MODELING AND CONTROL OF A MAGNETIC ROTOR-BEARING SYSTEM WITH MAGNETIC COUPLING

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ABSTRACT

This study deals with the modeling and nonlinear control of a magnetic rotor-bearing system. The system is taken from a high-speed spindle in a machine center. Instead of the commonly used uncoupled linear models, a magnetically coupled nonlinear model is considered here. A robust sliding mode controller is designed based on the coupled nonlinear model. It is found from numerical simulations that the closed-loop system not only achieves stabilization and high precision, but also is robust to parameter variations, impact loads, external disturbances and sensor noises.

I. INTRODUCTION

Bearings are used to support the rotor at precision position and sustain the forces transmitted from the rotor. Conventional bearings make contacts with the rotor, leading to the problems of lubrication and low rotating speed. Active magnetic bearings (AMB), on the other hand, make use of the electromagnetic force to provide non-contact supports, resulting in the advantages of long life and low maintenance cost, high rotating speed, and the elimination of complex lubrication systems. AMB has found wide applications in aerospace, physics, robotics and industry, even in special environments such as very low temperature and high vacuum situation (Lee, Ko and Hsiao, 1994). Modern industries demand high yield rate, which requires high rotor speed with the same or better level of accuracy and stiffness. In this regard, AMB is an inevitable substitute for conventional bearings.

However, AMB is an open loop unstable system subjected to the problems such as nonlinearities, parameter variations, magnetic coupling and saturation, external disturbances, modeling errors, and mass unbalances, etc. Moreover, these problems usually become worse as the rotor's rotating speed and/or the system's output horsepower increase. Hence, AMB needs sophisticated control technology in order to be useful. The controller provides the

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stiffness, damping, and accuracy of an AMB system. Controller is the heart of an AMB system. It is the controller that makes AMB more expensive than the conventional bearings.

The importance of AMB has led to many research investigations in the literature. Most previous studies considered magnetically uncoupled linear model (Nonami and Ito, 1996). In other words, two assumptions need to be satisfied. First, the magnetic force in x (y, respectively) direction depends only on the air gap in x (y, respectively) direction. Under this magnetic uncoupling assumption, AMB can be regarded as a combination of several single-input-single-output systems, which greatly simplifies the controller design. Second, the shaft displacement is assumed to be small, resulting in linear rotor dynamics and linear magnetic forces. With these assumptions in hand, a variety of control methods have been attempted for AMB, such as PID (Boehm et al., 1990), LQR (Kim and Lee, 1992), neural network (Ishimatsu et al., 1991), repetitive control (Curtelin et al., 1993), learning control (Higuchi, Otsuka and Mizuno, 1992), Q-parameterization theory (Mohamed et al., 1995), adaptive control (Lum, Coppola and Bernstein, 1996), H_∞ control (Shiau, Sheu and Yang, 1997), μ-synthesis (Nonami and Ito, 1996), fuzzy control (Weidemann and Xiao, 1994), and sliding mode control (Rundell, Drakunov and DeCarlo, 1996).

Although easy in analysis and design, the linear model has strict limitations in practice. A magnetic rotor-bearing system is in fact a highly nonlinear system with magnetic coupling. The problems of linear model will become more and more apparent as the rotor speed increases. Under high rotating speed, a little unbalance mass and/or disturbance can easily drive the system into the nonlinear region, and magnetic coupling can amplify this effect. When the system has large deviation away from the operating point, the performance of the controller based on uncoupled linear model will become worse, or even unstable. So the reliability is the problem of the uncoupled linear model. In this study we use the coupled nonlinear model of the magnetic rotor-bearing system.

Several control techniques could be employed for nonlinear AMB systems, including adaptive control, fuzzy control (Weidemann and Xiao, 1994), and sliding mode control (Sinha, Wang and Mease, 1991). Adaptive control is used to the system with uncertainties, but it needs to know the form of uncertainties and the controller is complicated. The design of fuzzy controller, on the other hand, does not require the exact model of the system, but it lacks systematic design steps and also has difficulties in stability analysis. Unlike adaptive control, sliding mode controller allows strong uncertainties of any form and is simple in structure. Also, it can be obtained systematically. Therefore, the sliding mode control method is adopted in the present work. The previous studies on AMB using sliding mode control method were either for linear model (Rundell, Drakunov and DeCarlo, 1996), or for simple, magnetically uncoupled nonlinear model (Sinha, Wang and Mease, 1991). It will be shown in this paper that sliding mode control can do even better under reasonable control current.

