NON-LINEAR CONTROL OF MAGNETIC BEARINGS FOR HIGH-SPEED FLYWHEELS

Z. Y. Wu,[1] P. H. Mellor,[1] P. E. Mason,[1] D. Howe[1]

ABSTRACT

Depending on the level of bias, the control system for an active magnetic bearing can be classified as either linear, non-linear or hybrid. For a given design of bearing, the minimum bias current which facilitates linear control, and which is therefore commensurate with low iron/copper losses and amplifier VA ratings, can be specified. However, non-biased, non-linear control is the most energy efficient mode, and, hence, is particularly appropriate for applications in which precise position control is not required, such as high-speed energy storage flywheels. The determination of the amplifier VA ratings and the effects of control current ripple are discussed, and a simple computer simulation is used to illustrate non-biased, non-linear controller design.

INTRODUCTION

In order to improve the linearity of the force-displacement characteristic of an active magnetic bearing, each pair of opposing electromagnets is usually pre-biased, by either DC currents or permanent magnets (Maslen et al., 1996). Biasing also improves the force to current sensitivity, and thus the dynamic performance. Nevertheless, currently, a tuning rule does not appear to have been established for this control parameter, and normally the bias field is simply set at half the saturation flux level. However, operation with bias may decrease the system efficiency, due to increased copper loss in the stator coils and eddy current and hysteresis losses in the rotor, which are particularly problematic when the rotor runs in a high vacuum, as is likely to be the case for a high-speed flywheel. Nevertheless, in such an application, a relatively large rotor displacement can usually be tolerated. Thus, it may be appropriate to employ a less responsive control action, by reducing or eliminating the bias current.

In this paper, the consequences of biasing an active magnetic bearing are investigated, with particular reference to the controller design, the force slew rate, the bearing power loss, and the system dynamics. The selection of the most appropriate bias for linear control and its elimination for non-linear control are considered, with regard to the amplifier VA rating, as well as the time variation of the control flux. Finally, results are presented from a computer

[1]Department of Electronic and Electrical Engineering, The University of Sheffield, Mappin Street, Sheffield S1 3JD, UK, tel.: ++44 114 2225195, fax.: ++44 114 2225196, e-mail: S.J.Gawthorpe@shef.ac.uk.

simulation of a non-biased non-linear controlled bearing, which, since it is conducive to low loss, is particularly appropriate for use in high-speed energy storage /peak power buffer flywheels for applications in electric vehicles.

Typically, such a flywheel unit for an urban electric vehicle would rotate at a maximum speed around 50,000 rpm, be capable of providing a recoverable energy of $\approx 350\,\mathrm{Wh}$ in slowing down to half speed, and have a peak power capability of some 40kW (Howe et al., 1995).

SIMPLE MAGNETIC BEARING

Each axis of a typical magnetic bearing usually comprises two opposing electromagnets, as shown in Fig. 1, in order that a bi-directional force can be exerted on the rotor. If the electromagnets are identical, and carry currents i_1 and i_2, respectively, the resultant force is:

$$F = k_f\left[\frac{i_1^2}{(x_0 + x)^2} - \frac{i_2^2}{(x_0 - x)^2}\right] \tag{1}$$

where

$$k_f = \frac{p-1}{4}\mu_0 N^2 A_p \tag{2}$$

and p is the number of poles per axis, A_p is the cross-sectional area of each pole, N is the number of turns on each coil, and x_0 is the nominal airgap length.

Usually, a bias current i_0 is used to linearise the force-control current and force-displacement characteristics around the equilibrium point, the coils being connected such that $i_1=i_0+i$ and $i_2=i_0-i$. Thus:

$$F = k_f\left[\frac{(i_0 + i)^2}{(x_0 + x)^2} - \frac{(i_0 - i)^2}{(x_0 - x)^2}\right] = k_i i + k_x x \tag{3}$$

where k_i is the bearing current sensitivity, and k_x is the bearing natural stiffness, i.e.

$$k_i = 4k_f\frac{i_0}{x_0^2} \tag{4}$$

$$k_x = -4k_f\frac{i_0^2}{x_0^3} \tag{5}$$

In this way, control via an inner current loop is usually used, as shown in Fig. 2.

