# NONLINEAR ROTORDYNAMIC BEHAVIOUR OF ROTORS ON ACTIVE MAGNETIC BEARINGS

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## ABSTRACT

A Jeffcott rotor is used to study the rigid body rotordynamic behavior of a rotor on active magnetic suspension. The nonlinearity introduced by the electromagnetic force actuator as well as by the driving power amplifier are expected to cause a softening type behavior for the rotor and anysotropic behavior of the bearing system. A nonlinear model is used to perform numerical time domain simulations for low bias current and the presence of lateral loading. The resuts obtained show that the nonlinearities introduced into the the system are usually not very strong, at least when the airgap is not very large, but the current saturation effects increase the softening character of the response of the system.

**Keywords:**

Active magnetic bearings, Rotordynamics, Nonlinear dynamics.

## INTRODUCTION

Active magnetic bearings are intrinsically nonlinear devices, not only because the force the actuators exert on the rotor is a function of the square of the current, but also because nonlinearities are present in the in the control loop and mainly in the power amplifiers. Nevertheless they are usually designed as linear devices, relying on the bias current superimposed to the control current to obtain a physical linearization of the current to force characteristic of the actuators and on working conditions which are far enough from saturations and other nonlinear phenomena. The linearized dynamics of active magnetic bearings is, at least for the basic control architectures, a consolidated engineering field ([1], [2]).

However, if large displacements are considered, a nonlinear behavior of the softening type has been found, e.g. with a backbone sloping to the left and with the presence of jumps, i.e. regions in which subharmonics of the synchronous whirling appear [3], [4]. The nonlinear behavior is stronger in the case of rotors operating with static lateral forces (e.g. horizontally operated) in which the bias current is lower than the current needed to

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counteract static forces. It is somehow customary to refer to bearings operating in a regime in which the bias current is higher than the current needed to counteract the static loads as class A bearings, while in class B actuators the bias current is smaller. In the former both the counterfaced coils are always active while in the latter case the coil exerting a force in the same direction of the static load is inactive, at least in case of small whirling orbits. Intermediate situations in which the coils are alternatively switched on and off during a whirling orbit, giving way to strong nonlinearities, are possible. In principle magnetic bearings working with a single active coil induce an anisotropic behavior of the system [7].

Strictly speaking, while the linearization of the behavior of class A bearings is substantially valid for small radial displacements, that of class B bearings never holds. However, it has been shown that the nonlinearities due to the actuators can be not much strong even in the case of class B bearings operating with journal orbits which are a substantial fraction of the air gap [8]. Such results have been obtained for a Jeffcott rotor running on active magnetic bearings controlled with a stationary position and velocity feedback, i.e an ideal PD controller with ideal sensors and ideal power amplifiers. Owing to the nonlinearity of the system, no general results have been obtained, as the analysis was based on the time-domain numerical integration in a number of cases.

The aim of the present paper is to extend the study the behavior of a Jeffcott rotor supported on magnetic bearings operating in class B or mixed regime, introducing a more realistic model of the control loop, to add the effect of the nonlinearities introduced by the latter to those which can be ascribed to the magnetic actuators only. The Jeffcott rotor is the simplest rotordynamic model that takes the main phenomena of unbalance and rigid body modes into account. Its behavior approximates realistically the actual one of a real rotor slowly accelerated through the rigid body critical speeds to operate in the subcritical flexible body speed range [5, 6].

## MODEL DESIGN AND IMPLEMENTATION

Using real coordinates, the equations of motion of a Jeffcott rotor supported by magnetic bearings and rotating at constant speed $\omega$ (Figure 1) are the usual ones [2]

$$\begin{cases} m\ddot{x} = m\epsilon\omega^2\cos(\omega t) + F_{c_x} + F_{n_x} \\ m\ddot{y} = m\epsilon\omega^2\sin(\omega t) + F_{c_y} + F_{n_y} \end{cases}, \tag{1}$$

where $\epsilon$ is the eccentricity and $F_c$ and $F_n$ are the control forces applied by the bearings and the static forces respectively.

