μ-SYNTHESIS CONTROLLER DESIGN FOR A 3 MW PUMP RUNNING IN AMBs

F. Lösch,[1] C. Gähler,[2] R. Herzog[3]

ABSTRACT

The dynamic properties of magnetically levitated pumps are determined to a large extent by the characteristics of their *seals*. These vary with throughput, pressure, and rotational speed and are not precisely known. Robustness of the AMB controller with respect to these plant variations and uncertainties is therefore a crucial issue.

μ-Synthesis is well suited for such control problems. Two points remain critical for practical applications: First, there is the choice of an uncertainty model that captures all uncertainty without being overly conservative. The second point is the selection of suitable weighting functions that express the size of the model uncertainty, system limitations, and performance goals and can be used in the controller design process.

In this paper, the robust controller design for a 3 MW boiler feed pump is presented (Lösch, 1997). The dynamics of this pump are discussed, and a special model reduction technique appropriate to these dynamics is presented. A systematic way for transforming the robustness and performance requirements to the μ-Synthesis setting is described. As the core of the paper, a new theorem for determining suitable weighting functions is developed and applied to the AMB pump. The controller designed on this basis proves to meet the specifications.

INTRODUCTION

THE PUMP

Pumps equipped with magnetig bearings offer a wide variety of advantages over conventional pumps. Among them are higher efficiency, lower maintenance and total costs, and the availability of status information. Figure 1 shows a cross-sectional view of the AMB pump considered. This pump was constructed in a diploma thesis in cooperation with the pump manufacturer KSB, Frankenthal (Ramb, 1994). It is a 5-stage centrifugal pump with a rotor

[1]International Center for Magnetic Bearings, ETH-Zentrum/CLA, 8092 Zürich, Switzerland, e-mail: loesch@ifr.mavt.ethz.ch.
[2]International Center for Magnetic Bearings, ETH-Zentrum/CLA, 8092 Zürich, Switzerland, e-mail: gaehler@ieee.org.
[3]Mecos Traxler AG, Gutstr. 36, CH-8400 Winterthur, Switzerland, e-mail: herzog@mecos.ch.

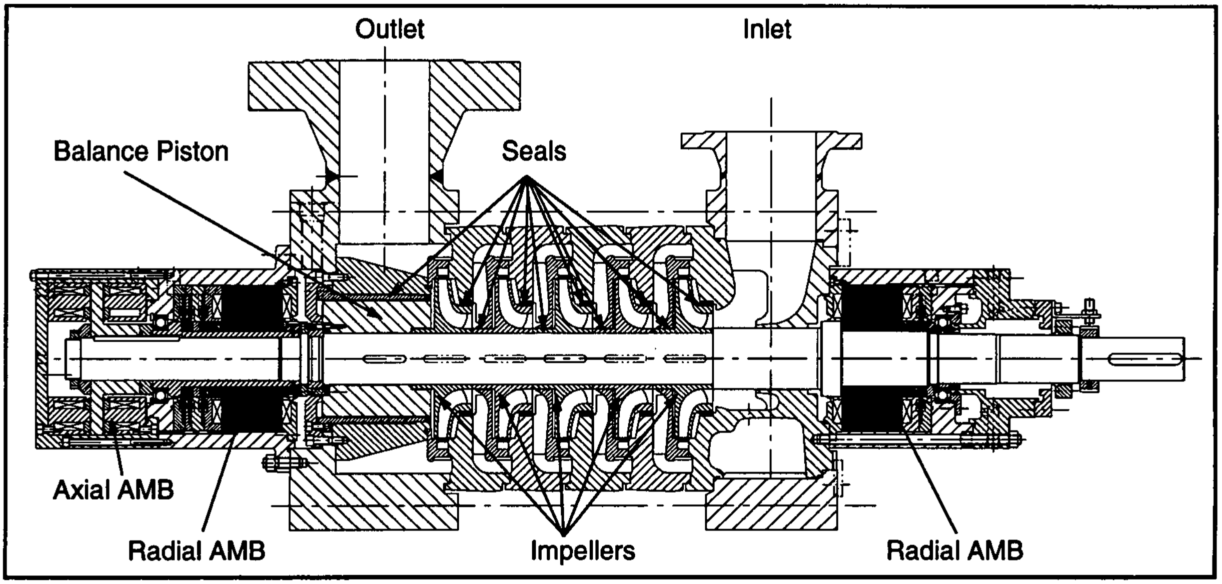


Figure 1: *Cross-sectional view of the pump*

length of 1.84m and a rotor weight of 220kg. It has a maximum power of 3MW and runs at a nominal speed of 6000rpm. The large axial thrust which is produced by the five impellers is counteracted by the balance piston near the high pressure end of the rotor. At the pump's outlet, there is a throttle that allows to adjust the throughput in a range from 0.19 to 1.2 times its nominal value.

