ROBUST SLIDING MODE CONTROL OF A PLANAR RIGID ROTOR SYSTEM ON MAGNETIC BEARINGS

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ABSTRACT

A planar rigid rotor supported on magnetic bearings is controlled using a robust sliding mode controller. Linearized magnetic bearings and mass unbalance are considered. A nonlinear sliding mode controller evaluates the magnetic bearing control currents which bring the rotor response to the sliding mode surface and cause it to remain within the boundary layer. A treatment of uncertainty limits is developed for unknown bearing open loop stiffness, actuator gain, and mass unbalance. A numerical simulation showed the desired system response when the rotor was subjected to large initial transient conditions.

INTRODUCTION

Magnetic bearings have been used for high speed rotors such as gyroscopes, beam choppers, energy storage flywheels and other applications. This paper discusses the dynamics and control of a magnetic bearing supported rigid rotor for applications where the magnetic bearings are supported in rigid supports. A key issue is the performance of the controller in the presence of uncertainty in bearing characteristics and rotor unbalance. The controller must be robust with respect to these uncertainties.

Many different types of controller approaches are now being employed for magnetic bearing controllers. Kanemistu, et al. [1994] discussed the use of Linear Quadratic Gaussian (LQG), H_∞, Time Delay Control (TDC), Sliding Mode Control (SMC), and Proportional Integral Derivative (PID) controllers in the magnetic levitation support of a flexible body. The H_∞ and PID controllers were found to have high phase and gain margins for excellent stability characteristics. The LQG controller was found to have the best robustness with regard to spillover in a higher order vibration mode. Theoretical results for the SMC controller indicated very good robustness in the low frequency range, poor robustness at high frequencies, and experimental results were inconclusive for the SMC due to an instability. Yamashita, et al. [1996] considered an advanced controller design using an H_∞ design for a flexible rotor and compared it to a PID controller design. The H_∞ design increased the magnetic bearing/controller stiffness to reduce rotor vibrations at low frequencies.

Sliding mode controllers were originally developed in the Soviet Union more than 30 years ago for flexible structures . The algorithm relies on very high speed switching between the control values and has been successfully applied to flexible structures [Sinha and Kao, 1991][Kao and Sinha, 1992]. Recent advances in high speed switching amplifiers have made sliding mode controls feasible in industrial systems.

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Sliding mode controllers select a surface in state space, typically a linear hypersurface, called the switching surface, and switch the control input on this surface [Spong and Vidyasagar, 1989][Sinha and Miller, 1995]. The control input is then chosen to guarantee that the trajectories near the sliding surface are directed toward the surface. Then, any control input will suffice to move the trajectory toward the surface. Once the system is on the surface, the closed loop dynamics are completely governed by the equations which define the surface. Since the parameters defining the surface are chosen by the designer, the closed loop dynamics will be independent of perturbations of the parameters of the system and robustness is achieved. These principles are applied to the magnetic bearing supported flywheel rotor.

The specific problems are 1) the determination of the control gains so that the rotor motion trajectory reaches the sliding surface in finite time, 2) the specification of the switching logic to constrain the dynamic rotor motions at the magnetic bearing locations is constrained to the surface, and 3) the determination of the rotor equations governing the surface to describe the dynamics of the system on the surface.

Nonami and Yamaguchi [1993] presented a detailed discussion of the use of a sliding mode control for magnetic bearings in a flexible rotor modeled as a rigid rotor. A linearized state space model was developed for the system and a switching width selected for the sliding mode control. Each feedback gain was switched at a specified percentage of linear feedback gain, with a value of approximately 10% found to be optimum. The results indicated that the sliding mode control was more robust, when a 10% change in rotor mass was considered, than the best PID controller found for the system, particularly when the rotor displacements were large.

Tian and Nonami [1994] presented a sliding mode control for a flexible rotor operating up to 40,000 rpm. A reduced order model of the rotor and bias current linearized magnetic bearings was employed with a treatment of external disturbances. A continuous time sliding mode controller which exhibited chattering behavior was replaced with a discrete time sliding mode controller to solve the problem.

Tian, et al. [1996] describes a high speed grinding spindle-magnetic bearing system using a bias linearized bearing and discrete time sliding mode control. A variable structure system (VSS) disturbance observer is employed to compensate for external disturbances. A switching manifold design was developed to overcome some difficulties which had been encountered when over conservative feedback gains cause chattering.