The objective of this study is to design a robust controller for a magnetically coupled nonlinear AMB system which consists of a rotor shaft supported by two magnetic bearings at two ends. The job of the controller is to maintain the shaft at the center position in the presence of model uncertainties, external disturbance and sensor noise. In other words, robustness is our major concern in addition to accuracy and stabilization. To this aim, a continuous version of sliding mode control method will be applied to this complicated, nonlinear, uncertain system.

The outline of this paper is as follows. After the introduction, we begin in Section II with a magnetic circuit analysis to obtain a relatively complete mathematical model for the AMB system. In Section III, this mathematical model is used to design a robust sliding mode controller. Next in Section IV, the analysis is verified by the simulations with parameter variations, impact loads, external disturbance, and sensor noise. Finally, conclusions are made in Section V.

II. MODELING OF THE AMB SYSTEM

The magnetic rotor-bearing system under study is composed of a rotor and two radial magnetic bearings. The structure is shown in Fig. 1. The horizontal rotor is assumed to be rigid and symmetric. The radial magnetic bearings are 4-pole electromagnetic bearings, and the winding structure is shown in Fig. 2. The radial motion is controlled actively by the magnetic bearings and the axial motion is constrained passively. In Fig. 2, $i_1 \sim i_4$ represent the coil currents, and $\Phi_1 \sim \Phi_4$ represent the magnetic fluxes at the air gaps #1~#4 respectively.

The magnetic flux Φ_i can be obtained in terms of rotor displacements and coil currents by the analysis of the magnetic circuit shown in Fig. 3. Then the magnetic forces at x and y directions can be obtained by Faraday's law as

$$F_x = \frac{1}{2\mu_0 A_g}\left(\Phi_4^{\,2} - \Phi_2^{\,2}\right) \text{ and } F_y = \frac{1}{2\mu_0 A_g}\left(\Phi_1^{\,2} - \Phi_3^{\,2}\right) \tag{1}$$

where μ_0 is the magnetic permeability of the air and A_g is the effective cross sectional area at air gap.

In electromagnetic bearings, each coil current is divided into two parts, i.e., the control current and the bias current, in such a way that

$$i_1 = i_{1b} + i_y, i_2 = i_{2b} - i_x, i_3 = i_{3b} - i_y, i_4 = i_{4b} + i_x, \tag{2}$$

where i_x and i_y are the control currents in the x and y directions, and i_{1b}~i_{4b} are the bias currents. The bias currents are constant and are used to suspend the rotor in the equilibrium point without any control currents. However, the bias currents alone can not stabilize the equilibrium point. When the rotor is not in the equilibrium point, the control currents are used to actively bring the rotor back to the equilibrium point.

The nonlinear magnetic forces can be well approximated by its linearized version if the bias currents are large enough [6]. The disadvantage of large bias currents is that the AMB will consume more energy and have the magnetic saturation problems. In this study, by considering the complete nonlinear magnetic force, we are not restricted on large bias currents. There are many possible choices of the bias currents, which can be determined by solving equation (1) with $F_x = F_y = 0$. It is found that different sets of bias currents may result in different performance, even with the same controller. How to choose an optimal set of bias currents is a current research topic and will not be pursued here. By trial and error, our choice of bias currents in this work is $i_{1b} = 4i_b$, $i_{2b} = 3i_b$, $i_{3b} = -2i_b$, $i_{4b} = 3i_b$, $i_b = \left(G\sqrt{3mg}\right)/\left(6N\sqrt{\mu_0 A_g}\right)$, where G is the nominal air gap and m is the mass of the rotor shaft.