Modes of Operation

Two different modes of operation can be distinguished:

- Normal operation: *Variable throughput* q, constant (nominal) speed U
- Run-up/run-down: *Variable speed* U, constant (low) throughput q

The run-up/run-down mode is relevant only for taking the pump into, or out of, service. In order to reduce the robustness requirements in favour of better performance, two different controllers have been designed for the two modes (gain scheduling). In this paper, we focus on the controller design for normal operation.

Seals

At ten different locations, the pump contains seals. There is one seal located on the low pressure side (right hand side) of each impeller. To the left of each of the four rightmost impellers, there is one additional seal. Finally, there is another seal on the balance piston near the high pressure end of the rotor. This seal is very long (about 200mm), and all of the pressure (up to 250bar) produced by the pump is acting on it. When the rotor rotates, the seals drastically influence eigenfrequencies, related damping, and eigenmodes of the pump. Moreover, they cause a coupling between the motions in the two radial planes. In particular, the

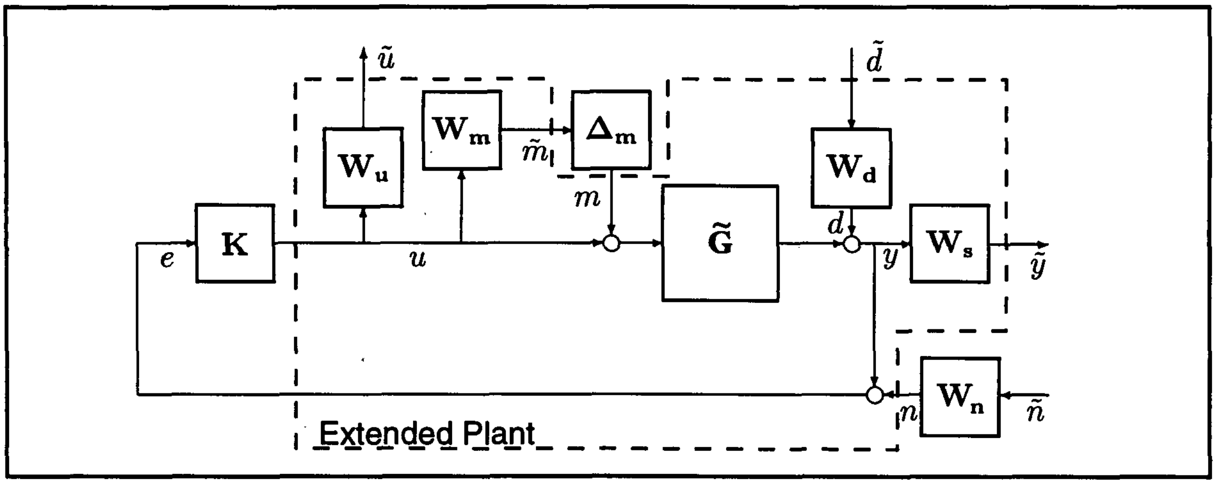


Figure 2: *Framework with reduced plant, multiplicative uncertainty, and weighting functions Note that all signals are vectors with 4 elements.*

influence of the long seal at the balance piston is huge. Its mass coefficient, for instance, is twice the rotor mass. The seals' mass, damping and stiffness coefficients vary with throughput, pressure, and rotational speed (Diewald, 1989). The related robustness requirements are a central issue in controller design for AMB pumps. μ-Synthesis is well suited for this problem.

μ-SYNTHESIS

μ-Synthesis (Balas et al., 1995; Zhou et al., 1996) is a method for designing controllers which guarantee that performance specifications are met not only for the *nominal plant* $\tilde{\mathbf{G}}$ but also for all plants **G** which differ in a certain limited way from $\tilde{\mathbf{G}}$ (*robust performance*). This capablitiy makes μ-Synthesis a very powerful tool, since it is able to put an end to the widespread trial and error in classical control. The method consists of the following steps:

1) Definition of the structure of the framework for the controller design (the so-called *extended plant*), such that all relevant aspects regarding performance criteria and plant uncertainty are captured. An example is shown in Figure 2.
2) Definition of appropriate frequency-dependent *weighting functions* **W**(s) for quantification of the plant uncertainty and of the performance goals corresponding to the criteria mentioned above.
3) Computation of a controller matching the performance and robustness specifications. This is done using the ***D-K**-iteration* which forms the computational core of the method.

Theory regarding step 3 (**D-K**-iteration) is well elaborated and supported by powerful software tools. However, little literature and guidelines exist regarding steps 1 and 2 where the real-world problem and its specifications should be transformed into the μ-Synthesis framework. Consequently, the control engineer is left alone with this important task. This seems to be a main reason why μ-Synthesis enters the area of real-world applications very slowly. In the following, we present a formalized way to approach this transformation.