Jiang and Zhao [1996] developed a variable structure control control using flux for a single DOF AMB system. It included voltage control and a switching power amplifier design. The control algorithm was found to be not sensitive to the operating point.

Nonami and Nishina [1996] considered a discrete time sliding mode control for a rotor supported in a permanent magnet biased magnetic bearing. A combination linear and nonlinear control algorithm was developed to eliminate chattering. A discrete time VSS observer was developed to estimate unmeasured state variables.

Rundell et al. (1996) developed a sliding mode controller for a vertical rotor to control rotational dynamics of the shaft. A sliding mode observer was developed for state and distubance estimation. Simulation results indicated the robustness of the controller.

Charara (1996) considered several methods of nonlinear control of magnetic levitation without bias flux. One of these approaches was the use of a sliding mode control of the switching amplifier directly to combine the sliding mode switching function with the control stage of the switching amplifier.

This work discusses the dynamics of a single mass rigid planar rotor supported on magnetic bearings. Figure 1 shows the geometry. The purpose is to conduct an analysis of robust sliding mode control on the rotor dynamics and evaluate the linear and nonlinear control force laws to be employed with the magnetic bearings. The effects of this type of control law on the uncertainty of magnetic bearing open loop stiffness, actuator gain, and rotor unbalance is discussed.

MAGNETIC BEARING FORCE

A double sided magnetic bearing has the net force given by

$$F = \frac{\mu_0 N^2 i_1^2 A}{4 g_1^2} - \frac{\mu_0 N^2 i_2^2 A}{4 g_2^2} = \frac{\mu_0 N^2 i_1^2 A}{4 (g_0 - u)^2} - \frac{\mu_0 N^2 i_2^2 A}{4 (g_0 + u)^2} \tag{1}$$

if the bearing upper and lower poles have the same pole face area, the same number of coil turns, and air gaps $g_1 = g_0 - u$ and $g_2 = g_0 + u$ [Allaire, et al., 1994, 1997].

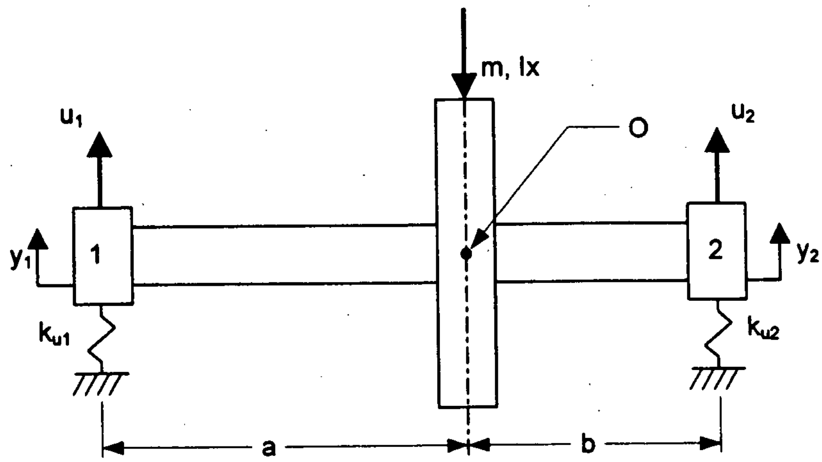

Figure 1. Schematic of Rigid Rotor on Magnetic Bearings In Planar Small Amplitude Motion

SYSTEM EQUATIONS OF MOTION

The rigid rotor is supported on magnetic bearings at the ends of the rotor (for simplicity in the analysis) with bearing span L, as shown in Figure 1. Here the magnetic bearings are modeled as connected to ground and the rotor is assumed to be in planar vibrations in the transverse direction. The rotor transverse displacements at the bearing locations are denoted as u_1 and u_2. The centroidal mass moment of inertia of the rotor is I_G and the rotor is assumed to have the disk center of gravity placed at a distance a from the left end and b from the right end.

The system equations of motion in terms of the rotor transverse displacements at the bearing locations are

$$\begin{bmatrix} \frac{bm}{2L} + \frac{I_{xG}}{L^2} & \frac{am}{2L} - \frac{I_{xG}}{L^2} \\ \frac{bm}{2L} - \frac{I_{xG}}{L^2} & \frac{am}{2L} + \frac{I_{xG}}{L^2} \end{bmatrix} \begin{Bmatrix} \ddot{u}_1 \\ \ddot{u}_2 \end{Bmatrix} = \begin{Bmatrix} F_1 \\ F_2 \end{Bmatrix} + \begin{Bmatrix} \frac{b}{L} U\omega^2 \cos\omega t \\ \frac{a}{L} U\omega^2 \cos\omega t \end{Bmatrix} \tag{2}$$

where the first term on the right gives the magnetic bearing forces and second term on the right represents the planar forces due to unbalance U. The phase angle of the unbalance can be taken as zero without loss of generality. The control forces can be produced without a bias current, as given in Eq. (3), or with a bias current, as given in Eq. (8).

