epsilon_0 - z)^2} - \frac{1}{2} \frac{\lambda i_{1zm}^2}{(\epsilon_0 + z)^2} + P_z \\
 (m_p \delta y^2 + I_x) \ddot{\phi} &= m_p \delta y \sin(\phi) \ddot{y} - m_p \delta y \cos(\phi) \ddot{z} \\
 \left. \begin{aligned}
 \frac{\lambda i_{1\eta p}}{\epsilon_0 - \eta} &= -\frac{\lambda i_{1\eta p} \dot{\eta}}{(\epsilon_0 - \eta)^2} - R i_{1\eta p} + E_{1\eta p} \\
 \frac{\lambda i_{1\eta m}}{\epsilon_0 + \eta} &= \frac{\lambda i_{1\eta m} \dot{\eta}}{(\epsilon_0 + \eta)^2} - R i_{1\eta m} + E_{1\eta m}
 \end{aligned} \right\} \text{with } \eta = y \text{ or } z.
 \end{aligned}
 \tag{9}$$

In Equation (9) the first and second terms are respectively perturbed by $\delta y \sin(\phi) \ddot{\phi} + \delta y \cos(\phi) (\dot{\phi})^2$ and $-\delta y \cos(\phi) \ddot{\phi} + \delta y \sin(\phi) (\dot{\phi})^2$. Usually, the modeling is done without considering the difference in position between G and O ; the unbalance perturbation is then an external force that is added in the mechanical expressions. But from our standpoint, it is clear that:

- the perturbation is a constant (if $\dot{\phi}$ is constant) in a reference C_a centered on O (2) turning with speed $\dot{\phi}$ and dephased by γ from C' . This attractive property will be used immediately to formulate a very simple control law. In C_a , the expression of the perturbation is

$$F_{p/C_a} = \delta y \begin{bmatrix} -\sin(\gamma) \ddot{\phi} + \cos(\gamma) \dot{\phi}^2 \\ -\cos(\gamma) \ddot{\phi} - \sin(\gamma) \dot{\phi}^2 \end{bmatrix}. \tag{10}$$

- the direction of this perturbation is \overrightarrow{OG} in permanent functioning as shown in Figure 3(a). The acceleration vector \overrightarrow{X} of O is equal to the sum of the command vector \overrightarrow{C} and the perturbation vector $\overrightarrow{F_p}$.

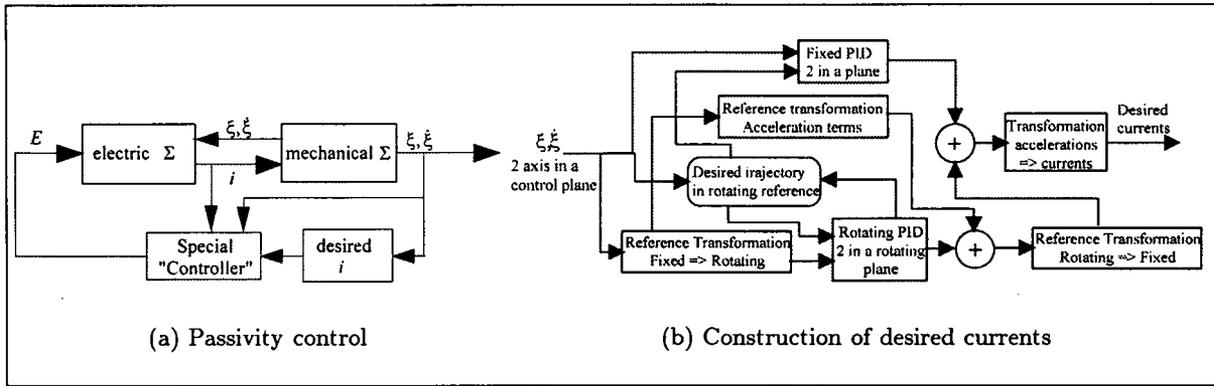


Figure 4: Control law

SIMULATION MODEL OF THE SHAFT WITH UNBALANCE

The simulation model is obtained using the same method. The center of gravity is not on the geometrical axis and the inertial rotating matrix I can be full. Equations cannot be shown here because there are too large; they contain a lot of coupling between all the generalized coordinates.

