# BEARINGLESS SINGLE-PHASE MOTOR WITH CONCENTRATED FULL PITCH WINDINGS IN EXTERIOR ROTOR DESIGN

### Siegfried Silber,<sup>1</sup> Wolfgang Amrhein<sup>1</sup>

# ABSTRACT

Active magnetic bearings as well as bearingless motors are more and more introduced into different fields of industrial applications. For many potential industrial applications, however, they are still too expensive.

This paper presents a new concept of a bearingless single-phase motor that leads to a considerable reduction in mechanical and electronic expenditure. This can be accomplished by integrating additional carrying windings into the stator design of a single-phase motor and by using concentrated full pitch windings instead of distributed windings.

This type of bearingless motor can be used in any application where there is no demand for a high starting torque.

#### **1. INTRODUCTION**

Increasing the rotational speed of an electric motor at a given output power can help to reduce its size. Thus a considerable reduction in mechanical expenditure can be achieved. At high rotational speeds, however, ball bearings can cause problems and undesired rotor vibrations may occur (Chiba and Fukao, 1994; Schweitzer, Bleuler and Traxler, 1994). Active magnetic bearings as well as bearingless motors are capable of overcoming these problems (Oshima et al., 1996). In this context bearingless means that carrying forces required for the suspension of the rotor are generated magnetically.

Normally, two motor-bearing parts are required for full stabilization of the rotor position. However, for selective applications where the length of the stator can be short in relation to its diameter, it is possible to stabilize three degrees of freedom passively (Bleuler et al., 1994; Schöb and Barletta, 1996; Post, 1997; Amrhein and Silber, 1998). Only three degrees of freedom (x-, y- coordinates of the rotor plane, rotation angle  $\delta$ ) need to be stabilized actively. A further simplification of the mechanical and electrical design will be obtained if the carrying windings are integrated into a single phase motor, and concentrated full pitch windings are used instead of distributed windings.

This paper examines the feasibility of a bearingless single phase motor with concentrated windings.

<sup>&</sup>lt;sup>1</sup>Johannes Kepler University Linz, Dept. of Power Electronics and Electrical Drives, Altenbergerstr. 69, A-4040 Linz, Austria.

# 2. FORCE AND TORQUE CALCULATION

Force and torque calculation for bearingless AC motors with sinusoidally distributed flux densities in the air gap and sinusoidal current density distribution have been proposed (Bichsel, 1990; Schöb, 1993; Schöb and Bichsel, 1994). In general, flux distribution in the air gap is not sinusoidal, neither is current density distribution. By utilizing a derivation based on the Fourier coefficients of these functions a simple solution for the force and torque calculation can be found.

The mechanical stress on the stator surface  $\sigma$  can be calculated by the Maxwell Stress Tensor  $T_m$  and the normal vector **n** 

$$\mathbf{T}_{m} = \begin{bmatrix} \mu H_{x}^{2} - \frac{1}{2} \mu H^{2} & \mu H_{x} H_{y} & \mu H_{x} H_{z} \\ \mu H_{y} H_{x} & \mu H_{y}^{2} - \frac{1}{2} \mu H^{2} & \mu H_{y} H_{z} \\ \mu H_{z} H_{x} & \mu H_{z} H_{y} & \mu H_{z}^{2} - \frac{1}{2} \mu H^{2} \end{bmatrix}$$

$$\boldsymbol{\sigma} = \mathbf{T}_{m} \mathbf{n}. \tag{1}$$

Under the condition  $\mu_{\text{Fe}} \gg \mu_0$ , the tangential component of the flux density in the air gap can be neglected and (1) can be simplified to

$$\sigma(\varphi) = \begin{bmatrix} \frac{B_n(\varphi)^2}{2\mu_0} \\ A(\varphi)B_n(\varphi) \\ 0 \end{bmatrix},$$
(2)

where  $B_n$  denotes the normal component of the flux density,  $\mu_0$  is the permeability of air and A is the current density distribution on the stator surface. The distribution of the flux density in the air gap and the current density distribution on the stator surface are both periodic functions and can therefore be represented by Fourier series

$$B_{n}(\phi) = \sum_{\mu = -\infty}^{\infty} b_{\mu} e^{j\mu\phi}, \qquad (3)$$

where  $b_{\mu}$  - the so-called complex Fourier coefficients - can be calculated by

$$b_{\mu} = \frac{1}{2\pi} \int_{-\pi}^{\pi} B_n(\phi) e^{-j\mu\phi} d\phi.$$
(4)