If the magnetic bearings are operated with a bias current, the bearing force equations are linearized and the bearing forces are given by

$$\begin{Bmatrix} F_1 \\ F_2 \end{Bmatrix} = -\begin{bmatrix} k_{u1} & 0 \\ 0 & k_{u2} \end{bmatrix} \begin{Bmatrix} u_1 \\ u_2 \end{Bmatrix} + \begin{bmatrix} k_{i1} & 0 \\ 0 & k_{i2} \end{bmatrix} \begin{Bmatrix} i_{c1} \\ i_{c2} \end{Bmatrix} \tag{3}$$

where the equations are still nonlinear in the force limit due to saturation. Steady state magnetic bearing currents, such as for constant unidirectional loading, are subtracted from the total current to leave the dynamic force terms. Then the system equations of motion are

$$\begin{bmatrix} \frac{bm}{2L}+\frac{I_{xG}}{L^2} & \frac{am}{2L}-\frac{I_{xG}}{L^2} \\ \frac{bm}{2L}-\frac{I_{xG}}{L^2} & \frac{am}{2L}+\frac{I_{xG}}{L^2} \end{bmatrix} \begin{Bmatrix} \ddot{u}_1 \\ \ddot{u}_2 \end{Bmatrix} + \begin{bmatrix} k_{u1} & 0 \\ 0 & k_{u2} \end{bmatrix} \begin{Bmatrix} u_1 \\ u_2 \end{Bmatrix} = \begin{bmatrix} k_{i1} & 0 \\ 0 & k_{i2} \end{bmatrix} \begin{Bmatrix} i_{c1} \\ i_{c2} \end{Bmatrix} + \begin{Bmatrix} \frac{b}{L}U\omega^2\cos\omega t \\ \frac{a}{L}U\omega^2\cos\omega t \end{Bmatrix} \tag{4}$$

for the rotor supported in magnetic bearings with bias currents.

In industrial magnetic bearing control systems, modal control is often employed to control rigid body modes. The design is considerably simplified by decoupling into two single input systems. Define the dynamically uncoupled coordinates as

$$\begin{Bmatrix} v_1 \\ v_2 \end{Bmatrix} = \begin{bmatrix} \frac{b}{L} & \frac{a}{L} \\ -\frac{1}{L} & \frac{1}{L} \end{bmatrix} \begin{Bmatrix} u_1 \\ u_2 \end{Bmatrix} \tag{5}$$

where v_1 is the displacement of the rotor mass center and v_2 is the angular displacement of the shaft. Then the equations of motion are

$$\boldsymbol{H}\ddot{\boldsymbol{v}}+\boldsymbol{h}=\boldsymbol{i}_c \tag{6}$$

where

$$H = \begin{bmatrix} \frac{m}{2k_{i1}} & -\frac{I_{xG}}{Lk_{i1}} \\ \frac{m}{2k_{i2}} & \frac{I_{xG}}{Lk_{i2}} \end{bmatrix}, \quad h = \begin{bmatrix} \frac{k_{u1}}{k_{i1}}v_1-\frac{ak_{u1}}{k_{i1}}v_2-\frac{b}{Lk_{i1}}U\omega^2\cos\omega t \\ \frac{k_{u2}}{k_{i2}}v_1+\frac{bk_{u2}}{k_{i2}}v_2-\frac{a}{Lk_{i2}}U\omega^2\cos\omega t \end{bmatrix} \tag{7}$$

Also, $\boldsymbol{i}_c=[i_{c1}\ i_{c2}]^T$ and $\boldsymbol{v}=[v_1\ v_2]^T$. The values $\boldsymbol{h}$ are the modal control forces.