Neglecting the saturation of the magnetic circuit, the control force due to the $j$-th electromagnet can be approximated by the expression

$$F_{c_j} = K_j\left(\frac{i_j}{c_j}\right)^2, \tag{2}$$

where $i_j$ and $c_j$ are respectively the current flowing in the coil and the air gap while $K_j$ is a coefficient which summarizes the electromagnetic characteristics of the actuator. As a first approximation, it can be considered as a constant, at least until saturation of the iron core is reached.

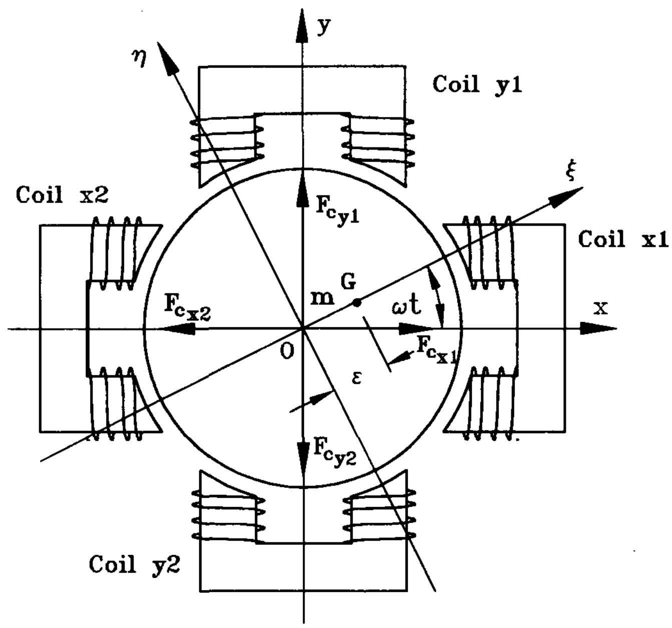


Figure 1: Sketch and notation of the Jeffcott rotor running on active magnetic bearings.

The total control force acting on the rotor can be expressed as

$$\begin{cases} F_{c_x} = K_x \left[ \left( \frac{i_{x1}}{c_{x_1}} \right)^2 - \left( \frac{i_{x2}}{c_{x_2}} \right)^2 \right] + \alpha K_y \frac{x}{c} \left[ \left( \frac{i_{y1}}{c_{y_1}} \right)^2 + \left( \frac{i_{y2}}{c_{y_2}} \right)^2 \right] \\ F_{c_y} = K_y \left[ \left( \frac{i_{y1}}{c_{y_1}} \right)^2 - \left( \frac{i_{y2}}{c_{y_2}} \right)^2 \right] + \alpha K_x \frac{y}{c} \left[ \left( \frac{i_{x1}}{c_{x_1}} \right)^2 + \left( \frac{i_{x2}}{c_{x_2}} \right)^2 \right] \end{cases}, \tag{3}$$

where $\alpha$ is a geometrical coupling coefficient which takes into account the crosscoupling between $x$ and $y$ axes which is present when the pole pieces are located around the rotor as in Figure 1 [3] and $c$ is the air gap, which is assumed to be constant.

When the bearing works near the center of the air gap this coupling can be neglected. For large displacements its effects can be larger and cause an increase of the amplitude of whirling at speeds lower than those for which the jump occurs. At higher speeds they reduce the amplitude of the response. In [3] is suggested that coupling can be beneficial when the eccentricity is large. In the present work this coupling will be neglected as it was done in [8], as the aim is to focus on the effect on the nonlinearities occurring even at small amplitudes; moreover in class B bearings the bias current is low and this reduces the importance of cross coupling.

As it is well known, magnetic bearings introduce a negative stiffness on the rotor that compels the use of an output feedback control loop whose basic dinamics are those of a PID controller to obtain positive stiffness, i.e stability, damping and static load compensation. The controller actually modulates the current the power amplifier force into the coils.