By combining the rotor dynamics and the magnetic forces, the equations of motions for the complete magnetic-rotor bearing system can now be obtained as

$$
\begin{aligned}
\ddot{x}_1 &= -\frac{\left(I_{zz}^c - I_{xx}^c\right)\omega_z\left(\dot{y}_1 - \dot{y}_2\right)}{2I_{yy}^c} + \left(\frac{1}{m} + \frac{L^2}{I_{yy}^c}\right)\left(F_{x1} + \Delta F_{x1}\right) + \left(\frac{1}{m} - \frac{L^2}{I_{yy}^c}\right)\left(F_{x2} + \Delta F_{x2}\right) \\
\ddot{x}_2 &= \frac{\left(I_{zz}^c - I_{xx}^c\right)\omega_z\left(\dot{y}_1 - \dot{y}_2\right)}{2I_{yy}^c} + \left(\frac{1}{m} - \frac{L^2}{I_{yy}^c}\right)\left(F_{x1} + \Delta F_{x1}\right) + \left(\frac{1}{m} + \frac{L^2}{I_{yy}^c}\right)\left(F_{x2} + \Delta F_{x2}\right) \\
\ddot{y}_1 &= \frac{\left(I_{yy}^c - I_{zz}^c\right)\omega_z\left(\dot{x}_2 - \dot{x}_1\right)}{2I_{xx}^c} + \left(\frac{1}{m} + \frac{L^2}{I_{xx}^c}\right)\left(F_{y1} + \Delta F_{y1}\right) + \left(\frac{1}{m} - \frac{L^2}{I_{xx}^c}\right)\left(F_{y2} + \Delta F_{y2}\right) - g \\
\ddot{y}_2 &= -\frac{\left(I_{yy}^c - I_{zz}^c\right)\omega_z\left(\dot{x}_2 - \dot{x}_1\right)}{2I_{xx}^c} + \left(\frac{1}{m} - \frac{L^2}{I_{xx}^c}\right)\left(F_{y1} + \Delta F_{y1}\right) + \left(\frac{1}{m} + \frac{L^2}{I_{xx}^c}\right)\left(F_{y2} + \Delta F_{y2}\right) - g
\end{aligned}
\tag{3}
$$

where x_1, y_1, x_2 and y_2 are the displacements at *magnetic bearing #1 and #2*; F_{x1}, F_{y1}, F_{x2} and F_{y2} are the resultant magnetic forces generated by the *magnetic bearing #1 and #2* respectively; ΔF_{x1}, ΔF_{y1}, ΔF_{x2} and ΔF_{y2} are the uncertainties or the external disturbances of F_{x1}, F_{y1}, F_{x2} and F_{y2}; L is half of the length of the rotor; ω_z is the rotation speed of the rotor, and I_{xx}^c, I_{yy}^c and I_{zz}^c are the mass moments of inertia with respect to the center of gravity.

III. SLIDING MODE CONTROLLER DESIGN

The magnetic rotor-bearing system derived in the previous section possesses strong uncertainties including modeling errors, parameter variations, and external disturbances. In order to function well under these uncertainties, robustness is our major concern in addition to the stability for the controller of AMB. Thus we shall design a robust controller via the sliding mode control method in this section.

The idea of sliding mode control is to design a sliding manifold such that the dynamics on this manifold will be asymptotically stable. By switching the control inputs, we can change the structure of the system in such a way that any initial condition of the system can be brought to the sliding manifold in finite time and then slide along the sliding manifold to the origin of the system. When the system is on the sliding mode, only the sliding manifold will affect the dynamics of the system and hence robustness can be achieved.

Let us define $\eta = [x_1 \quad y_1 \quad x_2 \quad y_2]^T$ and $\xi = [\dot{x}_1 \quad \dot{y}_1 \quad \dot{x}_2 \quad \dot{y}_2]^T$. Then the system in equation (3) can be rewritten in the regular form as

$$\dot{\eta} = \xi \tag{4}$$

$$\dot{\xi} = f(\eta,\xi) + G(\eta,\xi)[u + \delta(\eta,\xi,u)] \tag{5}$$

where $f(\eta,\xi)$ contains the magnetic and inertial forces, u is composed of control currents which serve as our control inputs, $\delta(\eta,\xi,u)$ is related to the uncertainties represented by ΔF_i's in equation (3).