THE MODEL

The dynamic model of the pump was derived as a combination of the models of its components, i.e. the rotor, the seals, and the magnetic bearings. The rotor was modelled using the Finite Element method. To account for the complex geometry and the large number of force insertion points (seals, impellers, bearings), 39 nodes had to be considered in the model (156 degrees of freedom). The impellers have been modelled as rigid disks. For the AMBs, a linear model was assumed (Schweitzer, 1994).

The rotor-stator interaction at the seals was described using a linearized dynamic model of order 2. The related mass, damping and stiffness matrices in this model are skew symmetric. The coefficients were calculated for different values of throughput, pressure, and rotational speed. Special software packages for this purpose are available for various types of seals (Diewald, 1989).

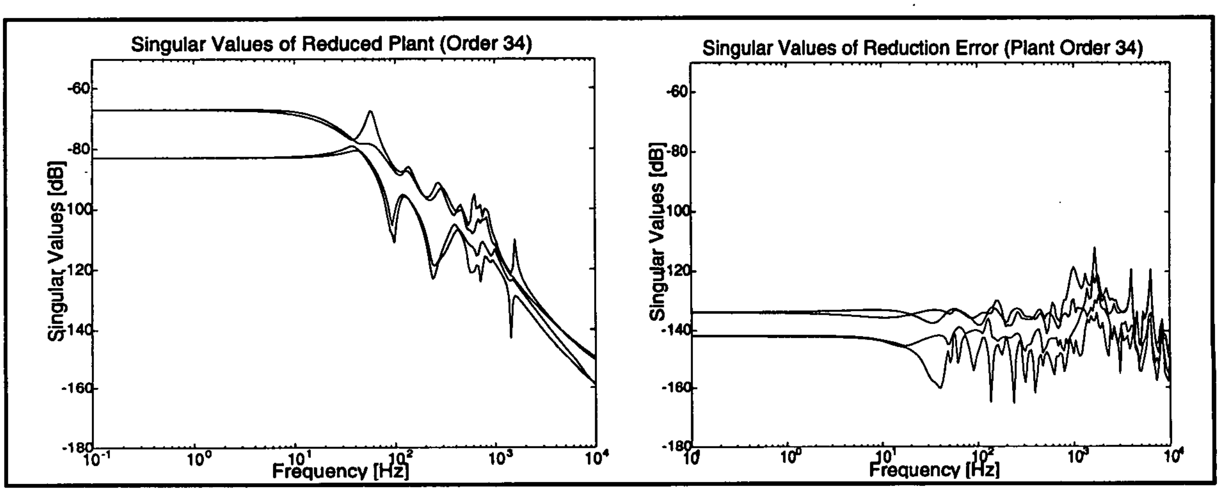
Since the throughput q varies during normal operation, a whole family of linear models is needed to describe this mode of operation. The models for six distinct values of q were chosen to represent this family (q = 0.19, 0.38, 0.54, 0.77, 1.0, 1.2). It is reasonable to assume that a controller that robustly stabilizes all of these plants will also robustly stabilize the plant at all other values of q and thus solve the given control problem. For each of the six operating points, a state space model of the complete plant including rotor, AMBs, and seals was derived. Each model has four inputs (control currents u), four outputs (rotor displacements y), and 312 states.

MODEL REDUCTION

Before attempting a model reduction, a nominal system representing all of the six systems mentioned above was created by taking the average of the seals' coefficients. The dynamic properties of the seals proved to be relevant for the choice of the model reduction method. Different model reduction techniques have been compared:

1) Modal reduction
2) Weighted model reduction according to Green and Limebeer (Green, Limebeer, 1995)
3) Weighted balanced truncation of the stable model part (Balas et al., 1995).

Figure 3: Reduced model and reduction error from weighted balanced truncation



Modal reduction, which is the standard choice in AMB machinery (Schweitzer, 1994), produced an inacceptable error larger than 20% of the plant's transfer function at 20Hz. The reason for this is the fact that the seals destroy the definiteness properties of the stiffness and damping matrices of the plant model. The Green/Limebeer method suffered from numerical problems caused by the high order of the original system. Application of the weighted balanced truncation method, however, resulted in a reduced model of order 34 with a reduction error of only 0.03% for all frequencies up to 30Hz and an error of 3% up to 750Hz. Both the reduced system and the reduction error are depicted in figure 3. The weighted balanced truncation algorithm comprises the following steps:

1) Decomposition of the model $\mathbf{G}$ into its stable part $\mathbf{G_s}$ and its antistable part $\mathbf{G_a}$.
2) Definition of square, stable, minimum phase frequency weighting functions $\mathbf{W_1}(s)$ and $\mathbf{W_2}(s)$ of small magnitude at frequencies where a small reduction error is required.
3) Pre- and postmultiplication of the stable part with the inverse conjugate transpose of the weighting functions: $\mathbf{G_w}(s) = \mathbf{W_1}(-s)'^{-1}\mathbf{G_s}(s)\mathbf{W_2}(-s)'^{-1}$
4) Reduction of the stable part of $\mathbf{G_w}$ by balanced truncation yields $\mathbf{G_{w,r}}$.
5) Reversion of step 3 on $\mathbf{G_{w,r}}$: $\mathbf{G_{s,r}}(s) = \mathbf{W_1}(-s)'\mathbf{G_{w,r}}(s)\mathbf{W_2}(-s)'$
6) Addition of the antistable part $\mathbf{G_a}$ to the stable part of $\tilde{\mathbf{G}}_{s,r}$ yields the reduced plant $\tilde{\mathbf{G}}$.

The order of the reduced plant $\tilde{\mathbf{G}}$ is given by $ord(\tilde{\mathbf{G}}) = ord(\mathbf{G_a}) + ord(\mathbf{G_{w,r}})$.

μ-SYNTHESIS CONTROLLER DESIGN

THEORETICAL BACKGROUND

Being a method for robust controller design, μ–Synthesis adresses three problems:

1) In the *robust stability problem*, the goal lies in finding a controller that stabilizes the plant in the face of all uncertainties belonging to a certain set of uncertainties.
2) In the *nominal performance problem*, one looks for a controller that achieves a certain behaviour of the closed loop including the nominal (undisturbed) plant.
3) The *robust performance problem* is the combination of the above. Here, one looks for a controller that guarantees a certain closed loop performance for the nominal system *and all disturbed systems*. It is this class the controller design problem for the pump belongs to.

We will now give a short description of how these problems are adressed in μ-Synthesis.

Robust controller design methods are based on the so-called *small gain theorem*, which states that the closed loop depicted in figure 4a is stable for all disturbances Δ with $\|\Delta\|_\infty \le \gamma$ if and only if the maximum singular value of $\mathbf{M}$ is strictly smaller than γ for all frequencies. From this follows that the peak of the maximum singular value $\bar{\sigma}(\mathbf{M}(j\omega))$ is the reciprocal of the size of the largest unstructured uncertainty Δ the system can handle without becoming instable. The definition of μ follows from an extension of this: When restricting the uncertainties modelled by Δ mentioned above to be of a certain structure (e.g. block-diagonal), μ is defined as the reciprocal of the size of the largest admissible uncertainty of that structure. It follows directly from this definition that $\mu(\mathbf{M}(j\omega))$ is alway smaller than $\bar{\sigma}(\mathbf{M}(j\omega))$, since only a subset of disturbances is considered. Consequently, the μ approach is less conservative than the unstructured (H_∞)-approach. Robust controller design aims at maximizing the closed