The power amplifier is internally fedback in order to make it work as a transconductance amplifier, i.e. to behave as a voltage driven current generator, the driving voltage being the output of the controller filter (Figure 2). The usually high inductive load of the electromagnetic coils is the cause of the voltage saturation of the power amplifier. Being $Z_L(s) = R + sL$ the coil impedance, with $R$ and $L$ the coil resistance and inductance

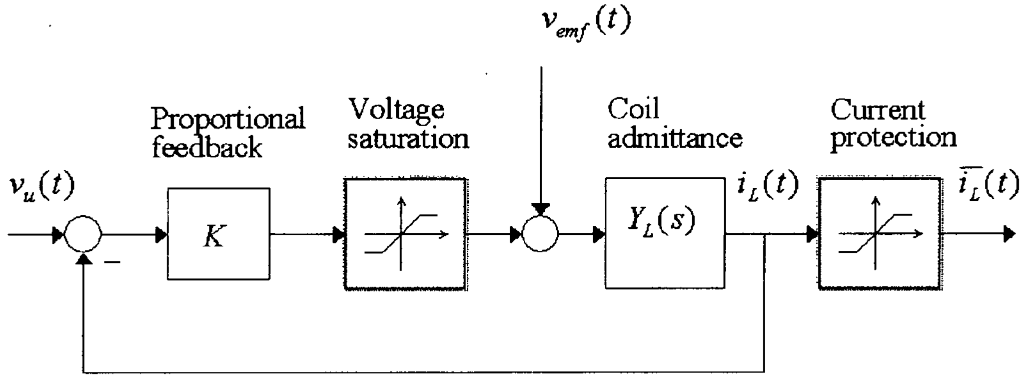


Figure 2: Transconductance amplifier feedback loop with nonlinearities.

respectively, the absolute value of the voltage across it is the following

$$|v_L(t)| = |R + j\omega L||i_L(t)| \tag{4}$$

which clearly cannot exceed that of the voltage external supply. To be noted that $\omega$ is the actual spin speed since the main dynamic control force is that generated to constrain the rotor subject to its unbalance forces. This phenomenon can be equivalently described as current slew rate limitation for the transconductance amplifier whose effect is usually maximum at the critical speeds crossings.

However, for a well-designed power amplifier, the effect of the current slew rate, as well as the back-electromotive force, are reduced by the current feedback loop. The nonlinear characteristic of the saturation function may be thought in terms of reduced slope, i.e. equivalent static gain reduction. On the other side, the decreased influence of a gain variation inside a feedback loop for the overall system is a well known result of classical control theory described, frequency by frequency, by the sensitivity function:

$$S_k^T(s) = \frac{\frac{\partial T}{T}}{\frac{\partial k}{k}} = 1 - T(s) \tag{5}$$

where $T(s) = \frac{L(s)}{1+L(s)}$ is the transmission function, i.e. that describing the dynamic behavior of the transconductance amplifier beetween modulating input voltage and load current output, with $L(s)$ the so-called loop function, i.e. the series product of all the functions found along the control loop.

In practice, the actual limiting function of the transconductance amplifier is its current protection which strongly depends on the engineering specification followed in the construction of the power amplifier electronics. Usually they follows from size, thermal and economic constraints on the active magnetic bearings. Again, for a well- designed system, the adjustable current protection should be the primary nonlinear factor in the active magnetic bearing control loop together with the quadratic characteristic of the magnetic force function. This is the case for the simulation model assumed in the following (Figure 3).

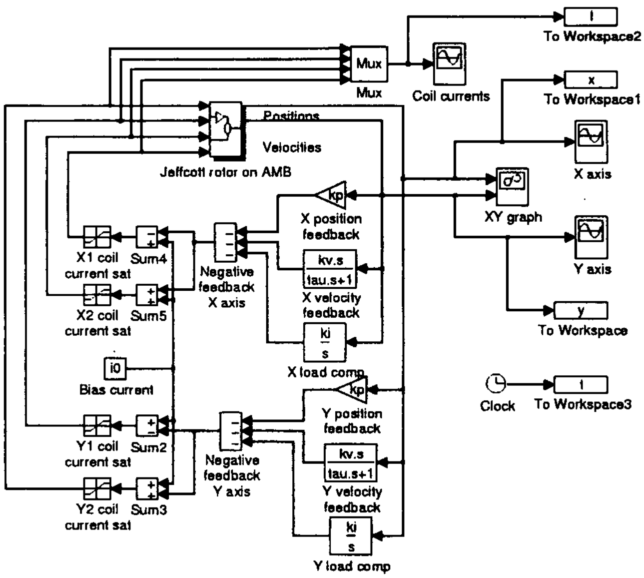


Figure 3: Functional block scheme for the Jeffcott rotor on active magnetic bearings.

