CONTROL DESIGN OF SENSORLESS MAGNETIC BEARINGS FOR RIGID ROTOR

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ABSTRACT

A model of axially-symmetric lateral motion of rigid rotor supported by two radial sensorless magnetic bearings was reduced to the modal model with complex variables. Next, this model was divided into two subsystems connected with two lateral modes: translation and rotation. The controller and observer was designed independently for each mode. This approach allows to express analytically the controller and observer gains in function of values of desired closed-loop poles and of plant model parameters. It was appeared that gyroscopic effects influences the controller and observer parameters.

INTRODUCTION

Depending on measurement system there are a few schemes to control magnetic suspension. They are called: current, voltage, magnetic flux, self-sensing, respectively (Schweitzer, Bleuler and Traxler, 1993). The self-sensing scheme is the cheapest one (Vischer, 1994). In this case the measurement system is reduced to easily realised measurement of currents in electromagnetic coils. The elimination of gap sensors causes that the rotating machinery structure is more compact.

Mueller et al. (1996) report on the first ever experiments to suspend a magnetic bearing rotor without position sensors in all four radial degrees of freedom. A stable rotational speed of 14400 rpm has been reached. Probably the speed limit was caused by using of rotor and magnetic bearing model in which the gyroscopic effect, and nonlinearities were omitted.

In this paper the global self-sensing control scheme for the rigid rotor is considered. In comparison with Mueller et al. (1996) the gyroscopic effect is taking into account. Since the axial movement of the rotor is independent from the lateral one we will analyse the control of four-degree-of-freedom lateral (radial) motion. The lateral motion is split into motion of two rigid rotor modes: translation and rotation ones what simplifies the control system design. Further simplification of the control system design can be obtained by introduction of complex variables (Gosiewski, 1997). Using the modal approach the control law in the analytical form will be found.

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MODEL OF THE PLANT

Usually the rotor and magnetic bearings are axially-symmetric. Therefore, introducing the complex variables and using well-established linearized model of self-sensing magnetic bearings (Kucera, 1997) we describe the lateral motion of the free rigid rotor as follows:

$$
\begin{aligned}
&m\ddot{r} = F_e + F_s, \\
&I_y\ddot{\phi} + jI_z\Omega\dot{\phi} = M_e + M_s, \\
&u_l = K_i \frac{dr_l}{dt} + (L_o + L_s)\frac{di_l}{dt} + Ri_l, \\
&u_r = K_i \frac{dr_r}{dt} + (L_o + L_s)\frac{di_r}{dt} + Ri_r.
\end{aligned} \tag{1}
$$

where: m - rotor mass, $I_x = I_y, I_z$ - inertia moments against the axes x,y,z, respectively, Ω - rotor angular velocity, R - coil resistance, L_s - leakage inductance, L_o -air-gap inductance, $r = r_x + jr_y$, $r_l = r_{lx} + jr_{ly}$, $r_r = r_{rx} + jr_{ry}$, - displacements of the rotor in mass centre, and in the (l- left, r -right) bearing planes, respectively, $\phi = \alpha + j\beta$ - angles of the rotor tilt, $F_e = F_{ex} + jF_{ey}$, $M_e = M_{ex} + jM_{ey}$ - electromagnetic forces and its moments reduced to the rotor mass centre, $F_s = F_{sx} + jF_{sy}$, $M_s = M_{sx} + jM_{sy}$ - external forces and its moments, $u_l = u_{lx} + ju_{ly}$, $u_r = u_{rx} + ju_{ry}$, $i_l = i_{lx} + ji_{ly}$, $i_r = i_{rx} + ji_{ry}$ - voltages, and currents, in left and right bearings, respectively.

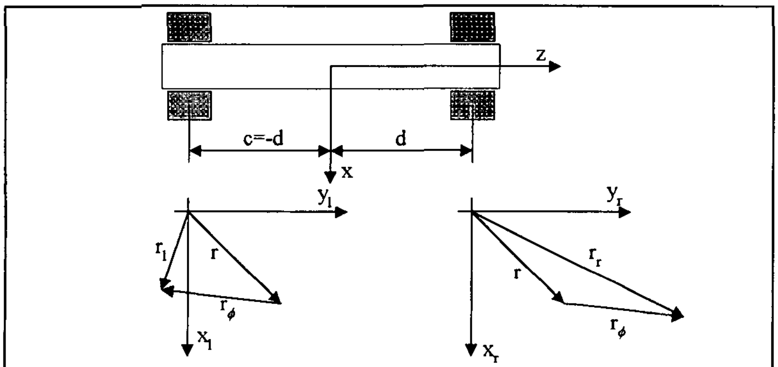

Figure 1. Rotor-magnetic bearings scheme with geometrical presentation of rotor modal and physical displacements in the left and right bearings.