ROBUST SLIDING MODE CONTROL

A significant problem in rotors supported in magnetic bearings concerns the uncertainty in magnetic bearing characteristics and rotor unbalance. Following Asada and Slotine [1986], the sliding mode control law is given by

$$i_c = \hat{H}r+\hat{h} \tag{8}$$

where $\hat{H}$ and $\hat{h}$ are estimates of H and h, respectively. Let $\hat{k}_{u1},\hat{k}_{i1},\hat{k}_{u2},\hat{k}_{i2},\hat{U}$ be estimates of $k_{u1},k_{i1},k_{u2},k_{i2},U$. The objective is to determine the control vector r which yields the desired sliding mode control for the rotor-magnetic bearing system. Then $\hat{H}$ and $\hat{h}$ are defined by (7) with the estimated values of the bearing open loop stiffness, actuator gain, and unbalance replacing the actual values in (7). From (6) and the estimated matrix $\hat{h}$, the equation of motion is

$$\dot{v} = H^{-1}\hat{H}r+H^{-1}\Delta h \tag{9}$$

where $\Delta h=\hat{h}-h$. Let $H^{-1}=[L_1\ L_2]^T$ and $\hat{H}=[H_1\ H_2]$. Then the first term on the right hand side of (9) is

$$H^{-1}\hat{H} = \begin{bmatrix} L_1\hat{H}_1 & L_1\hat{H}_2 \\ L_2\hat{H}_1 & L_2\hat{H}_2 \end{bmatrix} = \begin{bmatrix} \frac{1}{2}\left(\frac{k_{i1}}{\hat{k}_{i1}}+\frac{k_{i2}}{\hat{k}_{i2}}\right) & \frac{I_{xG}}{mL}\left(-\frac{k_{i1}}{\hat{k}_{i1}}+\frac{k_{i2}}{\hat{k}_{i2}}\right) \\ \frac{Lm}{4I_{xG}}\left(-\frac{k_{i1}}{\hat{k}_{i1}}+\frac{k_{i2}}{\hat{k}_{i2}}\right) & \frac{1}{2}\left(\frac{k_{i1}}{\hat{k}_{i1}}+\frac{k_{i2}}{\hat{k}_{i2}}\right) \end{bmatrix} \tag{10}$$

The second term on the right hand side in (9) is

$$H^{-1}\Delta h = \begin{bmatrix} L_1\Delta h \\ L_2\Delta h \end{bmatrix} = \begin{bmatrix} \frac{1}{m}A \\ \frac{L}{2I_{xG}}A \end{bmatrix} \tag{11}$$

where the coefficient A is

$$\begin{aligned} A = &\left(\hat{k}_{u1}\frac{k_{i1}}{\hat{k}_{i1}}-k_{u1}\right)(v_1-av_2)+\left(\hat{k}_{u2}\frac{k_{i2}}{\hat{k}_{i2}}-k_{u2}\right)(v_1+bv_2) \\ &-\left(\hat{U}\frac{k_{i1}}{\hat{k}_{i1}}-U\right)\frac{b}{L}\omega^2\cos\omega t-\left(\hat{U}\frac{k_{i2}}{\hat{k}_{i2}}-U\right)\frac{a}{L}\omega^2\cos\omega t \end{aligned} \tag{12}$$

The limits on the errors in bearing parameters and unbalance are given by

$$\frac{1}{\gamma_j}<\left|\frac{k_{ij}}{\hat{k}_{ij}}\right|<\gamma_j \quad , \quad |\hat{k}_{uj}-k_{uj}|<\delta_j \quad , \quad |\hat{U}-U|<\kappa \quad , \ j=1,2 \tag{13}$$

It should be noted that $\hat{k}_{ij}=\left(k_{ij}^{max}k_{ij}^{min}\right)^{1/2}$ if $k_{ij}^{min}\le k_{ij}\le k_{ij}^{max}$ and $\gamma_j>1$ for $j=1,2$.

It can be shown that $\epsilon^{min}<L_k\hat{H}_k<\epsilon^{max}$, $k=1,2$ where

$$\epsilon^{min} = \frac{1}{2}\left(\frac{1}{\gamma_1}+\frac{1}{\gamma_2}\right) \quad , \quad \epsilon^{max} = \frac{\gamma_1+\gamma_2}{2} \tag{14}$$