The control force on the $j$th axis ($j = 1, 2$), including the presence of the current protection, is thus analitically expressed as

$$F_{c_j} = K_j \left\{ \left[ \frac{\min(i_{j_p}, \max(i_{j_0} + i_j, 0))}{c - x_j} \right]^2 - \left[ \frac{\min(i_{j_p}, \max(i_{j_0} - i_j, 0))}{c + x_j} \right]^2 \right\}, \qquad (6)$$

where $i_{j_p}$ is the current limitation/protection for the single coil of the $j$-th axis, csubscript 1 refers to $x$-axis ($x_1 = x$) and subscript 2 to $y$-axis ($x_2 = y$)

Differently from what done in [8] where a static position and velocity (assumed as directly available), i.e. an ideal (noncausal) PD controller, was used, a more realistic PID controller is here adopted:

$$C_{PID}(s) = K_p + \frac{K_p}{T_i s} + \frac{K_d s}{\tau_d s + 1} \qquad (7)$$

where $K_p$ is the controller stationary gain, $T_i$ the integrative time constant, $K_d/K_p$ the derivative time constant and $\tau_d$ the time constant of the derivative causal pole. Note that most of the actual controllers used in real active magnetic suspensions are only a variation of the PID scheme with the derivative action carefully distribuited on the frequency range of interest [10].

## NUMERICAL SIMULATIONS

Owing to the nonlinear nature of the system and to the large number of parameters entering into the model, no general results can be given and each point in the multi-dimensional parameter space has its own typical behavior. In the following only few cases will be studied, in order to get an insight on the typical behavior of the system.

Consider the same Jeffcott rotor running in isotropic magnetic bearings studied in [8], having the following data: mass $m = 2.5$ kg, bearing constant $K_x = K_y = 1.5 \times 10^{-6}$ $Nm^2/A^2$, clearance $c = 0.5$ mm, unbalance grade $G = 63$ at 20000 rpm, corresponding to an eccentricity $\epsilon = 30$ $\mu$m. The data are taken from an experimental test machine which is used by the authors for basic research work on magnetic bearings [9]; however a value of the eccentricity well in excess of the actual one was assumed. The data of the PID controllers are: $K_p = 9400$, $T_i = 0.1$, $K_d = 8.4$ and $\tau_d = 1 \cdot 10^{-4}$.

**Simulation #1**

A first simulation has been run in conditions which are close to those previously studied: the rotor is assumed to be in horizontal position ($y$-axis in vertical direction) and a low bias current is used ($i_{x_0} = i_{y_0} = 0.5$ A). The power amplifier protection is set high enough (at a value of 5 A) to prevent the amplifier from saturation. In these conditions the only differences between the present solution and the previous one are due to the way in which the derivative action is originated and the presence here of an integrative action instead of an imposed compensation current.

The linearized critical speeds are 201 rad/s and 228 rad/s for the $xz$ and $yz$ planes respectively. The unbalance response from 120 to 500 rad/s has been computed by simulating the motion for a number of orbits at selected values of the speed. It is reported in Figure 4a, together with the results obtained using the ideal controller in [8] and the backbone and limit envelope computed using a series expansion of the bearing forces (always with an ideal PD controller).

The response computed using a more realistic model of the controller is quite close to that obtained previously, except near the crossing of the linearized critical speeds. It is clear that the damping action of the PID controller is smaller than that of the idealized PD controller, resulting in a larger and more elongated elliptical orbit at the crossing of the critical speeds, and in higher currents. This effect was predictable.