To simplify the consideration we assume that rotor mass centre has equal distance $|c| = |d|$ from both bearings as it is seen in Fig.1. It is useful to change co-ordinate ϕ by the co-ordinate $r_\phi = z\phi$. In the case $z = d$ it is $r_\phi = d\phi$ and this co-ordinate is located in the plane of right bearing. Modal displacements can be expressed by the rotor displacements in the bearing planes:

$$
\begin{bmatrix} r \\ r_\phi \end{bmatrix} = \begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix}^{-1} \begin{bmatrix} r_l \\ r_r \end{bmatrix}, \tag{2}
$$

We express the force F_e and moment M_e by forces acting in the bearing planes: $F_e = F_{rr} + F_{rl}, M_e = F_{\phi r} d - F_{\phi l} c = 2F_{\phi m} d$, where in our case: $F_{rr} = F_{rl} = F_{rm}$, and $|F_{\phi r}| = |F_{\phi l}| = F_{\phi m}$. It means that the modal forces generated by left and right bearings are:

$$\begin{bmatrix} F_{rm} \\ F_{\phi m} \end{bmatrix} = \begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix}^{-1} \begin{bmatrix} F_l \\ F_r \end{bmatrix}. \tag{3}$$

Since the modal electromagnetic forces can be presented in the linearized form:

$$\begin{aligned} F_{rm} &= K_s r + K_i i_{rm}, \\ F_{\phi m} &= K_s r_\phi + K_i i_{\phi m}, \end{aligned} \tag{4}$$

then the plant model can be split into two modal subsystems.

I. Translation motion state-space model:

$$\begin{aligned} \dot{\mathbf{x}}_1 &= \mathbf{A}_1 \mathbf{x}_1 + \mathbf{b}_1 u_1 + \mathbf{b}_{z1} f_{z1}, \\ y_1 &= \mathbf{c}_1 \mathbf{x}_1, \end{aligned} \tag{5}$$

where: $\mathbf{x}_1 = [r \quad \dot{r} \quad i_{rm}]^T$, $u_1 = u_{rm}$, $f_{z1} = F_{sm}$, $y_1 = i_{rm}$,

$$\mathbf{A}_1 = \begin{bmatrix} 0 & 1 & 0 \\ v_1 & 0 & v_2 \\ 0 & -v_3 & -v_4 \end{bmatrix}, \ \mathbf{b}_1 = [0 \quad 0 \quad v_5]^T, \ \mathbf{b}_{z1} = [0 \quad 2/m \quad 0]^T, \ \mathbf{c}_1 = [0 \quad 0 \quad 1],$$

and: $v_1 = \dfrac{2K_s}{m}$, $v_2 = \dfrac{2K_i}{m}$, $v_3 = \dfrac{K_i}{L_o + L_s}$, $v_4 = \dfrac{R}{L_o + L_s}$, $v_5 = \dfrac{1}{L_o + L_s}$.

II. Rotation motion state-space model:

$$\begin{aligned} \dot{\mathbf{x}}_2 &= \mathbf{A}_2 \mathbf{x}_2 + \mathbf{b}_2 u_2 + \mathbf{b}_{z2} f_{z2}, \\ y_2 &= \mathbf{c}_2 \mathbf{x}_2, \end{aligned} \tag{6}$$

where: $\mathbf{x}_2 = [r_\phi \quad \dot{r}_\phi \quad i_{\phi m}]^T$, $u_2 = u_{\phi m}$, $f_{z2} = F_{\phi s}$, $y_2 = i_{\phi m}$,

$$\mathbf{A}_2 = \begin{bmatrix} 0 & 1 & 0 \\ v_6 & -v_8 & v_7 \\ 0 & -v_3 & -v_4 \end{bmatrix}, \ \mathbf{b}_2 = [0 \quad 0 \quad v_5]^T, \ \mathbf{b}_{z2} = [0 \quad 2d^2/I_y \quad 0]^T, \ \mathbf{c}_2 = [0 \quad 0 \quad 1],$$

and: $v_6 = \dfrac{2d^2 K_s}{I_y}$, $v_7 = \dfrac{2d^2 K_i}{I_y}$, $v_8 = j(I_z \Omega)/I_y$.