In a vehicle-mounted flywheel, the dominant disturbances are those which result from gyroscopic effects arising from the motion of the vehicle, and high frequency forces which result from unbalance of the flywheel. Fig. 3 is typical of the bearing force which results when a vehicle is driven through a fairly severe slalom course at 40km/hr. For this application, the magnetic bearing has to have a maximum force capability of around 1600N, whilst its control system must be capable of preventing touch-down of the flywheel on the mechanical back-up bearing.

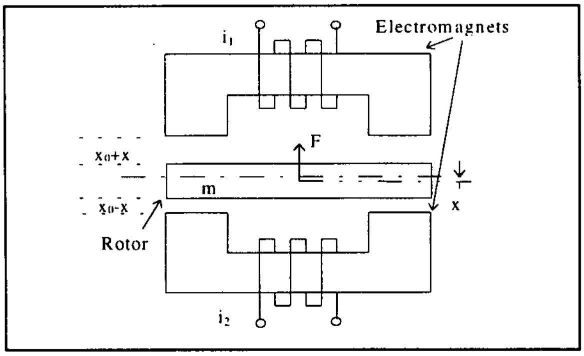

Fig. 1. Single axis active magnetic bearing

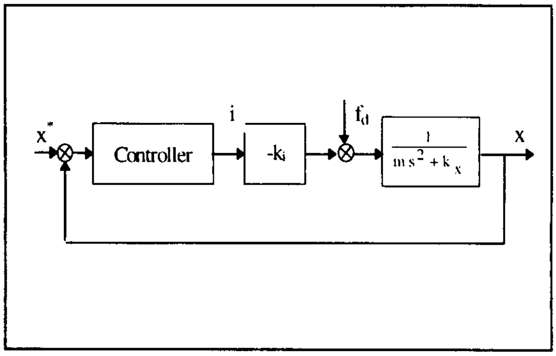

Fig. 2. Magnetic bearing control system

BIASING

Biasing is usually introduced as an additional parameter in the magnetic bearing control system. However, to date, it would appear that no rules have been established to tune this parameter. Therefore, more often than not, it is simply set at half the maximum coil current or half the saturation flux level. However, it is pertinent to consider the consequence of biasing the electromagnets in greater detail.

LINEAR AND NON-LINEAR CONTROL

There are three basic control strategies which depend on the level of bias which is employed, viz:

Linear Control

Although permanent magnets can be employed to provide the bias flux, and thereby reduce the copper loss in the coils (Maslen, 1996), it is more common to provide the bias by a dc current i_0, which is usually set at half the maximum coil current. The resulting coil currents are then given by:

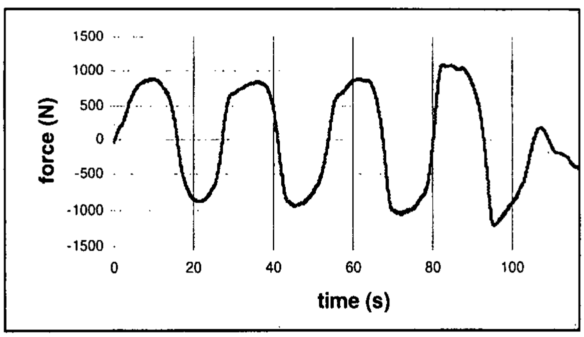

Fig.3 Typical EV flywheel magnetic bearing load, with vehicle on "slalom" test

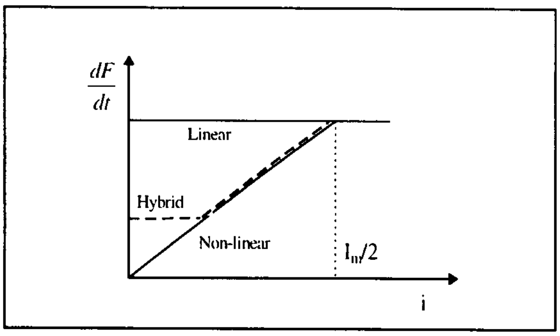

Fig. 4. Magnetic bearing force slew rate for different control schemes

$$\begin{cases} i_1 = i_0 + i \\ i_2 = i_0 - i \end{cases} \tag{6}$$

Such biasing effectively linearises the electromagnet characteristic, as given by Eqn. (3).