Furthermore, inequalities related to the limitations on the system model uncertainty cross terms are given by

$$\begin{aligned} |L_1\hat{H}_2| &< \frac{I_{xG}}{mL}\left[\max(\gamma_1,\gamma_2)-\frac{1}{\max(\gamma_1,\gamma_2)}\right] \\ |L_2\hat{H}_1| &< \frac{Lm}{4I_{xG}}\left[\max(\gamma_1,\gamma_2)-\frac{1}{\max(\gamma_1,\gamma_2)}\right] \end{aligned} \tag{15}$$

and for the coefficient A

$$\begin{aligned} |A| < &\left[\gamma_1\delta_1-\left(|\hat{k}_{u1}|+\delta_1\right)(\gamma_1-1)\right]|v_1-av_2|+\left[\gamma_2\delta_2-\left(|\hat{k}_{u2}|+\delta_2\right)(\gamma_2-1)\right]|v_1+bv_2| \\ &+\left[\kappa\gamma_1+\left(|\hat{U}|+\kappa\right)(\gamma_1-1)\right]\frac{b}{L}\omega^2|\cos\omega t|-\left[\kappa\gamma_2+\left(|\hat{U}|+\kappa\right)(\gamma_2-1)\right]\frac{a}{L}\omega^2|\cos\omega t| \end{aligned} \tag{16}$$

These terms are important to the sliding mode control algorithm.

The objective is to choose a sliding surface corresponding to each control input and then to select the control law so that a sliding motion exists on the intersection of the sliding surfaces. Define the two linear hyperspace surfaces s_i as

$$s_i = \dot{v}_i+2\lambda_i v_i+\lambda_i^2\int_0^t v_i(\tau)d\tau \quad , \quad i=1,2 \ \ and \ \ \lambda_i>0 \tag{17}$$

Here there are 2 linear sliding surfaces, one for each of the inputs. The parameters λ_i represent the slopes of the sliding surfaces in hyperspace. The condition to bring the system response within the boundary layers around the sliding hyperplanes and to remain inside them, called the reaching conditions, is

$$s_i\dot{s}_i < 0 \quad , \quad |s_i|>\phi_i \tag{18}$$

Here ϕ_i is the thickness of the boundary layer around the sliding hyperplane. The elements of the vector $\mathbf{r}$ which satisfy the conditions (18) are given by

$$r_i = G_i\left[\hat{r}_i-g_i sat\left(\frac{s_i}{\phi_i}\right)\right] \quad , \quad i=1,2 \tag{19}$$

where

$$G_i = \left(\epsilon_i^{max}\epsilon_i^{min}\right)^{-1/2} \quad , \quad \hat{r}_i = -2\lambda_i\dot{v}_i-\lambda_i^2 v_i \quad , \quad \beta_i = \left(\frac{\epsilon_i^{max}}{\epsilon_i^{min}}\right)^{1/2}$$
$$g_1 \ge \beta_1\left[\left(1-\beta_1^{-1}\right)|\hat{r}_1|+(L_1\Delta h)+|L_1\hat{H}_2||r_2|+\eta_1\right] \tag{20}$$
$$g_2 \ge \beta_2\left[\left(1-\beta_2^{-1}\right)|\hat{r}_2|+(L_2\Delta h)+|L_2\hat{H}_1||r_1|+\eta_2\right]$$

There are cross terms which depend upon $|r_1|$ and $|r_2|$ in (20) which require that g_1, g_2 have to be determined simultaneously. From (19),

$$|r_i| \le G_i|\hat{r}_i|+G_i k_i \quad , \quad i=1,2 \tag{21}$$

Also, from (20)

$$g_1-\beta_1|L_1\hat{H}_2|G_2 g_2 \ge \alpha_{11} \quad , \quad -\beta_2|L_2\hat{H}_2|G_1 g_1+g_2 \ge \alpha_{22} \tag{22}$$

where

$$\alpha_{11} = \beta_1\left[(1-\beta_1)^{-1}|\hat{r}_1|+(L_1\Delta h)+\eta_1+|L_1\hat{H}_2|G_2|\hat{r}_2|\right]$$
$$\alpha_{22} = \beta_2\left[(1-\beta_2)^{-1}|\hat{r}_2|+(L_2\Delta h)+\eta_2+|L_2\hat{H}_1|G_1|\hat{r}_1|\right] \tag{23}$$

The parameters g_i are given by (22) as

$$\begin{bmatrix} g_1 \\ g_2 \end{bmatrix} = \frac{1}{1-\beta_1\beta_2|L_1\hat{H}_2||L_2\hat{H}_1|G_1G_2}\begin{bmatrix} 1 & |L_1\hat{H}_2|G_2\beta_1 \\ |L_2\hat{H}_1|G_1\beta_2 & 1 \end{bmatrix}\begin{bmatrix} \alpha_{11} \\ \alpha_{22} \end{bmatrix} \tag{24}$$

For g_1, g_2 to be positive, $\beta_1\beta_2|L_1\hat{H}_2||L_2\hat{H}_1|G_1G_2<1$. When $\gamma_1=\gamma_2=\gamma$, the left hand side of this inequality is

$$\beta_1\beta_2|L_1\hat{H}_2||L_2\hat{H}_1|G_1G_2 = \left(\frac{\gamma_1^2-1}{2}\right)^2$$

Then, positive values for g_1, g_2 are obtained when $1<\gamma<\sqrt{3}$. If γ satisfies this condition, then the reaching condition is met. It is guaranteed that $|s_i|<\phi_i$ and $|v_i|<2\phi/\lambda_i$.