The orbit at the crossing of the critical speed referred to $xy$ plane (namely at 200 rad/s) is reported in Figure 4b: as already stated the system behaves in a very anisotropic way, with a damping barely sufficient to prevent backward whirling (the orbit almost degenerates to a straight line). The total currents ($i_{j_0} + i_j$) in the four coils are reported in Figures 4c and 4d. The fact that the system operates as a class B bearing in $yz$ plane is clear: the lower coil never operates. The coils in $xz$ plane are alternatively switched on and off.

The integrative action is sufficient to allow the rotor to operate in the geometrical center of the bearing, compensating the weight of the system.

All attempts to decrease substantially the value of the protection current resulted in incorrect working of the rotor: the current needed to carry the load is of about 2 A and a suitable margin is needed to work during the transient occurring at the beginning of each solution. With a protection current as low as 2.3 A the rotor can work in the high supercritical range, but far higher values are needed to work in the vicinity of the critical speed. No results of these attempts is reported here.

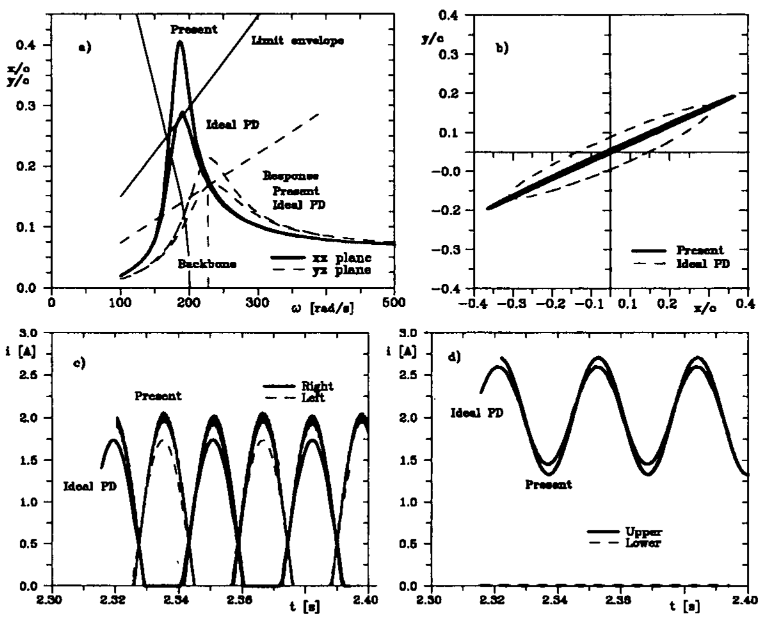

Figure 4: Simulation results for a class B bearing. (a): unbalance response, with backbone and limit envelope; (b): orbit at a speed of 200 rad/s; (c): total currents in the coils in $xz$ plane as functions of time at 200 rad/s; (d): as (c), but for $yz$ plane.

## Simulation #2

A second simulation has been performed by assuming that the rotor is still in horizontal position, but that $x$- and $y$-axes are at 45° from the vertical direction. A low bias current is again used ($i_{x_0} = i_{y_0} = 0.4$ A).

The unbalance response for the speed range 100 - 400 rad/s is shown in Figure 5. The response was computed using both the idealized PD controller and the present PID controller.

The numerical simulation was performed setting the protection current at 5 A, a value high enough for the protection never to act, and at 2.4 A. In the former case the response is only slightly nonlinear and differs from the idealized case only for the lower damping action, leading to a larger response at the crossing of the critical speed. Lowering the protection current the behavior gets more nonlinear, and at 2.4 A a strong softening pattern is clearly visible, with even a jump taking place. The full line refers to a simulation in which the speed is slowly increased, while the dashed line to one performed at decreasing speed. The jump occurs at different speeds. Another interesting feature is that in the computation performed at decreasing speed shows that the jump does not occur at the same speed for $x$ and $y$ axes.

The orbit at 200 rad/s, a speed close to the linearized critical speed, is shown in Figure 6a. It has been computed using both the idealized PD controller and the present PID controller: in the latter case the orbit is circular if a protection current of 5 A is used, while in the case of a protection currents of 2.4 A it takes an irregular shape.