Non-Linear Control

In this case, the bias current is set to zero, and only one of the electromagnets is active at any instant, one producing the force for a positive control signal and the other for a negative control signal. This leads to non-linear control, viz:

$$i_1 = \begin{cases} i, i \geq 0 \\ 0, i < 0 \end{cases}, \quad i_2 = \begin{cases} 0, i \geq 0 \\ i, i < 0 \end{cases} \tag{7}$$

Hybrid Control

In order to prevent cross-over distortion as in the non-linear control, a small bias current may be used, i.e.

$$i_1 = \begin{cases} i_0 + i, i_0 + i \geq 0 \\ 0, i_0 + i < 0 \end{cases}, \quad i_2 = \begin{cases} i_0 - i, i_0 - i \geq 0 \\ 0, i_0 - i < 0 \end{cases} \tag{8}$$

This strategy may be considered as a hybrid of the linear and non-linear control schemes, and can improve bearing efficiency, by reducing both the stator copper loss and the rotor iron loss, as compared to linear control.

The force slew rate characteristics for the different control schemes are illustrated in Fig. 4

POWER LOSS

When the bearing system is in steady-state, with only a steady load force F_d, the coil currents will be constant, whilst both the stator copper loss and the rotor iron loss can be considered to be approximately proportional to the square of the current, i.e.

$$P_{loss} = K_{loss}[(i_0 + i)^2 + (i_0 - i)^2] \tag{9}$$

and

$$F_d = \frac{4k_f}{x_0^2} i_0 i \tag{10}$$

The loss will be a minimum when:

$$i_0 = i \tag{11}$$

i.e. the optimal bias current for the highest efficiency is half the current which is necessary to produce the required restoring force. In other words, the most efficient operational mode is when only one coil is energised at a time. The dynamic related losses will be discussed later.

DYNAMIC RESPONSE

Consider the magnetic bearing of Fig. 1, having both its airgaps initially at the nominal value x_0, and the currents in the two coils equal to i_{10} and i_{20}, respectively. When a step load force F_d is applied, full voltage is assumed to be applied to the windings to cause the current in one coil to increase and the current in the other coil to decrease, so as to produce the necessary restoring force. In the following, the system dynamic response for both non-linear and linear control is considered.

Dynamic Response With Non-Linear Control

Assume the initial steady load force F_{d0} on the rotor is such that:

$$i_{20} \neq 0, i_{10} = 0 \tag{12}$$

When the load force changes to F_d in the opposite direction, the control currents vary as follows:

$$\begin{cases} i_1 = \dfrac{V}{L}t \\ i_2 = i_{20} - \dfrac{V}{L}t \end{cases} \text{and} \begin{cases} i_1 \leq I_m \\ i_2 \geq 0 \end{cases} \tag{13}$$

where I_m is the maximum current.

The motion of the rotor can be described by:

$$m\ddot{x} = F_d - F \tag{14}$$

where m is the mass of the rotor. Further, if the influence of the variation of the airgap length on the force and the coil inductance is neglected, the electromagnetic force F during three distinct stages of operation is given by:

$$F = \begin{cases} \dfrac{k_f}{x_0^2}[(\dfrac{V}{L}t)^2 - (i_{20} - \dfrac{V}{L}t)^2], & i_2 \geq 0 \\ \dfrac{k_f}{x_0^2}(\dfrac{V}{L}t)^2, & i_1 \leq I_m \,\&\, i_2 = 0 \\ \dfrac{k_f}{x_0^2}I_m^2, & i_1 = I_m \,\&\, i_2 = 0 \end{cases} \tag{15}$$

In practice, due to the displacement of the rotor, the electromagnetic force will be slightly higher than given by Eqn. (16). However, the simplification can impart some robustness on the dynamics when the control system is designed.