NUMERICAL EXAMPLE

An example rotor, with numerical values given in Table 1, was selected for analysis purposes. The uncertainty in the model is in the

An example rotor, with numerical values given in Table 1, was selected for analysis purposes. The uncertainty in the model is in the bearing open loop stiffnesses of 5% and actuator gains of 10% while the unbalance uncertainty was approximately 50%. The estimated model parameters are given in Table 1. Table 2 presents the control parameters selected by the sliding mode algorithm described in the above derivation. Fig. 2 showns the time response of the variable $v_1(t)$ from the initial value of -5x10^{-5} m with a steady state amplitude of less than 0.2x10^{-5} m. Fig. 3 is a plot of the $v_2(t)$ time transient response from an initial value of 5x10^{-5} rad with a steady state amplitude of less than 0.1x10^{-5} rad. The performance of the sliding mode parameters is shown in the next two plots. Fig. 4 gives the response of $s_1(t)$ from an initial value of -10x10^{-5} m/s while Fig. 5 shows the time transient response of $s_2(t)$ from the initial value of 10x10^{-3} rad/s. Fig. 6 gives the time transient control current values $i_{c1}(t)$ and $i_{c2}(t)$ during the control with a steady state amplitude of approximately 0.5 amps for each control current.

Table 1. System Parameters

Parameter	Value
m	30 kg
I_{xG}	0.30 kg-m^2
a	0.18 m
b	0.12 m
$\hat{k}_{u1}=\hat{k}_{u2}$	-1x10^6 N/m
$k_{u1}=k_{u2}$	-0.95x10^6 N/m
$\delta_1=\delta_2$	0.1x10^6 N/amp
$\hat{k}_{i1}=\hat{k}_{i2}$	100 N/amp
$k_{i1}=k_{i2}$	110 N/amp
$\gamma_1=\gamma_2$	1.2
U	0.0001 kg-m
$\hat{U}$	0.000154 kg-m
κ	0.0001 kg-m
ω	10,000 rpm

Table 2. Control Parameters

Parameter	Value
λ_1	100 rad/s
λ_2	100 rad/s
ϕ_1	0.005 m/s
ϕ_2	0.005 rad/s
η_1	0.1 m/s^2
η_2	0.1 rad/s^2

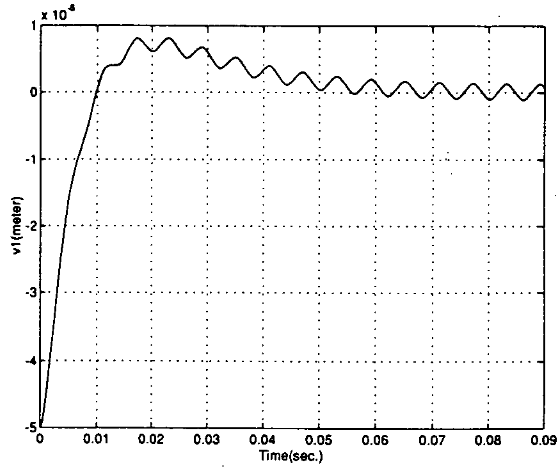

Figure 2. Time Transient Response of Sliding Mode Displacement Parameter v_1 For Rigid Rotor

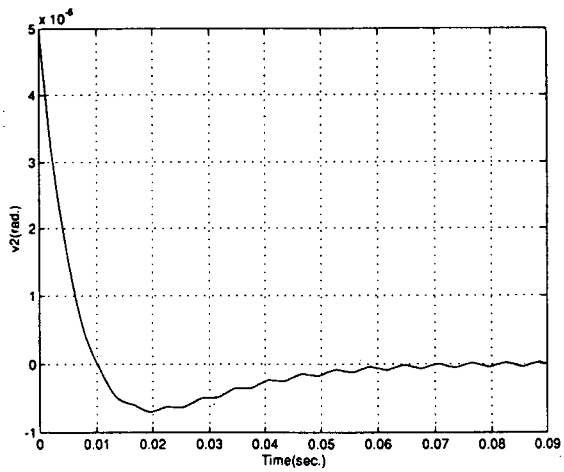

Figure 3. Time Transient Response of Sliding Mode Displacement Parameter v_2 For Rigid Rotor