Choosing a linear sliding manifold

$$\sigma = \xi + b\eta = 0,\ b>0 \tag{6}$$

and by a standard design procedure for the sliding mode control (Khalil, 1996), we can obtain the sliding mode control as

$$u = [G(\eta,\xi)]^{-1}\left[-f(\eta,\xi) - b\xi - \frac{\rho}{1-k} sat\left(\frac{\sigma}{\varepsilon}\right)\right]$$

where $\rho \geq 0$ and $0 \leq k < 1$ are related to the bounds on the uncertainties, ε is a small but positive constant controlling the accuracy and $sat(\cdot)$ is the saturation function. Note that the transient performance of the system can be tuned by the parameter b. Lager b will yield faster responses.

IV. SIMULATIONS AND DISCUSSIONS

In order to illustrate and verify the analysis presented in previous sections, an example AMB system is numerically investigated here. As a comparison, a linear state feedback controller based on pole assignment is also designed.

The system consists of two identical radial AMBs and a rigid rotor. The parameters described below are taken from a practical system that will be used in the high-speed spindle (2×10^4 rpm) for a machine center in a future project. The rigid rotor has mass of m=7.5kg, with length $2L$=410mm and diameter d=58mm. The effective cross-area of the air gap is $A_g = 10^{-3}\,\mathrm{m}^2$, with nominal air gap of G=0.3mm. The turns of the winding coil per pole is N=300 and the permeability of the air is $\mu_0 = 4\pi\times10^{-7}$ (Wb/A·m). Nominal rotating speed is set to $\omega_z = 2\times10^3$rad/s, corresponding approximately to 2×10^4 rpm. In addition, it is assumed to have a backup bearing at half of the air gap to prevent the damage of AMBs in case of failure. Therefore, The maximum displacement of the rotor in both x and y directions is 1.5×10^{-4} m. The parameters for the sliding mode controller are chosen as follows : b=400, ρ=10, k=0.5, and ε=0.0004. Recall that the transient response is determined by b, accuracy by ε, and ρ and k are related to the upper bounds for the uncertainties.

The numerical simulations shown below are based on the equation (3) with the same initial conditions but different uncertainties ΔF_i's. All initial conditions are taken to be

$$p(0)=-1.5\times10^{-4}[1 \quad 1 \quad 1 \quad 1 \quad 0 \quad 0 \quad 0 \quad 0],$$

where $p(t)=[x_1(t) \quad y_1(t) \quad x_2(t) \quad y_1(t) \quad \dot{x}_1(t) \quad \dot{y}_1(t) \quad \dot{x}_2(t) \quad \dot{y}_2(t)]$. The chosen initial condition is to simulate the worst case that the rotor is starting at the farthest position from rest, and to be brought to the center position. The problem of mass unbalance is not considered here and is the subject of a current research. Because all initial conditions at bearing #1 and bearing #2 are the same, the response at each bearing in x and y directions are similar. Hence, only the response in the x direction at bearing #1 will be shown below.

There are three cases of uncertainties considered. They are

A. No uncertainties are present. In other words, the mathematical model is perfect. The time response, control current, and state trajectory of x displacement at bearing #1 are shown in Figures 4 to 6.

B. Parameter variations are allowed. Two levels of variations of each system parameters ---18% and 30% variation of the original value---are considered. Figure 7 shows the time response of x displacement at bearing #1 under these variations.

C. Impact loads, external disturbances, and sensor noises are added on in addition to the parameter variations (10%). The impact load is used to simulate the situation when the cutting tool starts contacting the object. 50N impact loads are added in both x and y directions at t=0.03 sec and t=0.04 sec (when the rotor has been brought to a small neighborhood of the origin). The external noises are assumed to be present in the feedback signals (outputs of displacement sensors) and the control currents. Suppose also that they are uniformly distributed white noise. The maximum and minimum value of sensor noise and control noise is $\pm5\mu$m and $\pm0.05A$ respectively. The result is depicted in Figure 8.

It shows from Figure 4 that without any uncertainties, the sliding mode controller and the pole assignment controller both can work well. However, Figure 5 shows that pole assignment controller needs more control currents. By the state trajectory shown in Figure 6, one can clearly see how the sliding mode controller works. The system is first brought to a small neighborhood of the sliding manifold from its initial condition, and then slides along the sliding manifold toward the origin.