The coefficients  $b_{\mu}$  can be combined into the infinite vector

$$\mathbf{b}^{\mathrm{T}} = \begin{bmatrix} \cdots & \mathbf{b}_{-2} & \mathbf{b}_{-1} & \mathbf{b}_{0} & \mathbf{b}_{1} & \mathbf{b}_{2} & \cdots \end{bmatrix}.$$
(5)

Then the flux density in the air gap can be written as

$$\mathbf{B}_{n} = \mathbf{\Omega}^{\mathrm{T}} \mathbf{b}, \qquad (6)$$

with

$$\Omega^{\mathrm{T}} = \begin{bmatrix} \cdots & \mathrm{e}^{-2j\varphi} & \mathrm{e}^{-j\varphi} & 1 & \mathrm{e}^{j\varphi} & \mathrm{e}^{2j\varphi} & \cdots \end{bmatrix}.$$
(7)

The current density distribution on the stator surface A can also be represented by Fourier coefficients

$$\mathbf{A} = \mathbf{\Omega}^{\mathrm{T}} \mathbf{c} \mathbf{I}_{1} \tag{8}$$

$$\mathbf{c} = \begin{bmatrix} \vdots & \vdots & \vdots \\ c_{-2,1} & c_{-2,2} & \cdots & c_{-2,m} \\ c_{-1,1} & c_{-1,2} & \cdots & c_{-1,m} \\ c_{0,1} & c_{0,2} & \cdots & c_{0,m} \\ c_{1,1} & c_{1,2} & \cdots & c_{1,m} \\ c_{2,1} & c_{2,2} & \cdots & c_{2,m} \\ \vdots & \vdots & & \vdots \end{bmatrix}, \quad \mathbf{I}_{1} = \begin{bmatrix} \mathbf{I}_{11} \\ \mathbf{I}_{12} \\ \vdots \\ \mathbf{I}_{1m} \end{bmatrix},$$

where the term  $\Omega^{T}$ **c** represents a distribution function of the stator current **I**<sub>1</sub>. The mechanical stress on the stator surface can be rewritten by using Fourier coefficients as

$$\boldsymbol{\sigma} = \begin{bmatrix} \frac{1}{2\mu_0} \mathbf{b}^{\mathrm{T}} \boldsymbol{\Omega} \boldsymbol{\Omega}^{\mathrm{T}} \mathbf{b} \\ \mathbf{I}_1^{\mathrm{T}} \mathbf{c}^{\mathrm{T}} \boldsymbol{\Omega} \boldsymbol{\Omega}^{\mathrm{T}} \mathbf{b} \\ 0 \end{bmatrix}.$$
(9)

The total developed electromagnetic torque is obtained by the following integral

$$T = lr^{2} \int_{-\pi}^{\pi} \mathbf{I}_{1}^{\mathrm{T}} \mathbf{c}^{\mathrm{T}} \Omega \Omega^{\mathrm{T}} \mathbf{b} \, \mathrm{d}\boldsymbol{\varphi}, \tag{10}$$

and can be simplified to

$$T = lr^{2}I_{l}^{T}c^{T}mb$$

$$= 2\pi \begin{bmatrix} & \ddots & \ddots & \ddots \\ 0 & 0 & 1 & \ddots \\ \vdots & 0 & 1 & 0 & \ddots \\ \vdots & 1 & 0 & 0 \\ \vdots & \vdots & \ddots & \ddots \end{bmatrix}$$
(11)

It follows from the infinite matrix **m** that electromagnetic torque can only be produced if the flux density distribution and the current density distribution on the stator surface contain harmonics of the same orders.

The radial forces acting on the rotor are obtained as

m

$$\mathbf{F} = \begin{bmatrix} \mathbf{F}_{\mathbf{x}} \\ \mathbf{F}_{\mathbf{y}} \end{bmatrix} = \mathbf{1}\mathbf{r} \begin{bmatrix} \frac{1}{2\mu_{0}} \mathbf{b}^{\mathrm{T}} & -\mathbf{I}_{1}^{\mathrm{T}} \mathbf{c}^{\mathrm{T}} \\ \mathbf{I}_{1}^{\mathrm{T}} \mathbf{c}^{\mathrm{T}} & \frac{1}{2\mu_{0}} \mathbf{b}^{\mathrm{T}} \end{bmatrix}_{-\pi}^{\pi} \begin{bmatrix} \Omega \Omega^{\mathrm{T}} \cos \varphi \\ \Omega \Omega^{\mathrm{T}} \sin \varphi \end{bmatrix} d\varphi \mathbf{b}$$
$$\mathbf{F} = \mathbf{1}\mathbf{r} \begin{bmatrix} \frac{1}{2\mu_{0}} \mathbf{b}^{\mathrm{T}} & -\mathbf{I}_{1}^{\mathrm{T}} \mathbf{c}^{\mathrm{T}} \\ \mathbf{I}_{1}^{\mathrm{T}} \mathbf{c}^{\mathrm{T}} & \frac{1}{2\mu_{0}} \mathbf{b}^{\mathrm{T}} \end{bmatrix} \begin{bmatrix} \mathbf{f}_{1} \\ \mathbf{f}_{2} \end{bmatrix} \mathbf{b}, \qquad (12)$$