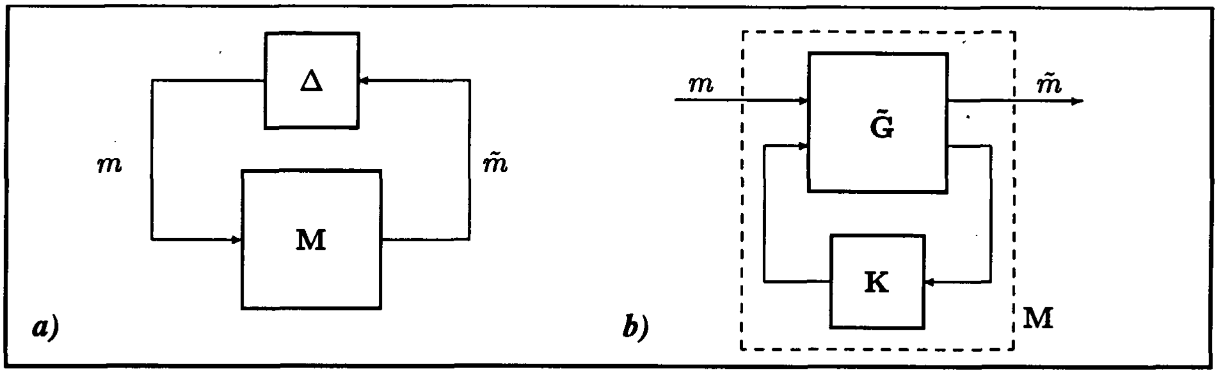


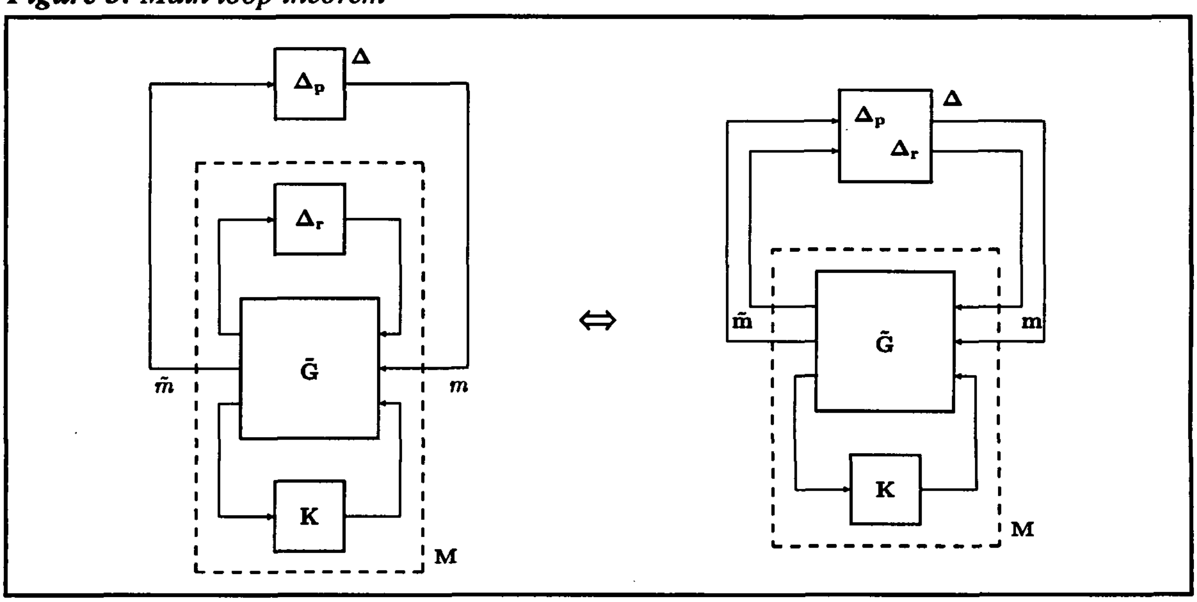
Figure 4: Small gain theorem and interpretations

loop's region of stability. For μ-Synthesis, this means choosing **K** such that the peak value of μ of the transfer function from m to $\tilde{m}$, $\mathbf{T}_{\tilde{\mathbf{m}}\mathbf{m}}$, is minimized. **M** is then replaced by the feedback configuration of **K** and $\tilde{\mathbf{G}}$ (figure 4b).

As we have shown, robustness requirements can be expressed as limits on certain transfer functions, and the *robust stabilisation problem* for uncertainties that can be expressed as a Δ-block is solved by minimizing these transfer functions. The same is true for performance objectives. Here, the controller is to ensure that certain signals remain smaller than some fixed values over a range of frequencies.

D-K-Iteration, which forms the core of today's μ-Synthesis, yields controllers that minimize the μ value of the transfer function $\mathbf{T}_{\tilde{\mathbf{m}}\mathbf{m}}$ shown in figure 4b. Therefore, this algorithm can be used to solve both the *robust stability* and *nominal performance* problems.
It is the so-called *main loop theorem* that states that the *robust performance problem* can also be addressed by μ-Synthesis. In essence, this theorem says that the robust performance problem (which is nothing but a nominal performance problem for a whole class of plants parametrized by an uncertainty Δ) is equivalent to a robust stabilisation problem for the nominal plant and an augmented uncertainty (see figure 5). A direct consequence from this is that *D-K*-Iteration can also be used to solve the robust performance problem.

Figure 5: Main loop theorem



APPLICATION ISSUES

Due to the above, μ-Synthesis is generally suitable for adressing the robust control problem. To use the algorithm for solving real-world problems, however, some additional considerations are necessary.

As we have seen, the μ-Synthesis algorithm minimizes the peak of the closed loop system's μ-plot over frequency, i.e. $\mu(\mathbf{M}(j\omega)) < \gamma$ for all ω where γ is as small as possible. (Using a suitable scaling of the plant, one can make 1 the critical value for γ; then if and only if $\mu(\mathbf{M}(j\omega)) < 1$ for all ω the design was successful. In the following, we assume this to be the case.) This is exploited for the purpose of transfer function shaping in the following way:

First, the relevant transfer functions for expressing the robustness and performance requirements must be identified. To this end, additional robustness and performance inputs and outputs must be introduced to the plant. Then, the problem-specific information about the amount of uncertainty and the performance requirements must be introduced. This is done via *weighting functions.* By postmultiplying a weighting function $\mathbf{W}(s)$ to a plant output, the controller resulting from a successful design attempt will achieve $\mathbf{W}(j\omega)^* \mathbf{W}(j\omega)\mathbf{M}(j\omega) < 1$ for all ω. Then, with $\mathbf{W}(s)$ being of the form $\mathbf{w}(s)^*\mathbf{I}$, this implies that $\mathbf{M}(j\omega) < \mathbf{w}^{-1}(j\omega)$ for all ω. Obviously, the transfer function is bounded by the inverse of the weighting function $\mathbf{w}$. Consequently, $\mathbf{W}$ can be used to shape $\mathbf{M}$ by choosing $\mathbf{w}$ to be large at frequencies where $\mathbf{M}$ is to be small. In a similar way, information on input signals (e.g. disturbances) is included by premultiplying weighting functions $\mathbf{W}(s)$ that are proportional to the amplitude of the signals.