The advantages of the sliding mode controller will become apparent as the parameter variations are introduced into the system. As one can see from Figure 7, although larger parameter variations will degrade system performance, the sliding mode controller still works quite well. This indicates its good robustness. On the contrast, the performance of the linear controller is strongly affected by the parameter variations. For the 30% variation, the linear controller can not even stabilize the system. From Figure 8, it is shown that under external disturbances, noises, impacts loads, and parameter variations, the rotor displacement can be kept in the order of 10^{-7}(m) by the sliding mode controller, whereas it will be in the order of 10^{-5}(m) if the linear controller is used.

From the simulation results, we conclude that with the proposed sliding mode controller, the rotor can be levitated from the worst position and reach the steady state within 0.03 sec with submicron steady state error (about 0.01 μm for cases A and B). Moreover, its performance is little influenced by the presence of parameter variations, impact loads, and noises, revealing its good robustness.

Finally, by examining the time responses of the control currents of the sliding mode control, one can find that peak control efforts occur during initial transient period and decay rapidly soon after, which is typical in control systems. Most peak control currents are below 0.2A and are within reasonable range. This implies that we still have space to tune the control parameters larger to enhance the system performance.

V. CONCLUSIONS

The modeling and control of an AMB system has been investigated analytically and numerically in this study. The system is adopted from a high-speed spindle system in a machine center, which is part of a future project. Unlike many previous studies, which considered only either linear model or uncoupled magnetic forces, a nonlinear model with magnetic coupling has been derived in this work by a magnetic circuit analysis. Using the magnetically coupled nonlinear model, the bias currents are determined and the control currents are designed via a nonlinear control method called sliding mode control. Numerical simulations with three different cases have also been performed to examine the validity of the controller design. The simulation results are also compared to those with linear state feedback controller based on pole assignment techniques.

It is found that the proposed robust sliding mode controller is more satisfactory than the pole assignment controller. The sliding mode control can yield better transient and steady state performance with smaller control effort. It can also stabilize the system in those cases where the linear controller fails. With the sliding mode controller, the rotor can be brought to the center position within 0.03sec with submicron steady state error from any admissible initial conditions, even from the worst position. Moreover, the performance is not affected by system's parameter variations, impact loads, external random disturbances, and sensor noises. Hence, the simulation results demonstrate the good robustness of the closed-loop system.

Also, within the reasonable range of control currents, the system performance can be easily enhanced (e.g. faster transient response) by tuning the control parameters lager.

Future work following this study includes the problem of mass unbalance, a systematic analysis of the bias currents, effects of magnetic saturation and elastic rotor. The experimental investigation is also under going. The present results can also be extended to the AMB system with 8 magnetic poles.

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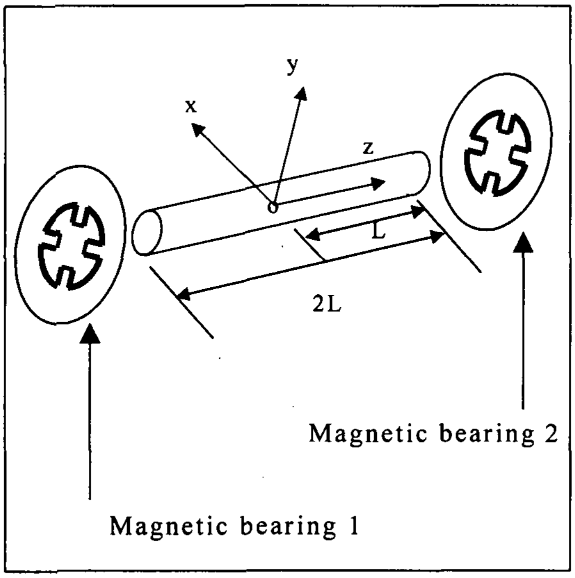