MODEL OF THE SHAFT WITHOUT UNBALANCE

Here G is supposed to be on the geometrical axis x of the rotor. The distance between G and O is δx . The inertial rotating matrix is diagonal.

$\dot{q} = [\dot{x}, \dot{y}_1, \dot{z}_1, \dot{y}_2, \dot{z}_2, \dot{\phi}, i_{1yp}, i_{1ym}, i_{1zp}, i_{1zmp}, i_{2yp}, i_{2yp}, i_{2zp}, i_{2zm}]$ are the generalized coordinate derivatives, with y_1 and z_1 the coordinates of the geometrical axis in the first control plane and y_2 and z_2 in the second control plane. An equation equivalent to Equation (8) is obtained

$$\begin{bmatrix} D_{2m}(q) & 0 \\ 0 & D_{2e}(q) \end{bmatrix} \begin{bmatrix} \ddot{q}_m \\ \ddot{q}_e \end{bmatrix} + C_2(q, \dot{q}) \begin{bmatrix} \dot{q}_m \\ \dot{q}_e \end{bmatrix} - \begin{bmatrix} 0 \\ \frac{\partial \mathcal{R}}{\partial \dot{q}} \end{bmatrix} = BF = \begin{bmatrix} 0 \\ E \end{bmatrix} \quad (11)$$

This model will be useful to compute the desired currents in the following section.

CONTROL LAW

The voltage control law $[E_{1yp}, E_{1ym}, E_{1zp}, E_{1zm}, E_{2yp}, E_{2ym}, E_{2zp}, E_{2zm}]$ is obtained by using an interesting property of Lagrangian systems which is a consequence of passivity (Desoer and Vidyasagar, 1975). A special kind of *controller* which can make globally converge the system's state is built. In this *controller*, it is possible to impose the values of the desired currents. This desired currents will be chosen to have the stability of the desired equilibrium point (Figure 4).

THE VOLTAGE PART

Consider the system with the following form

$$D(q) \ddot{q} + C(q, \dot{q}) \dot{q} + R\dot{q} = E. \quad (12)$$

The *controller* which contains the desired state variables is given by

TABLE I: NOMENCLATURE

G	center of Gravity	O	Geometric center
x_g, y_g, z_g	Position center of gravity	ϕ	Rotating position
i_α	Current (in coil α)	ϵ_0	Nominal air-gap
m_p	Rotor mass	I	Inertial rotating Matrix
R	Resistance	r, K_p, K_d, K_i	Coefficient control
C	Fixed reference	C', C_a	Rotating references
\mathcal{L}	Lagrangian value	T	Kinetic energy
T_m	Mecanical kinetic energy	T_e	Electical kinetic energy
U	Potential energy	\mathfrak{R}	Dissipation energy
q	Generalized coordinates	q_m	Mecanical gen. coordinates
q_e	Electrical gen. coordinates	γ	Offset angle
$E = BF$	Voltage control	$\delta x, \delta y, \delta z$	Position of G in rotor reference
λ	Force coefficient of the coils	P_y, P_z	Gravity force

$$D(q) \ddot{q}_d + C(q, \dot{q}) \dot{q}_d + R \dot{q}_d + r \dot{\tilde{q}} = E \quad (13)$$

$$\tilde{q} = q_d - q.$$

From the difference between Equations (12) and (13), and applying the following Lyapunov function

$$H = \frac{1}{2} \dot{\tilde{q}}^t D \dot{\tilde{q}} \quad (14)$$

the asymptotic convergence can be demonstrated, because the following property is a consequence of the Lagrangian structure:

$$\dot{\tilde{q}}^t (\dot{D} - 2C) \dot{\tilde{q}} = 0. \quad (15)$$

The derivative of the Lyapunov function is

$$\dot{H} = -\dot{\tilde{q}}^t (r + R) \dot{\tilde{q}} \quad (16)$$

with $(r + R)$ strictly positive definite. The voltage control law is the left part of Equation (13).