It can be proved that in response to the step load reversal the maximum rotor displacement will not occur during the first stage, and may occur during the second stage, in which case it can be found as:

$$x - x_0 = \frac{8 + 12\beta - 4\beta^3 - \beta^4}{3} \cdot \frac{k_f F_d^2}{mV^2} \approx 5\frac{k_f F_d^2}{mV^2} \tag{16}$$

where

$$\beta = 2\cos\frac{\pi}{9} - 1 \tag{17}$$

Here, for simplicity, but without loss of generality, it has been assumed that $|F_{d0}|=|F_d|$. However, if the load force is sufficiently high such that:

$$i_{20} > \frac{1}{1+\beta} I_m \tag{18}$$

the maximum rotor displacement will occur until the final stage when the bearing current is at its maximum. The corresponding maximum displacement is:

$$x - x_0 = \frac{k_f F_m^2}{9mV^2} \cdot A \tag{19}$$

where F_m is the maximum force capability of the bearing, and A is a function of the load ratio ($\lambda = \frac{i_{20}}{I_m}$) as shown in Fig. 5.

As indicated by Eqn. (19), for a given magnetic bearing and amplifier design, the maximum tolerable load force will be constrained by the specified maximum rotor displacement x_m. Clearly, if the amplifier voltage is not limited, the imposed load force on the bearing could be as high as the maximum force capability.

Dynamic Response With Linear Control

The same initial load force and states are assumed as for the non-linear case. When the step load force F_d is applied, the rotor motion may again be subdivided into three stages. However, if the bias is sufficiently high in comparison with the load force (e.g. $k_i i_0 > 3F_d$), the maximum rotor displacement will occur during the first stage. Thus, the other cases need not be considered. After some manipulation, the maximum rotor displacement can be found as:

$$x - x_0 = \frac{4F_d^3 x_0^2}{3mV^2 i_0^2} \tag{20}$$

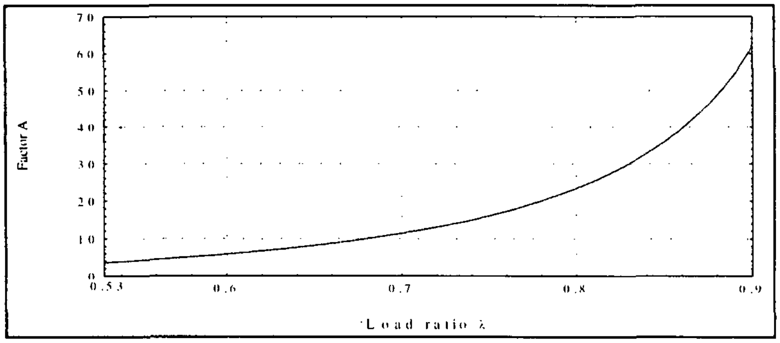

Fig. 5. Rotor maximum displacement factor A Vs the load ratio λ.

Clearly, the higher the bias current, the less the displacement of the rotor. In other words, for a given magnetic bearing, for which the step load force and the amplifier voltage have been specified, the minimum bias current can be determined according to the allowable maximum rotor displacement x_m, i.e.:

$$(i_0)_{min} = \frac{F_d x_0}{V}\sqrt{\frac{4}{3m}\cdot\frac{F_d}{x_m}} \tag{21}$$

Comparison Of Linear And Non-Linear Control Dynamic Performance

Considering a load force F_d and a bias current i_0, which satisfy:

$$k_i i_0 > 3F_d \tag{22}$$

the maximum rotor displacement for the non-linear and linear control strategies are related by:

$$\frac{(x - x_0)_{nonlinear}}{(x - x_0)_{linear}} > \frac{45}{16} \tag{23}$$

As expected, biased linear control results in a better dynamic response than non-biased non-linear control. However, when the load force is sufficiently high, the maximum rotor displacement for both linear and non-linear control will converge to the same value.

Again, by way of example, consider non-biased non-linear control applied to the magnetic bearings of the flywheel energy storage/peak power buffer unit referred to earlier. In the system considered, the maximum allowable rotor displacement is x_m=0.2mm and the rotor mass is ≈ 10kg, whilst the bearing amplifiers are each rated at 72V, 20A. If, initially, a steady load force of 1000N had been applied to the rotor in one direction and the coil currents had controlled the rotor to be at its equilibrium position, then when a load force of 1000N is applied in the opposite direction, the bearing controller will act to apply full voltage to the coils, and the currents will change until the bearing force in the initial direction is decreased to zero whilst that in the opposite direction is increased to its maximum value of 1600N. The maximum rotor displacement can be calculated to be 0.15mm, which is within the specified limit of 0.2mm.