with

$$\mathbf{f}_{1} = \pi \begin{bmatrix} & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots \\ & 0 & 0 & 1 & 0 & 1 & \ddots \\ & \ddots & 0 & 1 & 0 & 1 & 0 & \ddots \\ & \ddots & 1 & 0 & 1 & 0 & 0 & \\ & \ddots & \end{bmatrix}, \quad \mathbf{f}_{2} = \mathbf{j}\pi \begin{bmatrix} & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots \\ & 0 & -1 & 0 & 1 & 0 & 1 \\ & \ddots & 0 & -1 & 0 & 1 & 0 & 0 \\ & \ddots & \end{bmatrix}.$$

# **3. MECHANICAL DESIGN**

For the bearingless operation of a motor (without ball or sliding bearings) controllable carrying force components in radial and axial direction must be developed to stabilize the position of the rotor. For special fields of applications, however, where there is no demand for stiff stabilization in axial direction and where a small inclination of the rotor is allowed, it is possible to stabilize three degrees of freedom passively by means of permanent magnets. For these applications it is only the radial position of the rotor in x and y direction that needs to be stabilized actively. Thus at least two independent bearing windings are required for force generation in radial direction. If these bearing windings are integrated into a single phase motor only one additional torque winding is required. Figure 1 shows the cross section of the bearingless motor with permanent magnet excitation with external rotor.



Figure 1: Bearingless motor with three phases



Figure 2: Bearingless motor with four concentrated windings

In figure 1 the currents denoted by  $i_a$  and  $i_b$  are responsible for generating carrying forces in radial direction, whereas the current denoted by  $i_m$  generates electromagnetic torque.

Independent bearing force components and electromagnetic torque can also be produced by using an arrangement of the windings as shown in figure 2. Thus mechanical expenditure can be reduced. However, the carrying currents and the motor currents are no longer decoupled, but the required phase currents  $i_1$  to  $i_4$  may be expressed in terms of  $i_a$ ,  $i_b$  and  $i_m$ 

$$i_{1} = i_{a} - i_{b} - i_{m}$$

$$i_{2} = i_{a} + i_{b} + i_{m}$$

$$i_{3} = -i_{a} + i_{b} - i_{m}$$

$$i_{4} = -i_{a} - i_{b} + i_{m}.$$
(13)

Figure 3 shows the radial carrying forces in x and y direction acting on the rotor as a function of the rotor angle  $\delta$ , when phase a is fed with a constant current of 4A and the currents  $i_b$  and  $i_m$  are assumed to be zero. The vector of the stator currents then becomes

$$\mathbf{I}_{1} = \begin{bmatrix} 4\\4\\-4\\-4 \end{bmatrix}. \tag{14}$$

The solid line shows the values calculated by FEM program, the dashed line is calculated by applying the proposed analytical method. If only the first two harmonics are taken into account, the total error will become large (dash-doted line).



Figure 3: a) x-axis, b) y-axis component of the radial force (solid line: FEM result; dashed line: analytical method, dash-doted line: flux density in the air gap and current density distribution are assumed to be sinusoidal)

# 4. CARRYING FORCE CONTROL

Different control schemes for bearingless motors with sinusoidally distributed flux densities in the air gap and sinusoidal current distributions have already been proposed. In the case of a bearingless motor with concentrated windings neither the flux density in the air gap nor the current density distribution are sinusoidal. Therefore a special control scheme for the carrying currents is required.

In the air gap of a bearingless motor the motor flux is superimposed by the flux due to the torque windings as well as the flux due to the carrying windings. Thus the torque and carrying force production can not be considered as decoupled. This fact has already been dealt with (Schöb and Bichsel, 1994). In the case of a permanent magnet excited bearingless motor with a large air gap the influence of the torque winding on the radial bearing forces is very small and can therefore be neglected.