An example of a plant with additional inputs and outputs and weighting functions is shown in figure 2. For this plant, the robust performance problem is to find a controller $\mathbf{K}$ s.t. $\mu(\mathbf{T}(\mathbf{G},\mathbf{K})(j\omega)) < 1$ for all ω, where $\mathbf{T}(\tilde{\mathbf{G}},\mathbf{K})$ is defined as follows:

$$\begin{bmatrix} \tilde{m} \\ \tilde{y} \\ \tilde{u} \end{bmatrix} = \mathbf{T}(\tilde{\mathbf{G}},\mathbf{K}) \begin{bmatrix} m \\ d \\ n \end{bmatrix} = \begin{bmatrix} \mathbf{W_m} & & \\ & \mathbf{W_s} & \\ \mathbf{W_u} & & \end{bmatrix} \mathbf{T_1}(\tilde{\mathbf{G}},\mathbf{K}) \begin{bmatrix} \mathbf{I} & & \\ & \mathbf{W_d} & \\ & & \mathbf{W_n} \end{bmatrix} \begin{bmatrix} m \\ \tilde{d} \\ \tilde{n} \end{bmatrix},$$

$$\text{where } \mathbf{T_1} \text{ is defined by } \begin{bmatrix} u \\ y \end{bmatrix} = \mathbf{T_1}(\tilde{\mathbf{G}},\mathbf{K}) \begin{bmatrix} m \\ d \\ n \end{bmatrix}.$$

In summary, two steps are necessary to apply the theory to real-world problems like the pump: First, a suitable framework with robustness and performane inputs and outputs must be defined. Then, weighting functions reflecting the information on disturbances, on uncertainty and on performance requirements must be chosen. We will now describe these two steps in detail for the pump.

DEFINITION OF FRAMEWORK FOR CONTROLLER DESIGN

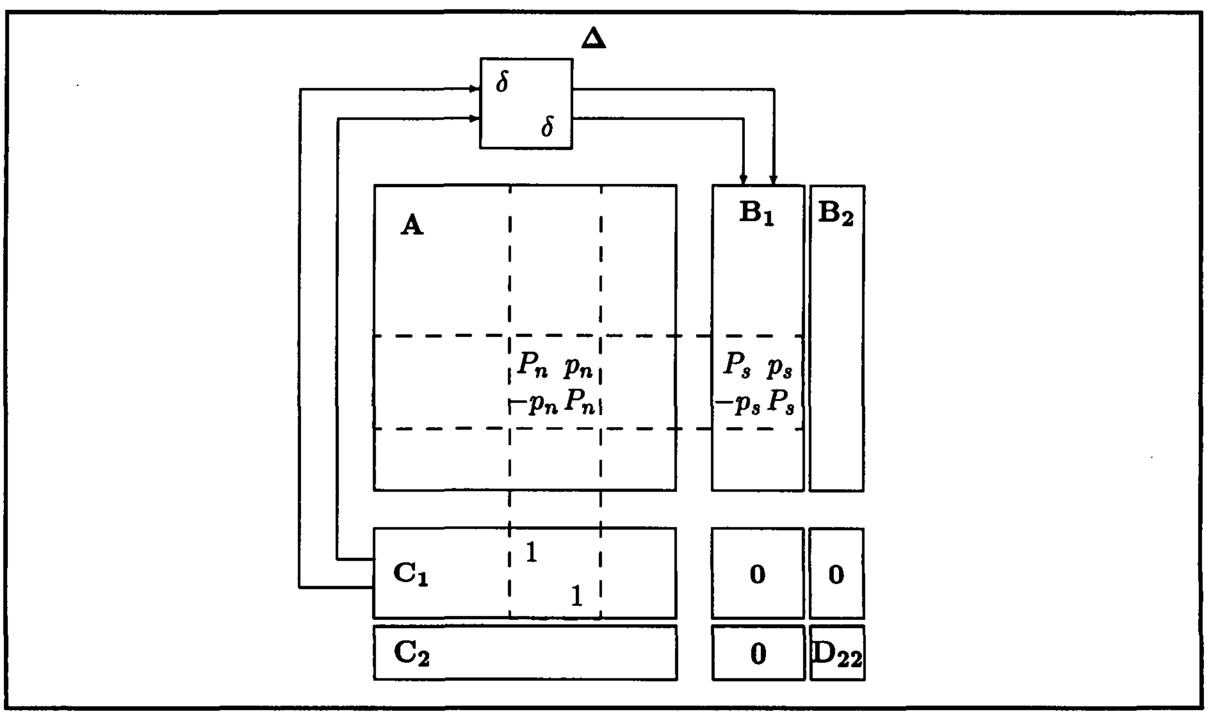
Robustness: Model of the System's Uncertainty