THE CURRENT PART

The desired currents are computed to stabilize the desired equilibrium point. Consider that the accelerations of the mechanical coordinates can be chosen. PID controllers can be used to obtain these accelerations. Knowing that constant perturbations (gravity) and rotating perturbation have to be cancelled, the sum of two kinds of PID will be used for each axis. First, standard PID's will regulate the difference between a real position and the desired position in the fixed reference C . The convergence is ensured by choosing appropriate values for K_p, K_d and K_i .

$$\ddot{\varrho}_d = K_p \varepsilon_{\varrho} + K_d \dot{\varepsilon}_{\varrho} + K_i \int \varepsilon_{\varrho} dt; \quad \varrho_d : \text{desired position} \quad (17)$$

$$\varepsilon_{\varrho} = \varrho_d - \varrho; \quad \varrho : y_1, z_1, y_2, \text{ or } z_2 \text{ (positions in the fixed references)}$$

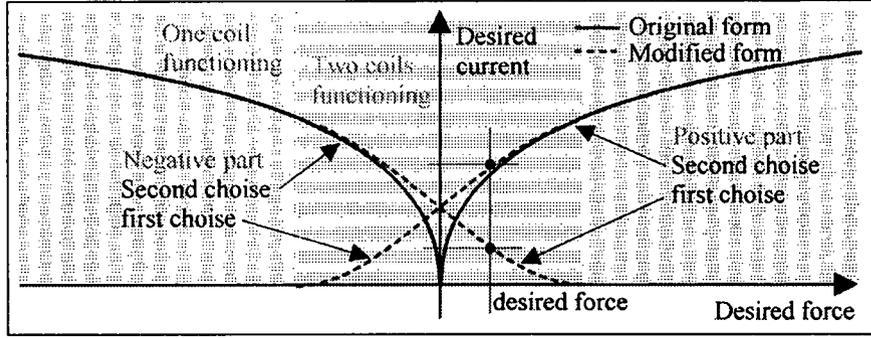


Figure 5: Current choise

A second PID is used on each axis of the rotating reference C_a . Writing

$$\ddot{\varrho}_{ad} = K_{ap}\varepsilon_{a\varrho} + K_{ad}\dot{\varepsilon}_{a\varrho} + K_{ai} \int \varepsilon_{a\varrho} dt; \quad \varrho_{ad} : \text{desired position}$$

$$\varepsilon_{a\varrho} = \varrho_{ad} - \varrho_a; \quad \varrho_a : y_{a1}, z_{a1}, y_{a2}, \text{ or } z_{a2} \text{ (positions in the rotating references } C_a) \quad (18)$$

the convergence is ensured by choosing appropriate values for K_{ap} , K_{ad} and K_{ai} . The constant (or slowly variable) perturbation in this reference will be cancelled by the Integrator term without knowing anything about the unbalance. Knowing the position of the rotor independently of the initial condition is sufficient. The desired accelerations in C_a are known, but we need it in the fixed reference C . We have the following reference transformation

$$\begin{bmatrix} y_i \\ z_i \end{bmatrix} = ROT[\phi + \gamma] \begin{bmatrix} y_{ai} \\ z_{ai} \end{bmatrix}; \quad ROT[\phi + \gamma] = \begin{bmatrix} \cos(\phi + \gamma) & -\sin(\phi + \gamma) \\ \sin(\phi + \gamma) & \cos(\phi + \gamma) \end{bmatrix}$$

with $i = 1$ or 2 (19)

and then the desired accelerations in the fixed reference are

$$\begin{bmatrix} \ddot{y}_{id} \\ \ddot{z}_{id} \end{bmatrix} = ROT[\phi + \gamma] \left(\begin{bmatrix} \ddot{y}_{aid} - y_{ai}\ddot{\phi}^2 \\ \ddot{z}_{aid} - z_{ai}\ddot{\phi}^2 \end{bmatrix} + \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} \left(\begin{bmatrix} 2\dot{y}_{ai}\dot{\phi} + y_{ai}\ddot{\phi} \\ 2\dot{z}_{ai}\dot{\phi} + z_{ai}\ddot{\phi} \end{bmatrix} \right) \right)$$

with $i = 1$ or 2 . (20)

For each axis of the 2 fixed references (2 for the 2 conrol plane), we have now 2 desired accelerations, one with the fixed PID and one with the rotating PID. It is possible to sum these two accelerations because there actions are linear.

$$\ddot{q}_{md} = \ddot{q}_{fr} + \ddot{q}_{rr}$$

\ddot{q}_{fr} : acceleration from PID in fixed reference (21)