Figure 1. Magnetic rotor-bearing syster

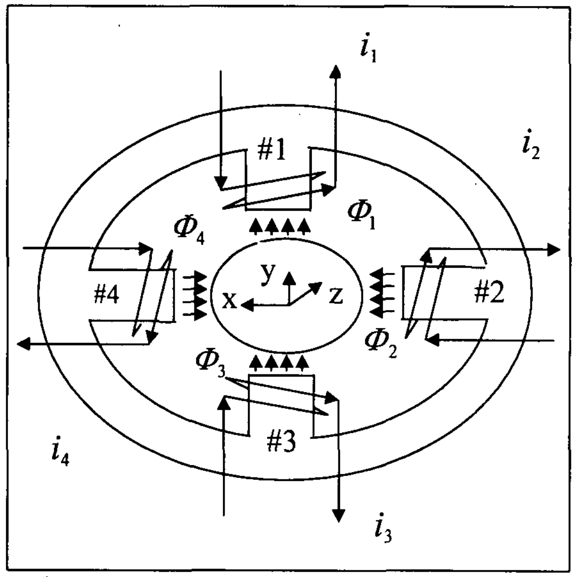

Figure 2. Electromagnetic bearing

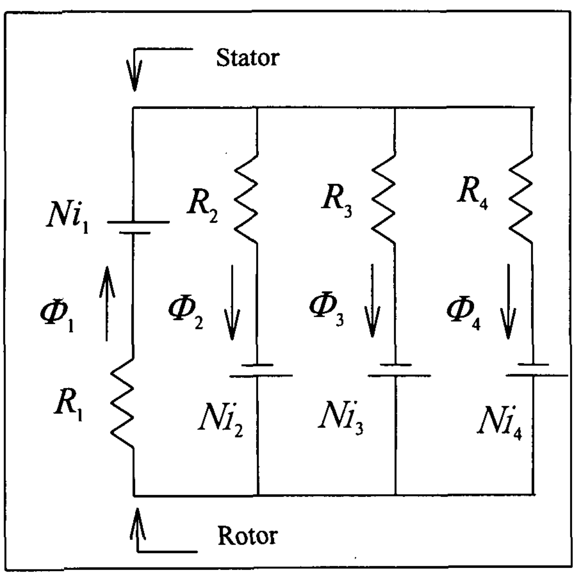

Figure 3. Magnetic circuit

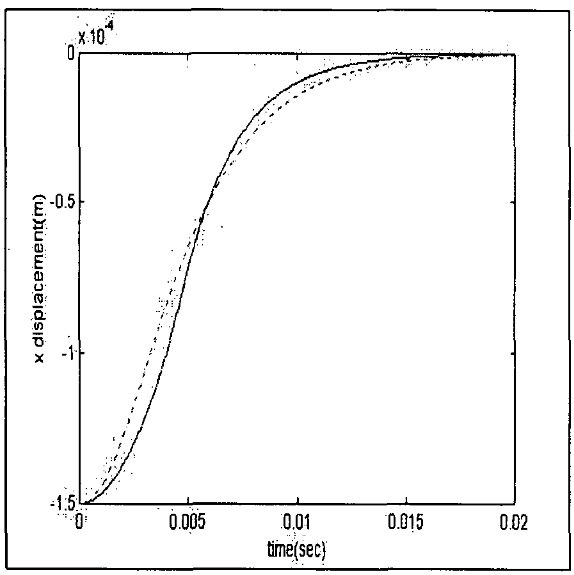

Figure 4. x displacement at bearing #1, case A

Sliding Mode Controller
Pole Assignment Controller

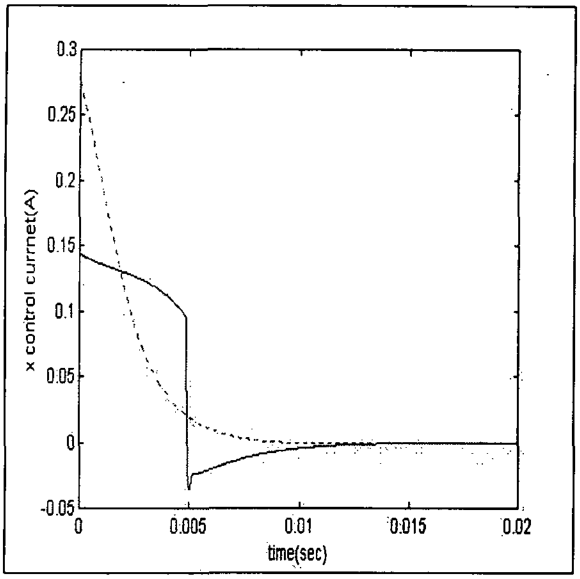

Figure 5. x control current at bearing #1, case A

Sliding Mode Controller ———

Pole Assignment Controller – – –

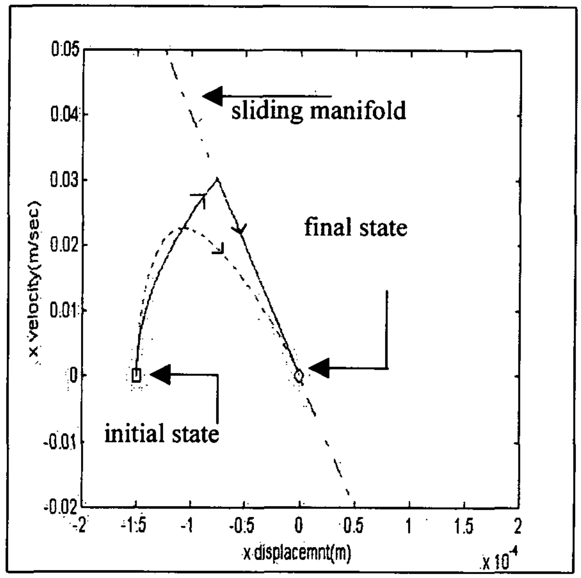