CONTROL OF TRANSLATION MOTION

The controller will be designed in typical for self-sensing control schemes with a state-feedback control law and an observer. We assume the control law in the form:

$$u_1 = -\mathbf{k}_1 \hat{\mathbf{x}}_1, \tag{7}$$

where: $\mathbf{k}_1 = [k_1 \quad k_2 \quad k_3]$ is the controller gain matrix. According to the pole-placement method we assume the following eigenvalues (poles): p_1, p_2, p_3 of the closed-loop system. For these eigenvalues the elements of the gain matrix are as follows:

$$k_3 = -(p_1 + p_2 + p_3 + v_4)/v_5,$$
$$k_2 = \frac{p_1 p_2 + p_1 p_3 + p_2 p_3}{v_2 v_5} + \frac{v_1 - v_2 v_3}{v_2 v_5}, \quad k_1 = \frac{v_1(k_3 + v_4) - p_1 p_2 p_3}{v_2 v_5}.$$

As it results from above formulae if we will choose the poles as real or complex conjugate the elements of the gain matrix are also real. It means that the control of the translation motion for x and y directions can be designed independently.

In our case the state equation of the full-order observer is as follows:

$$\dot{\hat{\mathbf{x}}}_1 = \mathbf{F}_1 \hat{\mathbf{x}}_1 + \mathbf{b}_1 u_1 + \mathbf{h}_1 y_1, \tag{8}$$

where: $\mathbf{F}_1 = [\mathbf{A}_1 - \mathbf{h}_1 \mathbf{c}_1]$ is the state matrix of the observer. For the chosen eigenvalues (poles): p_{10}, p_{20}, p_{30} of $\mathbf{F}_1$ the elements of the matrix $\mathbf{h}_1 = [h_1 \quad h_2 \quad h_3]^T$ are calculated as:

$$h_3 = -(p_{10} + p_{20} + p_{30} + v_4),$$
$$h_2 = \frac{-(p_{10} p_{20} + p_{10} p_{30} + p_{20} p_{30}) + v_2 v_3 - v_1}{v_3}, \quad h_1 = \frac{p_{10} p_{20} p_{30} - v_1(h_3 + v_4)}{v_1 v_3}.$$

The elements of the observer matrix are real for real or complex conjugate poles chosen. It means that the observer for the translation motion can be designed for x and y directions independently.

CONTROL OF ROTATION MOTION

Similarly to the translation motion we will calculate the gain matrices of controller and the observer to design the control system for rotation motion. The control law has the form:

$$u_2 = -\mathbf{k}_2 \hat{\mathbf{x}}_2, \tag{9}$$

where elements of the controller gain matrix $\mathbf{k}_2 = [k_4 \quad k_5 \quad k_6]$ for the assumed closed-loop poles: p_4, p_5, p_6 are as follows:

$$k_6 = -(p_4 + p_5 + p_6 + v_4 + v_8)/v_5,$$

$$k_5 = \frac{p_4 p_5 + p_4 p_6 + p_5 p_6}{v_7 v_5} + \frac{v_6 - v_7 v_3 - v_8(k_6 + v_4)}{v_7 v_5}, \quad k_4 = \frac{v_6(k_6 + v_4)}{v_7 v_5} - \frac{p_4 p_5 p_6}{v_7 v_5}.$$

Since the value v_8 is a complex number the above elements of the gain matrix are also complex numbers for any chosen poles. It means that the controller has cross gains between the axes x and y in the given bearing plane.