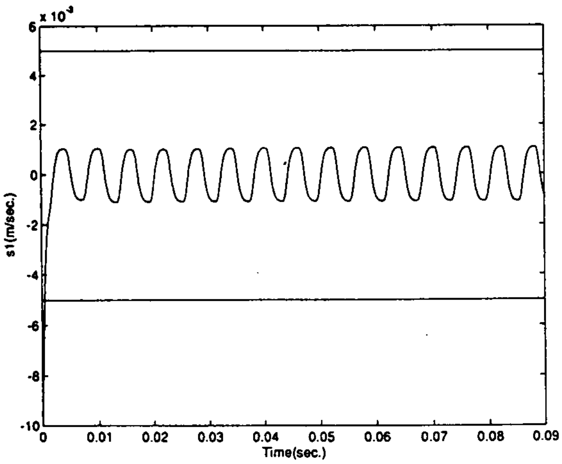

Figure 4. Time Transient Response of Sliding Mode Hyperspace Plane Parameter S_1 For Rigid Rotor

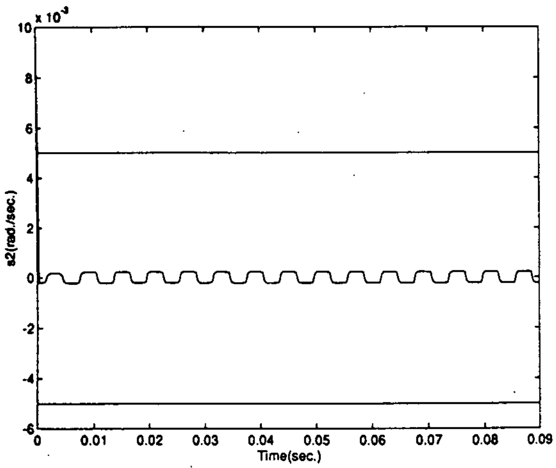

Figure 5. Time Transient Response of Sliding Mode Hyperspace Parameter S_2 For Rigid Rotor

CONCLUSIONS

A robust sliding mode controller has been developed for a planar rigid rotor on two magnetic bearings with linearized bias currents and mass unbalance. The sliding mode controller insures that the system response will move to a hyperplane, within a boundary layer. Estimates of uncertainty limits are obtained for magnetic bearing open loop stiffness, actuator gain, and mass unbalance. The sliding mode control was implemented for an example rigid rotor system on indentical magnetic bearings. The controller produced the desired system response when subjected to large initial transient conditions.

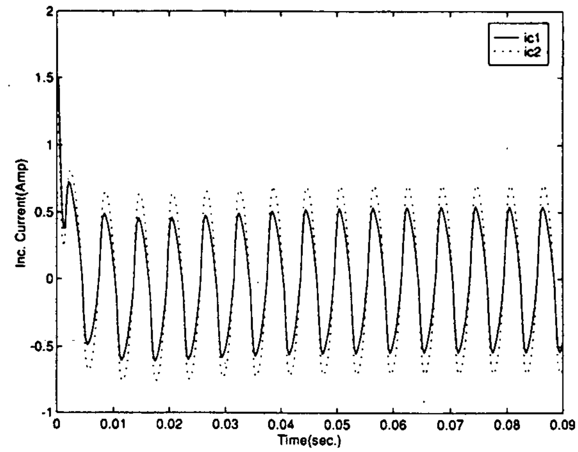

Figure 6. Magnetic Bearing Control Currents for Sliding Mode Control of Rigid Rotor Subject to Unbalance

REFERENCES

Allaire, P. E., Maslen, E. H., Humphris, R. R., Knospe, C. R., and Lewis, D. W., (1994), "Magnetic Bearings," Chapter in CRC Handbook of Lubrication and Tribology, Vol. III, E. R. Boozer, Ed., pp. 577-600.

Allaire, P. E., Fittro, R. L., Maslen, E. H., and Wakefield, W. C., "Measured Force/Current Relations in Solid Magnetic Thrust Bearings," Presented at ASME Turbo-Expo - Land, Sea,& Air, Houston, Texas, June 5-8, 1995, Journal of Engineering for Gas Turbines and Power, Trans. ASME, January, 1977.