DETERMINATION OF AMPLIFIER VA RATING

In the literature (Schweitzer, 1994), the required VA rating of the amplifiers is usually determined such that the bearing is able to generate a specified sinusoidally time-varying force, which is 180^0 out of phase with the imposed disturbance force. With this strategy of complete force compensation, the required maximum amplifier output voltage is related to the bias current in the linear control mode. The higher the voltage, the lower the bias current, with the required voltage tending to infinity when the bias is zero (non-linear control). However, complete compensation is not always essential. An example is when the bearing is supporting a flywheel, for which precise control of the airgap is not necessary. Nevertheless, the amplifier voltage should be high enough to enable the bearing to react sufficiently quickly in order to prevent the rotor displacement from exceeding the specified maximum value.

Although, in practice, a step load force will hardly ever occur, it is appropriate to consider this for the determination of the required maximum voltage. For simplicity, the maximum step load force is assumed to be higher than half the bearing force capacity, i.e. the maximum rotor displacement occurs during stage 3 for a non-linear control system. The required amplifier VA rating can be derived as:

$$\text{VA rating} = \frac{F_m x_0}{3}\sqrt{\frac{F_m A}{x_m m}} \tag{24}$$

Thus, as would be expected, the larger the load force and the stiffer the bearing, then the higher must be the amplifier rating. Again, for the magnetic bearing of the flywheel energy storage/peak power buffer unit referred to earlier, if F_d=1000N, x_0=0.5mm, x_m=0.2mm, and m=10kg, the required amplifier VA rating is 1.1kVA per axis.

CONTROL FLUX (CURRENT) FLUCTUATION

Linear and non-linear control schemes result in different control actions, which may induce losses in the rotor. Since high frequency flux fluctuations can have a significant influence on the rotor loss, the effect of the rotor unbalance force needs particular consideration. Control action is required to compensate for this, and typically, feedforward control would be used to reject this synchronous disturbance.

The ratio of the maximum current fluctuations required to generate a sinusoidal force for linear and non-linear control is:

$$\frac{\Delta i_{nonlinear}}{\Delta i_{linear}} = \frac{2i_0}{i_{pk(nonlinear)}} \geq 1 \tag{25}$$

Thus, if the bias current is sufficiently high, linear control introduces a much lower current ripple than non-linear control. However, if the high frequency disturbing force is due to rotor unbalance, and feedforward control is used to generate an electromagnetic force of the same frequency, the time-varying coil currents will be time-invariant with respect to the rotor.

Thus, the current ripple will not induce eddy current loss in the rotor. In contrast, the bias current will induce rotor loss.

COMPUTER SIMULATION OF NON-LINEAR CONTROL

A magnetic bearing without biasing is highly non-linear. However, there are several ways of overcoming this problem, one of the simplest being to tune the controller for the worst case and accept degraded performance under other operating conditions, viz. robust control. However, if the non-linear characteristic is known a-priori, it can be compensated for by simply generating the inverse of the non-linearity. For example, the square-root linearisation method with displacement compensation can be employed. Another approach to compensating for non-linearities is to simply divide the operating range into several small ranges in each of which the process is approximated by a linear model. Satisfactory control over the full operating range can then be obtained by changing the controller parameters as appropriate, so-called gain scheduling.

A MATLAB/SIMULINK® computer simulation model has been created for a non-linear PD controller for the magnetic bearing referred to earlier, the proportional gain and derivative coefficients being scheduled simply according to the control error as:

$$\begin{cases} k_p = \dfrac{k_{p1}}{k_{p2} + |e|} \\ k_d = \dfrac{k_{d1}}{k_{d2} + |\dot{e}|} \end{cases} \tag{26}$$

For comparison, a linear PD controller has also been designed for this non-biased bearing system. In both cases, the controller parameters were tuned by trial and error to obtain the best performance. Fig. 6 shows the simulated results when a step load force of 200N is applied at t=0.15s. It demonstrates that a controller design for a non-biased bearing has the capability of reference tracking and disturbance rejection, though the gain-scheduled non-linear controller appears to be more suitable.