For rotation motion the state equation of the full-order observer is as follows:

$$\dot{\hat{\mathbf{x}}}_2 = \mathbf{F}_2 \hat{\mathbf{x}}_2 + \mathbf{b}_2 u_2 + \mathbf{h}_2 y_2, \tag{10}$$

where $\mathbf{F}_2 = [\mathbf{A}_2 - \mathbf{h}_2 \mathbf{c}_2]$ is the state matrix of the observer. For the chosen poles for the observer: p_{40}, p_{50}, p_{60} we have the elements of the matrix $\mathbf{h}_2 = [h_4 \quad h_5 \quad h_6]^T$:

$$h_6 = -(p_{40} + p_{50} + p_{60} + v_4 + v_8),$$

$$h_5 = -\frac{p_{40} p_{50} + p_{40} p_{60} + p_{50} p_{60}}{v_3} + \frac{v_8(h_6 + v_4) - v_6 + v_7 v_3}{v_3}, \quad h_4 = \frac{p_{40} p_{50} p_{60}}{v_6 v_3} - \frac{(h_6 + v_4) v_8}{v_6 v_3},$$

Since v_8 is a complex number the elements of the matrix $\mathbf{h}_2$ are also complex numbers. It means that to design the observer for rotation motion we should use the cross-coupled signals measured in x and y directions.

GLOBAL CONTROL

Completing the equations (5-10) into one matrix equation we obtain the global model of the closed loop system:

$$\begin{bmatrix} \dot{\mathbf{x}} \\ \dot{\hat{\mathbf{x}}}_1 \\ \dot{\mathbf{x}}_2 \\ \dot{\hat{\mathbf{x}}}_2 \end{bmatrix} = \begin{bmatrix} \mathbf{A}_1 & -\mathbf{b}_1\mathbf{k}_1 & 0 & 0 \\ \mathbf{h}_1\mathbf{c}_1 & \mathbf{A}_1 - \mathbf{h}_1\mathbf{c}_1 - \mathbf{b}_1\mathbf{k}_1 & 0 & 0 \\ 0 & 0 & \mathbf{A}_2 & -\mathbf{b}_2\mathbf{k}_2 \\ 0 & 0 & \mathbf{h}_2\mathbf{c}_2 & \mathbf{A}_2 - \mathbf{h}_2\mathbf{c}_2 - \mathbf{b}_2\mathbf{k}_2 \end{bmatrix} \begin{bmatrix} \mathbf{x}_1 \\ \hat{\mathbf{x}}_2 \\ \mathbf{x}_2 \\ \hat{\mathbf{x}}_2 \end{bmatrix} + \begin{bmatrix} \mathbf{b}_{z1} & 0 \\ 0 & 0 \\ 0 & \mathbf{b}_{z2} \\ 0 & 0 \end{bmatrix} \begin{bmatrix} F_{sm} \\ F_{\phi m} \end{bmatrix},$$

$$\begin{bmatrix} i_{rm} \\ i_{\phi m} \end{bmatrix} = \begin{bmatrix} \mathbf{c}_1 & 0 & 0 & 0 \\ 0 & 0 & \mathbf{c}_2 & 0 \end{bmatrix} \begin{bmatrix} \mathbf{x}_1 \\ \hat{\mathbf{x}}_2 \\ \mathbf{x}_2 \\ \hat{\mathbf{x}}_2 \end{bmatrix}, \tag{11}$$

It is so called „modal" global system. In this model the modal complex variables: $F_{sm}, F_{\phi m}$ are inputs while the modal complex variables: $i_{rm}, i_{\phi m}$ are the outputs. In the real variables it is a four-input four-output system with 24-element state vector.

The global model can be expressed in the physical complex input and output variables (forces and currents in the bearing planes):

$$
\begin{bmatrix} \dot{\mathbf{x}} \\ \dot{\hat{\mathbf{x}}}_1 \\ \dot{\mathbf{x}}_2 \\ \dot{\hat{\mathbf{x}}}_2 \end{bmatrix} = \begin{bmatrix} \mathbf{A}_1 & -\mathbf{b}_1\mathbf{k}_1 & 0 & 0 \\ \mathbf{h}_1\mathbf{c}_1 & \mathbf{A}_1 - \mathbf{h}_1\mathbf{c}_1 - \mathbf{b}_1\mathbf{k}_1 & 0 & 0 \\ 0 & 0 & \mathbf{A}_2 & -\mathbf{b}_2\mathbf{k}_2 \\ 0 & 0 & \mathbf{h}_2\mathbf{c}_2 & \mathbf{A}_2 - \mathbf{h}_2\mathbf{c}_2 - \mathbf{b}_2\mathbf{k}_2 \end{bmatrix} \begin{bmatrix} \mathbf{x}_1 \\ \hat{\mathbf{x}}_2 \\ \mathbf{x}_2 \\ \hat{\mathbf{x}}_2 \end{bmatrix} + \begin{bmatrix} \mathbf{b}_{z1} & 0 \\ 0 & 0 \\ 0 & \mathbf{b}_{z2} \\ 0 & 0 \end{bmatrix} \begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix}^{-1} \begin{bmatrix} F_l \\ F_r \end{bmatrix},
$$