The various sources of uncertainties considered in the model can be split into two groups: Uncertainty stemming from the FE modelling, from numerical errors in the calculation of the seals' coefficients, from linearization, from manufacturing errors, and from the model reduction is *non-parametric* and must be modelled in a "lumped" (multiplicative) fashion. Uncertainty originating from the variation of the seals' damping and stiffness coefficients as the throughput q changes is *parametric*. To avoid conservatism, this uncertainty can be modelled by pulling the uncertain parameters out of the plant. This is outlined in figure 6 for the two parameters representing the damping or stiffness coefficients of one single seal.

In figure 6, the system without uncertainty is given by the matrices ($\mathbf{A}$,$\mathbf{B_2}$,$\mathbf{C_2}$,$\mathbf{D_{22}}$). The nominal parameters are contained in the matrix $\mathbf{A}$. The modelling of parametric uncertainty requires one extra input and one extra output on the plant for each uncertain parameter. Via the additional outputs which are represented by the matrix $\mathbf{C_1}$, the velocities and displacements of the degrees of freedom corresponding to the varying damping and stiffness coefficients leave the plant. These signals are then all multiplied by the same scalar δ which assumes values between -1 and 1 and represents the variation in throughput q. Then, the signals are multiplied by damping and stiffness scalings P_s and p_s representing the variation of the individual coefficents and the resulting forces are reinjected into the plant at the appropriate locations via the additional inputs defined by matrix $\mathbf{B_1}$.

This approach allows for explicit modelling of the variation in throughput. However, it assumes all coefficients to change linearly with q, which is not exactly the case. Even worse,

***Figure 6:** Parametric uncertainty model*



with extra inputs and outputs on the plant, the model reduction problem becomes much more difficult - the number of transfer functions that must be represented accurately by the reduced model increases quadratically with the number of inputs and outputs of the plant. Furthermore, with the ***D-K***-Iteration the resulting controller order increases with the number of inputs and outputs of the plant. Finally, the uncertain mass coefficients cannot be included in the parametric uncertainty model since the mass matrix enters the problem inversely. In consideration of these facts, we decided to incorporate the variation of the seals' coefficients in the multiplicative uncertainty together with the uncertainty coming from the other parameters. The uncertainty model chosen is depicted in figure 7.

Performance Objectives

Additional inputs and outputs on the framework are required to specify the performance objectives. To avoid reduction problems, all disturbance forces physically acting on the pump have been modelled as disturbances acting on the pump's output. Therefore, one additional input, d, was added to the framework directly behind the system. To account for sensor noise, the input n was introduced right before the controller.

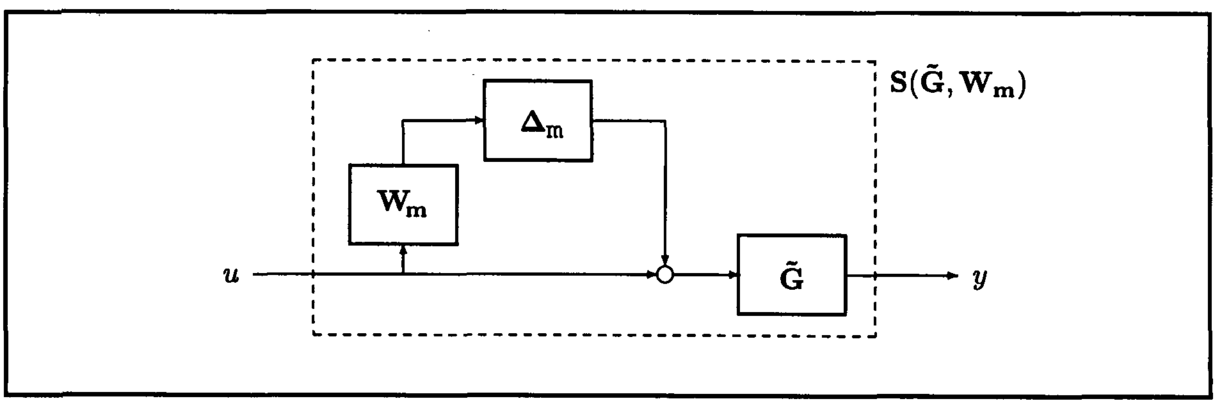
The most important performance requirement is that the rotor's displacement y may under no circumstances exceed the radius of the the air gap between rotor and magnetic bearings. To ensure that this goal will be met, an output $\tilde{y}$ is added. Finally, to model amplifier saturation, the output $\tilde{u}$ was added between controller and plant.