In general, the carrying forces acting on the rotor of the bearingless motor can be written as a nonlinear (non sinusoidal) function

$$\begin{bmatrix} \mathbf{F}_{\mathbf{x}} \\ \mathbf{F}_{\mathbf{y}} \end{bmatrix} = \mathbf{F}(\mathbf{x}, \mathbf{I}_{1}, \delta)$$
(15)

with

 $\mathbf{x} = \begin{bmatrix} \mathbf{x} \\ \mathbf{y} \end{bmatrix}, \quad \mathbf{I}_1 = \begin{bmatrix} \mathbf{i}_a \\ \mathbf{i}_b \\ \mathbf{i}_m \end{bmatrix}.$ 

Solving equation (15) for the stator current  $I_1$  is not possible in any case, but linearization about  $\mathbf{x}_0$  and  $\mathbf{I}_{10}$  may be stated as

$$\mathbf{F}(\mathbf{x},\mathbf{I}_{1},\delta) = \mathbf{F}(\mathbf{x}_{0},\mathbf{I}_{10},\delta) + \frac{\partial \mathbf{F}}{\partial \mathbf{x}}(\mathbf{x}_{0},\mathbf{I}_{10},\delta)\Delta \mathbf{x} + \frac{\partial \mathbf{F}}{\partial \mathbf{I}_{1}}(\mathbf{x}_{0},\mathbf{I}_{10},\delta)\Delta \mathbf{I}_{1}.$$
 (16)

In case of equilibrium the first term in equation (16) becomes zero. The second term describes the change of the force acting on the rotor when the rotor position is displaced. In the closed control loop displacement of the rotor can be assumed to be very small, therefore this term can be neglected. If only the third term is taken into account, equation (16) can be written as

$$\begin{bmatrix} F_{x} \\ F_{y} \end{bmatrix} = \begin{bmatrix} \frac{\partial F_{x}}{\partial i_{a}}(\mathbf{x}_{0}, \mathbf{I}_{10}, \delta) & \frac{\partial F_{x}}{\partial i_{b}}(\mathbf{x}_{0}, \mathbf{I}_{10}, \delta) \\ \frac{\partial F_{y}}{\partial i_{a}}(\mathbf{x}_{0}, \mathbf{I}_{10}, \delta) & \frac{\partial F_{y}}{\partial i_{b}}(\mathbf{x}_{0}, \mathbf{I}_{10}, \delta) \end{bmatrix} \begin{bmatrix} \Delta i_{a} \\ \Delta i_{b} \end{bmatrix} + \begin{bmatrix} \frac{\partial F_{x}}{\partial i_{m}}(\mathbf{x}_{0}, \mathbf{I}_{10}, \delta) \\ \frac{\partial F_{y}}{\partial i_{m}}(\mathbf{x}_{0}, \mathbf{I}_{10}, \delta) \end{bmatrix} \Delta i_{m}.$$
(17)

For the proposed permanent magnet excited motor with a large air gap the influence of i<sub>m</sub> on the bearing forces is very small. Thus i<sub>m</sub> can be neglected and a very simple solution for the radial bearing forces is obtained as

$$\begin{bmatrix} F_x \\ F_y \end{bmatrix} = T(\delta) \begin{bmatrix} \Delta i_a \\ \Delta i_b \end{bmatrix}$$
(18)

with

$$\mathbf{T}(\delta) = \begin{bmatrix} \frac{\partial F_{\mathbf{x}}}{\partial i_{a}} (\mathbf{x}_{0}, \mathbf{I}_{10}, \delta) & \frac{\partial F_{\mathbf{x}}}{\partial i_{b}} (\mathbf{x}_{0}, \mathbf{I}_{10}, \delta) \\ \frac{\partial F_{\mathbf{y}}}{\partial i_{a}} (\mathbf{x}_{0}, \mathbf{I}_{10}, \delta) & \frac{\partial F_{\mathbf{y}}}{\partial i_{b}} (\mathbf{x}_{0}, \mathbf{I}_{10}, \delta) \end{bmatrix}.$$

Further simplification can be obtained if the operating point of the motor is chosen at  $x_0=0$ . Then the matrix  $T(\delta)$  becomes non-singular and skew symmetric. Thus  $T(\delta)$  has an inverse and equation (18) can be rewritten

$$\begin{bmatrix} i_{a} \\ i_{b} \end{bmatrix} = \mathbf{T}^{-1}(\delta) \begin{bmatrix} F_{x} \\ F_{y} \end{bmatrix}$$

$$\begin{bmatrix} i_{a} \\ i_{b} \end{bmatrix} = \mathbf{k}(\delta) \begin{bmatrix} F_{x} \\ F_{y} \end{bmatrix}.$$
(19)

By utilizing equation (19) and (13) a very simple position controller can be designed with the help of linear time-invariant methods.

Since the design of controllers for magnetic bearing systems has been investigated intensively, it will not be discussed in this paper (Herzog, 1991).