$$
\begin{bmatrix} i_l \\ i_r \end{bmatrix} = \begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} \mathbf{c}_1 & 0 & 0 & 0 \\ 0 & 0 & \mathbf{c}_2 & 0 \end{bmatrix} \begin{bmatrix} \mathbf{x}_1 \\ \hat{\mathbf{x}}_2 \\ \mathbf{x}_2 \\ \hat{\mathbf{x}}_2 \end{bmatrix}. \qquad (12)
$$

We use the same state vector in the modal and physical models of the closed-loop control systems. Therefore both models has the similar structure and are presented in Fig. 2. There are shown the modal and physical input and output signals (before and after matrix transformation, respectively).

CLOSED-LOOP POLES

In the case of the pole-placement method for MIMO systems there is a problem how to choose the closed-loop poles and control law to save the control energy and to score the control aim. It is much simpler to design the control system using the modal approach (SISO) which is proposed in this paper. Silverberg and Morton (1989) proved that for lightly damped systems the modal controlling force is near minimal when the closed-loop system frequencies are kept almost unchanged in the comparison to open-loop frequencies (in mechanical systems it means that the vibration modes are kept unchanged) while the damping of modes (negative real parts of the eigenvalues) should be chosen to assure the stability and proper decay time of the transient process. For strong damping the optimal frequencies are moved in the direction of smaller values.

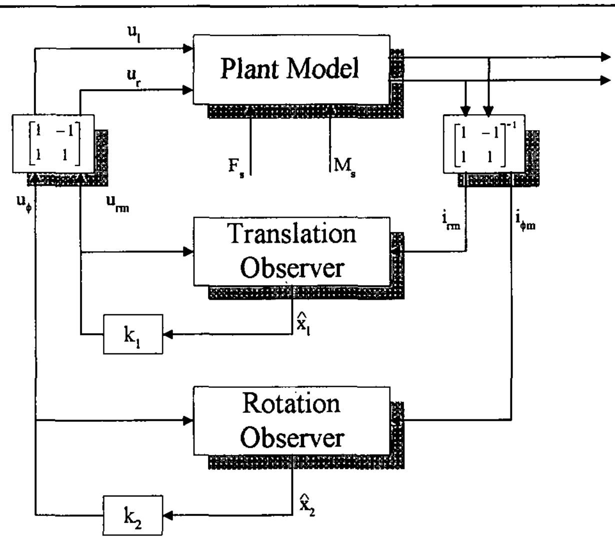

Figure 2. Closed-loop system with uncoupled vibration modes.

From the other hand (Kucera, 1997) has shown that the system is not observable in the range of low and high frequencies. Therefore the closed-loop poles should be located in the middle frequencies. That range of frequencies is indicated by open-loop poles.

The open loop poles are: $\pm j\sqrt{-2K_s/m}$, $-R/(L_o+L_s)$; for the translation motion and: $\cong j\left[\pm\sqrt{-\frac{2d^2K_s}{I_y}}+\frac{I_z}{2I_y}\Omega(1\pm\frac{1}{2})\right]$, $-R/(L_o+L_s)$; for the rotation motion. The first formula on the rotation motion poles is valid when second component is much smaller than first one (it means that angular velocity is low). For higher angular velocities that formula is more compound.

OPEN-LOOP SYSTEM IDENTIFICATION

The self-sensing magnetic bearings with linear controller are very sensitive to model uncertainty. Therefore the model should be carefully identify before the start up phase and next periodically identify during the operation. It can be realised by state observer used in the control loop.

In the first stage it is useful to use the position sensor and controller PD^2 since the open-loop state and control matrices in that control scheme are the same as in the self-sensing one. Differences are in the measurement matrices. For example for the translation motion there is:

$$u_1=-(a_2\ddot{y}+a_1\dot{y}+a_o y)+r=-\mathbf{k}_1\mathbf{x}_i+\mathbf{r},$$
$$\mathbf{k}_1=\begin{bmatrix}a_o+a_2v_1 & a_1 & a_2v_2\end{bmatrix},$$
$$\mathbf{c}_1=\begin{bmatrix}1 & 0 & 0\end{bmatrix},$$

where y is output (displacement) and r is a persistent excitation.