\ddot{q}_{rr} : acceleration from PID in rotating reference

The relation between the accelerations and the desired currents is obtained using the mechanical part of Equation (11). The inertial matrix is multiplied by the desired accelerations and terms of $C_2 \dot{q}$ which do not contain currents ($C_{2 \text{ part } 1} \dot{q}$) are added as follow:

TABLE II: SIMULATION VALUES

δy	$= 10^{-5} m$	m_p	$= 1 kg$
R	$= 0.5 \Omega$	λ	$= 1.16 \cdot 10^{-6}$
ϵ_0	$= 0.3 \cdot 10^{-3} m$	I_x	$= 0.05 kg.m^2$
$I_y = I_z$	$= 0.1 kg.m^2$	$I_{xy} = I_{yz} = I_{xz}$	$= 1.10^{-5} kg.m^2$
γ	$= \pi/4 rad$	Sampling Period	$= 200 \cdot 10^{-6} s$

$$D_{2m} \ddot{q}_{md} + C_{2 \text{ part1}} \dot{q} = \mathcal{F}(\mathcal{I}_{id}) \quad (22)$$

where $\mathcal{F}(\mathcal{I}_{id})$ is an affine function with respect to \mathcal{I}_{id} which has the following form

$$\mathcal{I}_{id\eta\kappa} = \frac{i_{\kappa\eta p}^2}{(\epsilon_0 - g(\eta))^2} - \frac{i_{\kappa\eta m}^2}{(\epsilon_0 + g(\eta))^2} \quad (23)$$

η : y or z and $\kappa = 1$ or 2

p : current in coil to create positive force

m : current in coil to create negative force

$$\mathcal{I}_{id} = [\mathcal{I}_{idy1} \ \mathcal{I}_{idy2} \ \mathcal{I}_{idz1} \ \mathcal{I}_{idz2}]^T = \mathcal{F}^{-1}(D_{2m} \ddot{q}_{md} + C_{2 \text{ part1}} \dot{q}) \quad (24)$$

To obtain the desired current, it is necessary to invert the matrix \mathcal{F} . In each equation $\mathcal{I}_{id\eta\kappa}$, two current values have to be found with only one acceleration. This degree of freedom is used to smooth the behavior of the square root function used to compute the current from $\mathcal{I}_{id\eta\kappa}$ (Equation (23)). For example, if the desired forces (right part of Equation (24)) is positive then the the current of the negative part of the axis will be chosen as a polynomial interpolation between zero and a fixed chosen value (see Figure 5). When one of the two currents is chosen, the other is easy to compute (Equation 25).

$$i_{\kappa\eta p} = \sqrt{(\epsilon_0 - g(\eta))^2 \left(\mathcal{I}_{id\eta\kappa} + \frac{i_{\kappa\eta m}^2}{(\epsilon_0 + g(\eta))^2} \right)} \quad (25)$$

SIMULATIONS

In the simulations, the shaft model is implemented. The rotor speed is shown on Figure 6(a). Figure 6(a) shows the response of the system in the fixed reference. The same positions are shown in Figure 6(b) for the rotating reference and in Figure 6(c) for the control plane. It can be seen that the gravity effect on z axis is cancelled at the beginning of the simulation. This action of Integral term in the fixed PID give the values of 1A on the currents i_{z1p} and i_{z2p} (Figure 7(a)). Until 0.7 seconds, the desired position is zero; the situation obtained is shown in Figure 3(b). At 0.7 seconds the reference position changes from zero to a circle whose center is a point of the inertial axis (Figure 3(b)). Following this desired position, the desired accelerations in the rotating reference become zero. Figure 7(a) and 7(b) show the electric variables. The low value for the requiral current when the rotor turns about its axis of inertia is the result of the term $C_{2 \text{ part1}} \dot{q}$. At 4 seconds, a constant perturbation torque is applied to the z axis to test the response of the fixed PID when the rotor speed is not zero.