Asada and Slotine (1986), Robot Analysis and Control, Wiley.

Charara, A., (1996), "Nonlinear Control of a Magnetic Levitation System Without Premagnetization," IEEE Transactions on Control Systems Technology, Vol. 4, No. 5, pp. 513-523.

Jiang, W., and Zhao, H. B., (1996), "Variable Structure Control for Active Magnetic Bearings," Proceedings of the Fifth International Symposium on Magnetic Bearings, August 28-30, 1996, Kanazawa, Japan, pp. 215.

Kanemitsu, Y., Ohsawa, M., and Marui, E., (1994), "Comparison of Control Laws for Magnetic Suspension," Proceedings of Fourth International Conference on Magnetic Bearings, Zurich, Switzerland, August, pp. 13-18.

Kao, C. K., and Sinha, A., (1992), "Coupled Modal Sliding Mode Control of Vibration in Flexible Structures," J. of Guidance, Vol. 15, No. 1, Jan-Feb, pp. 65-72.

Nonami, K., and Nishina, K., (1996), "Discrete Time Sliding Mode Control with Simple VSS Observer of Zero-Power Magnetic Bearing Systems," Proceedings of the Fifth International Symposium on Magnetic Bearings, August 28-30, 1996, Kanazawa, Japan, pp. 221-226.

Ponsart, J. C., Lottin, J., and Mouille, P., (1996), "Non Linear Control of Active Magnetic Bearings: Digital Implementation," Proceedings of the Fifth International Symposium on Magnetic Bearings, August 28-30, 1996, Kanazawa, Japan, pp. 77-82.

Rundell, A. E., Drakunov, S. V., and DeCarlo, R. A., (1996), "A Sliding Mode Observer and Controller for Stabilization of Rotational Motion of a Vertical Shaft Magnetic Bearing," IEEE Transactions on Control Systems Technology, Vol. 4, No. 5, pp. 598-608.

Sinha, A., (1990) "On the Design of Magnetic Suspension Systems," ASME-GT-90-239.

Sinha, A., and Kao, C. K., (1991), "Independent Modal Sliding Mode Control of Vibration of Flexible Structures," Journal of Sound and Vibration, Vol. 147, No. 2, pp. 352-358.

Sinha, A., and Miller, D. W., (1995), "Optimal Sliding-Mode Control of a Flexible Spacecraft Under Stochastic Disturbances," Journal of Guidance, Control, and Dynamics, Vol. 18, No. 3, May-June, pp. 486-492.

Spong, M. W., Vidyasgar, M. (1989), Robot Dynamics and Control, Wiley, 1989.

Tian, H., and Nonami, K., [1994], "Robust Control of Flexible Rotor-Magnetic Bearing Systems Using Discrete Time Sliding Mode Control," Proceedings of the Fourth International Symposium on Magnetic Bearings, August, Zurich, Switerland, pp. 47-52.

Tian, H., Yamamoto, M., Kuwahara, O., Hamabe, K., (1996), "Robust Control of High Speed Spindle-Magnetic Bearing System Using Sliding Mode Control and VVS Disturbance Observer," Proceedings of the Fifth International Symposium on Magnetic Bearings, August 28-30, 1996, Kanazawa, Japan, pp. 209-214.

Yamashita, K., Allaire, P. E., and Knospe, C. R., (1996) "Rotor Disturbance Attenuation using H_∞ Controller for Active Magnetic Bearings," Proceedings of the Fifth International Symposium on Magnetic Bearings, August 28-30, Kanazawa, Japan, pp. 227-232.

Voronkov, V. S., Gryaznov, K. V., Gryaznova, N. Y., (1994), "Robustness of the Stabilizing System in the Magnetic Suspension of a Rotating Shaft," Proceedings of the Fourth International Symposium on Magnetic Bearings, August, Zurich, Switerland, pp. 41-45.

NOMENCLATURE

A = Pole Face Area
F = Magnetic Bearing Force
g = Magnetic Gap Length
I = Bearing Coil Current
I_{xG} = Rotor Tranverse Moment of Inertia
k_u = Magnetic Actuator Gain
k_u = Magnetic Open Loop Stiffness
L = Rotor Length Between Bearings
m = Rotor Mass
N = Number of Coil Turns
s = Hyperspace Parameters
U = Rotor Unbalance
v = Sliding Mode Displacements