The identification procedure is as follows.

- Design of discrete model and replacement of existing observer by dead-beat observer.
- Experiment and split of the measured signals into their modal parts.
- Calculation of closed-loop system Markov parameters between the exciting signal and state signals obtained by observers for translation and rotation modes..
- Calculation of open-loop system Markov parameters.
- Calculation of dynamic system realisation $\{\tilde{\mathbf{A}},\tilde{\mathbf{B}},\tilde{\mathbf{C}}\}$,
- Calculation of physical system realisation $\{\hat{\mathbf{A}},\hat{\mathbf{B}},\hat{\mathbf{C}}\}$.

More details about identification procedure will be presented in (Gosiewski, Falkowski,1998)

Periodic identification can be realised by observer designed for self-sensing control scheme.

NUMERICAL EXAMPLE

We assumed that the rotor and magnetic bearings have got the following parameters: $m=1.8[kg]$, $J_x=0.0032[kgm^2]$, $J_y=0.0024[kgm^2]$, $x_o=0.001[m]$, $d=0.125[m]$

$i_o = 0.35[A]$, $R = 17.5[om]$, $L_o = 0.184[mH]$ $L_s = 0.086[mH]$, $K = 4.64 * 10^{-5}$, $K_i = 64.8[A/m]$, $K_s = 2268[N/m]$.

For non-rotating rotor (Ω=0) the open-loop doubled poles are: -64.8, ±j224.5 - for translation motion, and: -64.8, ±j538.4- for rotation motion. For rotating rotor with small angular velocity a part of poles of the rotation motion are changing according to approximated formulae ±j(538.4+Ω), ±j(538.4+0.333Ω), The number of full-order observer poles equals the number of plant poles and values of poles should assure the faster decay of observer dynamics than the transient dynamics of the plant. The regulator was designed for non-rotating rotor according to procedure LQR. The closed-loop doubled poles are: -160.6, -112.8±j87.5 -for translation motion, and: -349,4, -207.4 ±j104.2- for rotation motion. As we can see the increase of damping causes the reduction of frequencies of the closed-loop system with optimal control. It agrees with result of (Silverberg and Morton, 1989).

The observer poles are assumed as negative real numbers with values in the range <1200, 2500>. Choosing the observer poles we should take into account that a part of the open-loop poles increase with the increase of the rotor angular velocity.

The closed-loop system with LQR controller and assumed observer was excited by step function (voltage: 1V). The controller and observer are designed for non-rotating rotor. The rotor displacement caused by translation mode excitation is shown in Figure 3. Dynamics of translation mode does not change for different rotor angular velocities. The rotor displacement in bearing plane caused by rotation mode excitation is shown in Figure 4 for Ω

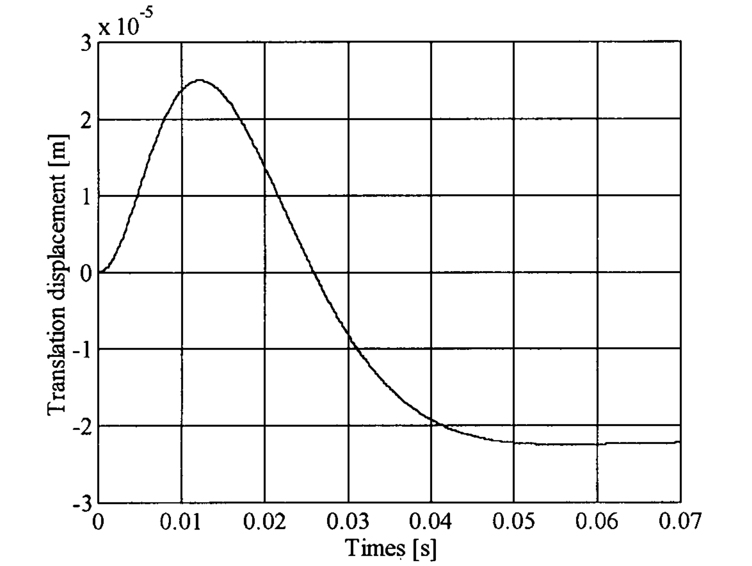

Figure 3. Rotor displacement in bearing plane to the step excitation of translation mode.