The time histories of the total currents in the coils at 200 rad/s for the three cases sre plotted in Figure 6 b, c and d.

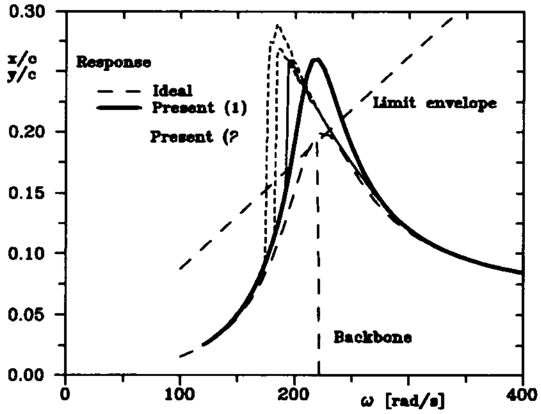


Figure 5: Simulation results for a class B bearing, working with the weight carried by the coils of both $x$ and $y$ axes. Unbalance response, with backbone and limit envelope, computed using both the idealized PD controller and the present PID controller. Case (1) refers to a protection current of 5 A, case (2) to a current of 2.4 A.

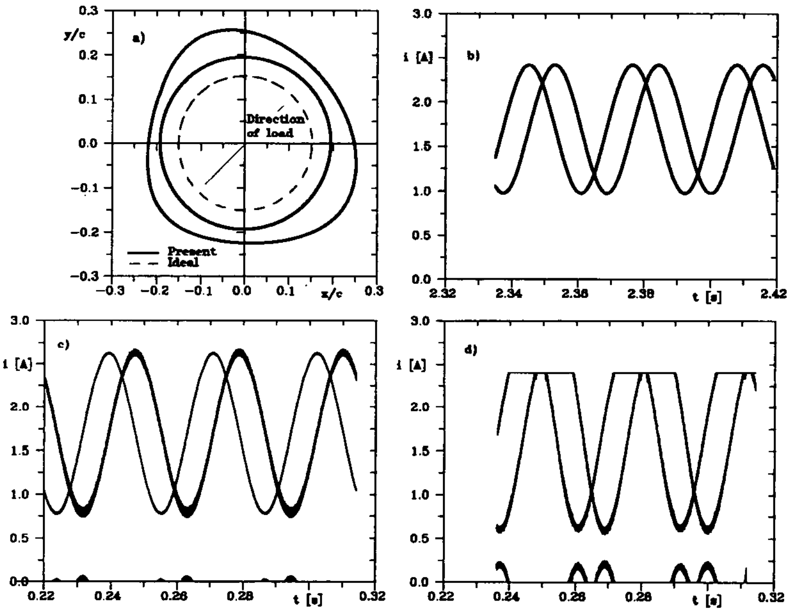


Figure 6: Simulation results for the same bearing of figure 5. (a): orbit at a speed of 200 rad/s; computed using both the idealized PD controller and the present PID controller. In the latter case the orbit is circular if a protection current of 5A is used, while in the case of a protection currents of 2.4 A it takes an irregular shape. Total currents in the coils as functions of time at: (b) ideal PD controller, (c) and (d), PID controller with protection current of 5 A and 2.4 A.

## CONCLUSIONS

A time domain study of the whirling of a Jeffcott rotor supported by active magnetic bearing with a PID controller has been performed through numerical integration of the equations of motion.

The results of some simulations confirmed that the greatest effect is linked to the saturation effect induced by the current limit/protection of the power amplifier, which increases the softening behavior of the system, even with jumps taking place.

When the protection of the amplifiers does not act to limit the current the results very near to the linear ones and quite similar to those obtained through an ideal PD controller with static compensation current, except for the fact that the damping action is weaker. Owing to the smaller damping effect, the amplitude of the response is higher in the zone close to resonance, while outside this speed range little difference can be found.

The study gives some insight into the expectable rotordynamic behavior during slow critical speed crossing for rotors suspended with underdamped rigid body modes.

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