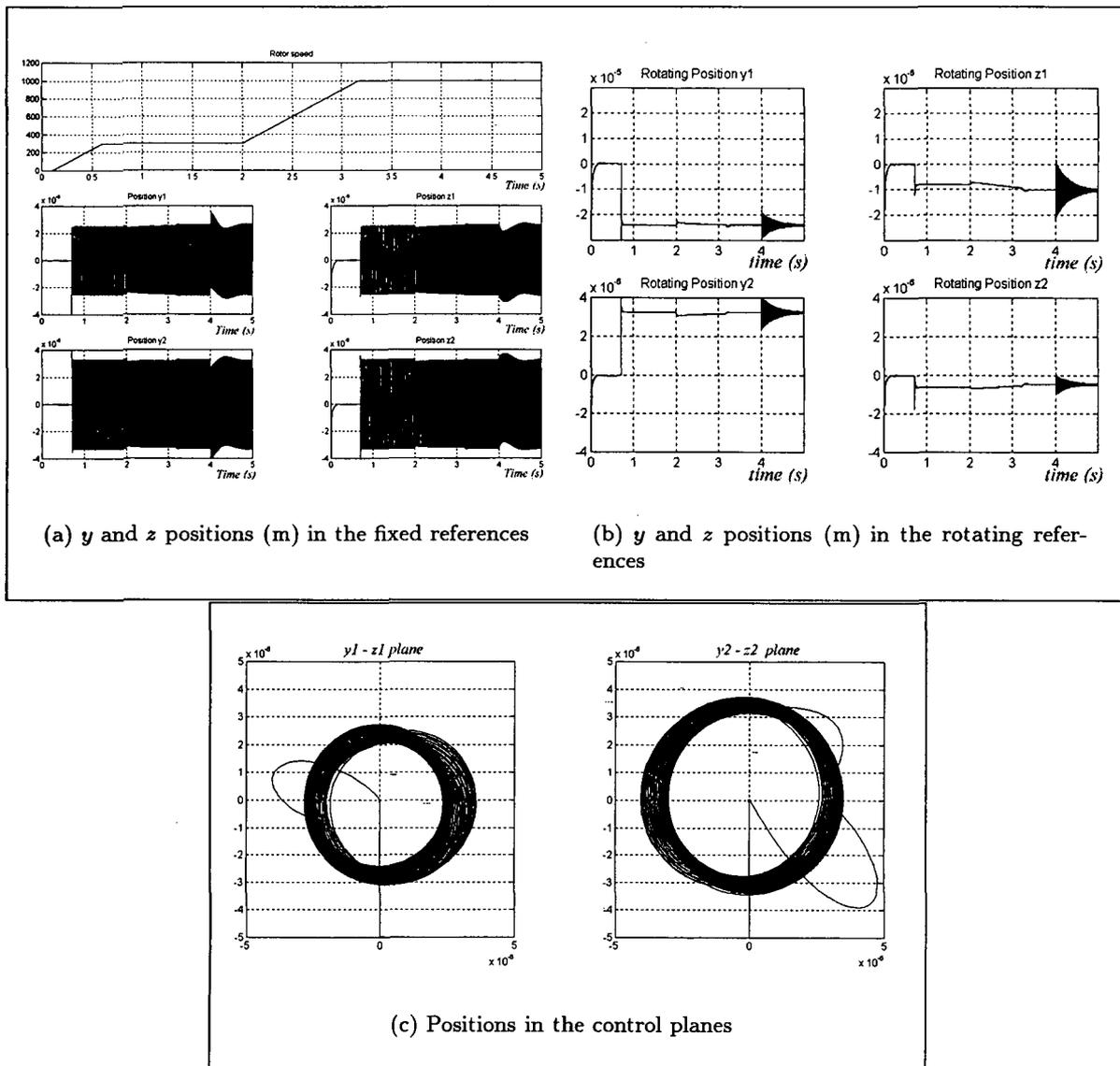


Figure 6: Rotor speed (rd/s) and positions y_1, z_1, y_2, z_2 (m)

CONCLUSION

In this paper, a nonlinear control law based on a rotating reference is proposed to cancel unbalance. This control law allows the rotor to turn about any desired rotor axis in the x direction (if we except the energy aspect), in particular the geometrical axis or the axis of inertia. At the present time, only simulations are available, but experimental tests are under construction.

REFERENCES

Boldea, I., Marcov, N., Trica, A., and Schulhoff, G. 1990 . High-speed induction motor drive with active magnetic bearings and sliding mode decentralized control. *Electr. Machines and Power Syst.*, pages 375–382.