#### **5. EXPERIMENTAL RESULTS**

Figure 4 presents the first prototype of the bearingless single-phase motor with four concentrated windings with external rotor. A digital signal processor is used for position and torque control. The motor was tested under steady state as well as dynamic conditions up to a rotational speed of 3000 rpm. To prove the feasibility of the proposed control scheme, two analytically calculated entries of the matrix  $\mathbf{k}(\delta)$  are compared with the correspondent measured values. Figure 5 shows the scaled calculated (dashed lines) and measured (solid lines) values of the matrix elements  $k_{11}$  and  $k_{21}$  as a function of the rotor angle.

The major advantage of the proposed control scheme is that it is possible to implement independent force controllers for the x and y directions (Figure 6). Theoretically, a disturbance in x direction only causes a deviation from the desired x position but not from the y position. The calculation of the matrix  $\mathbf{k}(\delta)$ , however, is based on a simplified mathematical model. Therefore it is obvious that a small coupling between the axes will occur. The system response to impulse disturbance in x direction for two different rotor angles is shown in figure 7. The small coupling between the x and y axes confirms the sufficient accuracy of the transformation matrix  $\mathbf{k}(\delta)$ .



Figure 4: Rotor and stator of the bearingless single-phase motor



Figure 5: a) k<sub>11</sub> entry, b) k<sub>21</sub> entry of the transformation matrix as a function of the rotor angle (solid line: measured values, dashed line: calculated values)



Figure 6: Block diagram of the controller



Figure 7: System response to impulse disturbance in x direction for different rotor angles (solid line: xdirection, dashed line: y-direction)

# **6. CONCLUSION**

In this paper a new concept for a bearingless-single phase motor has been presented. For bearing force and torque production only four concentrated windings are required. The proposed mechanical design, however, leads to non-sinusoidal flux and current density distributions. Therefore a special force control scheme is required to decouple the x and y directions of the rotor plane which has also been presented in this paper.

Tests with the first prototype of the bearingless single-phase motor show good results in motor and magnetic bearing performance.

#### 7. ACKNOWLEDGMENTS

The project was supported by the Laboratory for Electrical Engineering Design (EEK) of Swiss Federal Institute of Technology, Zurich (ETH Zurich) and Sulzer Electronics AG, CH-Winterthur.

#### REFERENCES

Amrhein, W. and S. Silber, 1998. "Bearingless single-phase brushless DC-motor", Proc. SPEEDAM, Sorrento, Italy.

Bichsel, J. 1990. "Beiträge zum lagerlosen Elektromotor", Dissertation, ETH Zürich.

Bleuler, H., H. Kawakatsu, W. Tang, W. Hsieh, D. K. Miu, Y. Tai, F. Moesner and M. Rohner. 1994. "Micromachined active magnetic bearings", Proc. 4th Int. Symp. Magn. Bearings, Zurich, Switzerland, 349-352. Chiba, A. and T. Fukao. 1994. "The maximum radial force of induction machine type bearingless motor using finite element analysis", Proc. 4th Int. Symp. Magn. Bearings, Zurich, Switzerland, 333-338.

Herzog, R. 1991. "Ein Beitrag zur Regelung von magnetgelagerten Systemen mittels positiv reeller Funktionen und H<sup> $\infty$ </sup> - Optimierung", Dissertation, ETH Zürich.

Ohishi, T., Y. Okada and K. Dejima. 1994. "Analysis and design of a concentrated wound stator for synchronous-type levitated motor", Proc. 4th Int. Symp. Magn. Bearings, Zurich, Switzerland, 201-206.

Oshima, M., S. Miyazawa, T. Deido, A. Chiba, F. Nakamura and T. Fukao. 1996. "Characteristics of a permanent magnet type bearingless motor", IEEE Trans. Ind. Applicat., Vol. 32, No. 2: 363-369.

Post, C. 1997. "Konstruktion eines lagerlosen Einphasenmotors", Diplomarbeit. Johannes Kepler Universität Linz.

Schöb, R. 1993. "Beiträge zur lagerlosen Asynchronmaschine", Dissertation, ETH Zürich.

Schöb, R. and J. Bichsel. 1994. "Vector control of the bearingless motor", Proc. 4th Int. Symp. Magn. Bearings, Zurich, Switzerland, 327-332.

Schöb, R. and N. Barletta. 1996. "Principle and application of a bearingless slice motor", Proc. 5th Int. Symp. Magn. Bearings, Kanazawa, Japan, 313-318.

Schweitzer, G., H. Bleuler and A. Traxler, 1994. "Active magnetic bearings", vdf, Zurich ISBN 3 7281 2132 0.