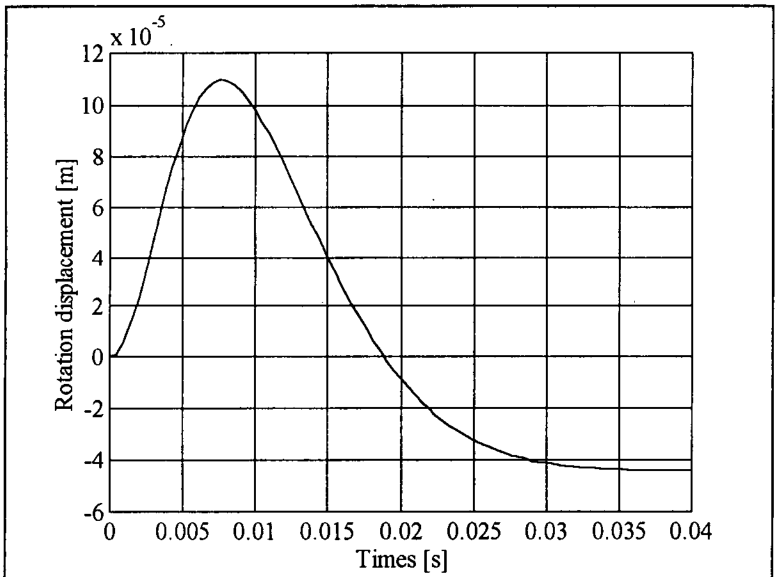

Figure 4. Rotor displacement in bearing plane to the step excitation of rotation mode for Ω = 0 [rad/s].

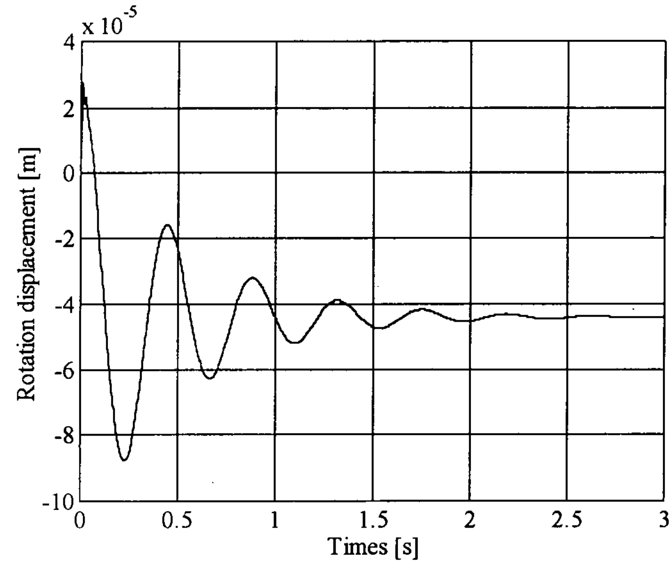

Figure 5. Rotor displacement in bearing plane to the step excitation of rotation mode for Ω = 1256 [rad/s].

= 0 [rad/s] and in Figure 5 for Ω = 1256 [rad/s] = 12000 [rev/min]. We see that giroscopic effects strongly influence the closed-loop dynamics. It was found the closed-loop system is unstable for angular velocities crossing the value Ω = 1865 [rad/s] = 17820 [rev/min]. If we reduce the values of the observer poles the system is unstable for lower angular velocity.

CONCLUSIONS

Sensorless magnetic bearings are desired in many applications. Since they are open-loop unstable and closed-loop nonminimal-phase the very precise model of the plant is necessary in order to find an observer-controller structure to stabilise system. The full model (with gyroscopic effect) of rigid rotor lateral motion and of two radial magnetic bearings has been developed in the paper.

Symmetry of bearings and rotor has allowed to reduce the real model to complex model with complex variables. Next, this model was divided into two subsystems connected with two lateral modes: translation and rotation. The controller and observer was designed independently for each mode. This approach allows to express analytically the controller and observer gains in function of values of desired closed-loop poles and of plant model parameters. It was appeared that gyroscopic effects strongly influences the controller and observer parameters. It is desired to design the adaptive control (as a gain scheduling versus angular velocity) to adjust the controller.

Such approach gives us the possibility to shape the rotor dynamics in the desired manner. So more there are possibility to obtain the information about external forces and moments (for example about inertial excitation in measurement instruments, Gosiewski and Falkowski (1996)) acting on the rotor by the measurement of the modal displacement and currents in the magnetic bearing planes.

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