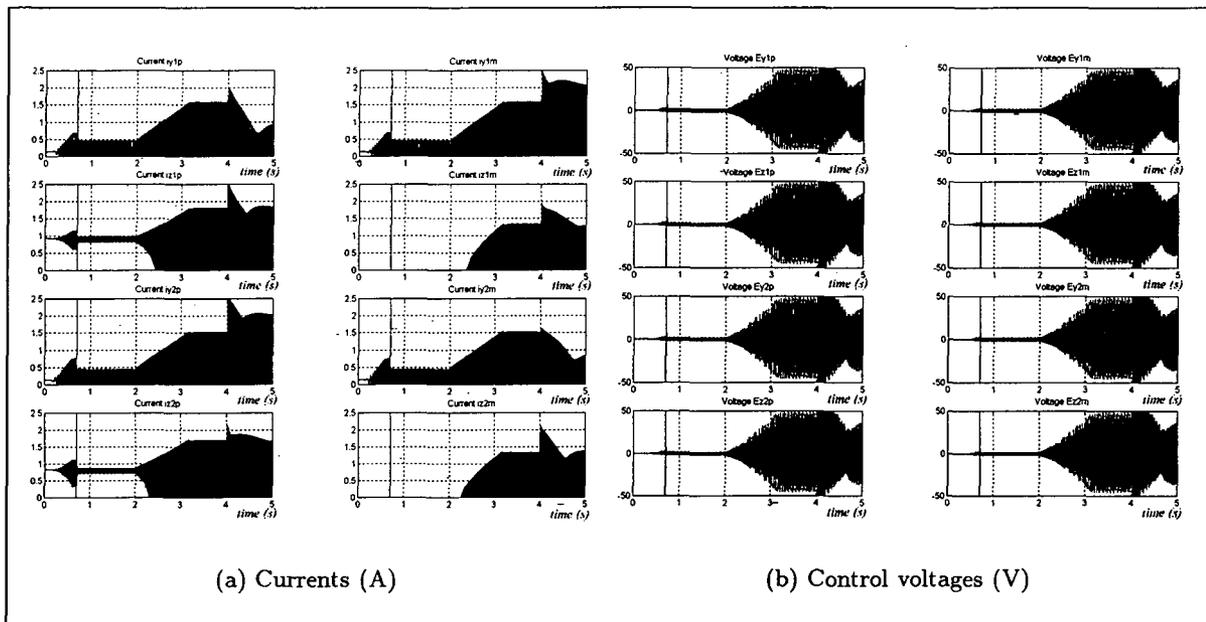


Figure 7: Desired currents and voltage control

Charara, A., Miras, J. D., and Caron, B. 1996 . Nonlinear control of a magnetic levitation system without premagnetization. *IEEE Transactions on Control Systems Technology*, 4(5):513–523.

Cho, D., Kato, Y., and Spilman, D. 1993 . Sliding mode and classical controllers in magnetic levitation system. *IEEE Control Systems Mag.*, pages 42–48.

Desoer, C. A. and Vidyasagar, M. 1975 . *Feedback systems: Input-Output Properties*. Academic Press New York.

Herzog, R., Bühler, P., Gähler, C., and Larssonneur, R. 1996 . Unbalance compensation using generalized notch filters in the multivariable feedback of magnetic bearings. *IEEE Transactions on Control Systems Technology*, 4(5):580–586.

Knospe, C. R. 1991 . Stability and performance of notch filter control for unbalance response. In *Proc.Int. Symp. Magn. Suspension Technol.*, NASA Langley Research Center.

Lum, K.-Y., Coppola, V., and Bernstein, D. 1996 . Adaptive auto-centering control for an active magnetic bearing supporting a rotor with unknown mass imbalance. *IEEE Transactions on Control Systems Technology*, 4(5):587–597.

Nonami, K. and Yamagushi, H. 1991 . Robust control of magnetic bearing systems by means of sliding mode control. In *Proc.Int. Symp. Application Electromagnetic Forces - ISEM'91*, Sendai, Japan.

Rundell, A., Drakunov, S., and Carlo, R. D. 1996 . A sliding mode observer and controller for stabilization of rotational motion of a vertical shaft magnetic bearing. *IEEE Transactions on Control Systems Technology*, 4(5):598–608.

Schweitzer, G., Bleuler, H., and Traxler, A. 1994 . *Active Magnetic Bearings: Basics, Properties and Applications of Active Magnetic Bearings*. v.d.f. Edition, Zürich.

Spong, M. W. and Vidyasagar, M. 1989 . *Robot Dynamics and Control*. Wiley New York.