CONCLUSIONS

Due to the non-linear electromagnetic characteristics of active magnetic bearings, a bias (by either dc current or permanent magnet) is usually introduced, for linearisation and for overcoming zero force cross-over distortion. Depending on the bias level, the bearing control system can be classified as linear, non-linear or hybrid. Linear control has the advantages of simplifying the controller design, and having good dynamic performance and relatively small control flux fluctuations. However, it causes additional losses in the system, particularly iron loss in the rotor. For a given design of magnetic bearing and the associated amplifiers, an appropriate bias level can be determined for linear control, based on the maximum allowable

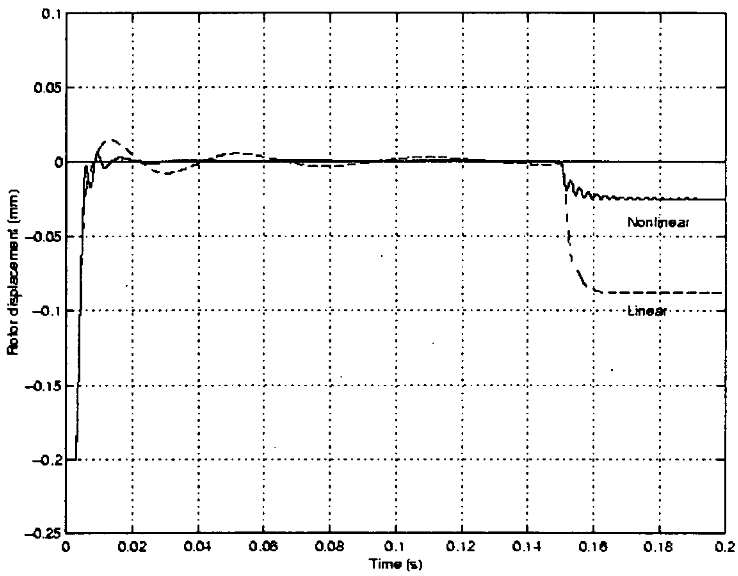

Fig. 6. Non-biased magnetic bearing control simulations with nonlinear PD and linear PD control. Step load force of 200N applied at t=0.15s.

rotor displacement and the applied load force, rather than by simply setting it at half the saturation flux level, as is common practice. However, non-linear control is more appropriate when high efficiency is required. Although the dynamic performance which is achieved with non-linear control is not as good as that with biased linear control, because of the relatively low force slew rate, the rotor displacement can be easily maintained within a specified range. Thus, it is particularly suitable for applications for which precise rotor position control is not required, such as energy storage flywheels. The required amplifier VA rating can be determined from a consideration of the dynamic response. Since the dynamic response for non-linear control converges to that for linear control as the load force is increased, ultimately the amplifier VA rating will be irrelevant to the control mode. The effectiveness of non-linear control has been demonstrated by computer simulations, and although the control current will have a relatively high ripple, the high frequency disturbance due to rotor unbalance can be compensated for by feedforward control at the same frequency. The current ripple is then effectively time-invariant with respect to rotation of the rotor.

REFERENCES

Maslen, E.H. and Allaire, P.E. et.al. 1996 "Magnetic bearing design for reduced power consumption". Journal of Tribology. 118. pp.839-845.

Howe, D. Mellor, P.H. and Peel, D.J. 1995. "High speed electro-mechanical energy storage systems for electric vehicles". Invited paper, IEEE/KTH Stockholm Power Tech Conference. Stockholm, Sweden, June, pp. 50-56.

Post, R.F. Fowler, T.K. and Post, S.F. 1993 "A high-efficiency electro-mechanical battery". Proceedings IEEE, Vol.81, No.3. pp.462-474.

Schweitzer,G. Et.al. 1994 "Active magnetic bearings - basics, properties and applications of active magnetic bearings". Vdf Hochschulverlag AG an der ETH Zurich.