Figure 6. state trajectory at bearing #1, case A

Sliding Mode Controller ———

Pole Assignment Controller – – –

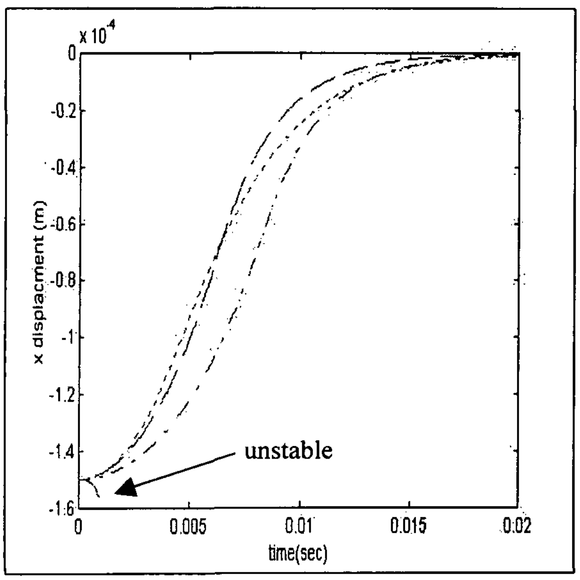

Figure 7. x displacement at bearing #1, case B

Sliding Mode Controller

18% variation — — 30% variation – · – · ·

Pole Assignment Controller

18% variation – – – 30% variation ———

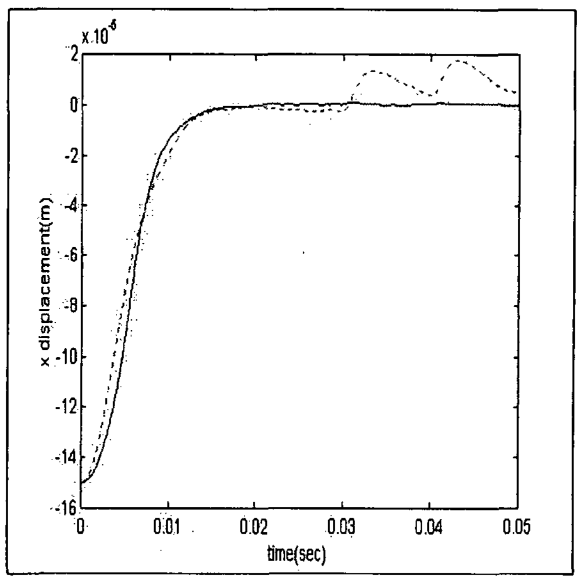

Figure 8. x displacement at bearing #1, case C

Sliding Mode Controller ———

Pole Assignment Controller – – –