SELECTION OF WEIGHTING FUNCTIONS

Uncertainty Weight $\mathbf{W_m}$

The weighting function $\mathbf{W_m}$ describes the magnitude of the model uncertainty over frequency. It has to be chosen large enough to cover all model uncertainty (from FE modelling, numerical errors in the calculation of the seals' coefficients, linearization, manufacturing, model reduction, and from parameter variations) in order to achieve robust stability. However, choosing $\mathbf{W_m}$ larger than necessary degrades performance and may even yield a problem

Figure 7: *Model of multiplicative uncertainty*



without solution, although the plant could actually be robustly stabilized. Consequently, it is necessary to choose $\mathbf{W_m}$ large enough but with as little conservatism as possible. The following new theorem provides a basis for this:

Theorem 1: Consider the system in figure 7. Let $\tilde{\mathbf{G}}$ denote the nominal system, $\mathbf{G}$ an arbitrary disturbed system, and $\mathbf{S}(\tilde{\mathbf{G}}, \mathbf{W_{m,i}})$ the set of all systems $\tilde{\mathbf{G}}(\mathbf{I} + \Delta_m \mathbf{W_{m,i}})$ with the same number of unstable poles as $\tilde{\mathbf{G}}$ and $\|\Delta_m\|_\infty \le 1$. Then for each frequency ω, the diagonal weighting function just large enough so that $\mathbf{G}$ is contained in $\mathbf{S}(\tilde{\mathbf{G}}, \mathbf{W_{m,i}})$ is given by

$$\mathbf{W_m}(j\omega) = \bar{\sigma}(\tilde{\mathbf{G}}^{-1}(j\omega)(\mathbf{G}(j\omega) - \tilde{\mathbf{G}}(j\omega))) \times \mathbf{I_n}$$

(A short proof is provided in Appendix A).

This theorem allows to calculate the magnitude of the weighting function required to just capture a certain uncertainty *exactly.* For noninvertible systems $\tilde{\mathbf{G}}$ as in our case, the values for $\mathbf{W_m}$ can be calculated *pointwise* over frequency. Then, for any pair of systems $\tilde{\mathbf{G}}$ and $\mathbf{G}$, this theorem yields a suitable uncertainty weight. The final uncertainty weight to be used in the design is obtained by applying the theorem to all combinations of $\tilde{\mathbf{G}}$ and $\mathbf{G}$, where $\tilde{\mathbf{G}}$ always is the nominal system used in the design and $\mathbf{G}$ varies over all disturbed systems. Pointwise maximisation then yields the magnitude of the weighting function required to account for all uncertainties considered. A stable, real-rational upper bound is then chosen for $\mathbf{W_m}$.

In the above theorem, $\tilde{\mathbf{G}}$ and $\mathbf{G}$ are not required to be the actual nominal and physical systems in the design. In fact, it can be applied to *any* pair of systems $\tilde{\mathbf{G}}$ and $\mathbf{G}$. In the following, we exploit this capability to illustrate the size of the individual errors. To do so, we proceed in two steps: First, a weighting function $\mathbf{W_{m,i}}$ is calculated for each individual error. To this end, $\tilde{\mathbf{G}}$ and $\mathbf{G}$ must be chosen appropriately. In the second step, the individual weighting functions are added to yield a weighting function that covers the total uncertainty to be expected. $\mathbf{W_m}$ is then chosen as a stable, real-rational upper bound.

To cover the model reduction and flux variation errors, the theorem is successively applied with the reduced nominal plant for $\tilde{\mathbf{G}}$ and each of the six full-order systems for $\mathbf{G}$. The resulting uncertainty weighting functions are displayed in figure 8. A suitable weight to cover these errors would be the maximum of the six weighting functions shown.

To get a weighting function that covers possible errors in the model of seals (numerical errors, manufacturing imprecisions, wear), the seal's coefficients of the six full order systems have been varied by 10% in various ways. For each case, the above theorem has been applied. The resulting weighting functions for one of the six full order plants are displayed in figure 9.

The linear model of the pump is an approximation that works quite well for the range of low frequencies up to about 100Hz. However, in the high frequency range beginning at 500Hz, the behaviour of the system is not linear. To model this error, the weighting function from figure 10 has been designed.

For the remaining errors (FE-modelling and actuator model), no additional weighting functions have been introduced. Because these errors are small (high-resolution FE-Model, good linearity properties of bearings), the following calculation of the overall weight $\mathbf{W_m}$ will introduce enough conservatism to account for small additional errors.

After having captured all the individual errors, the weighting function $\mathbf{W_m}$ that covers the worst case combination of all these errors has to be designed. To this end, the total error has to

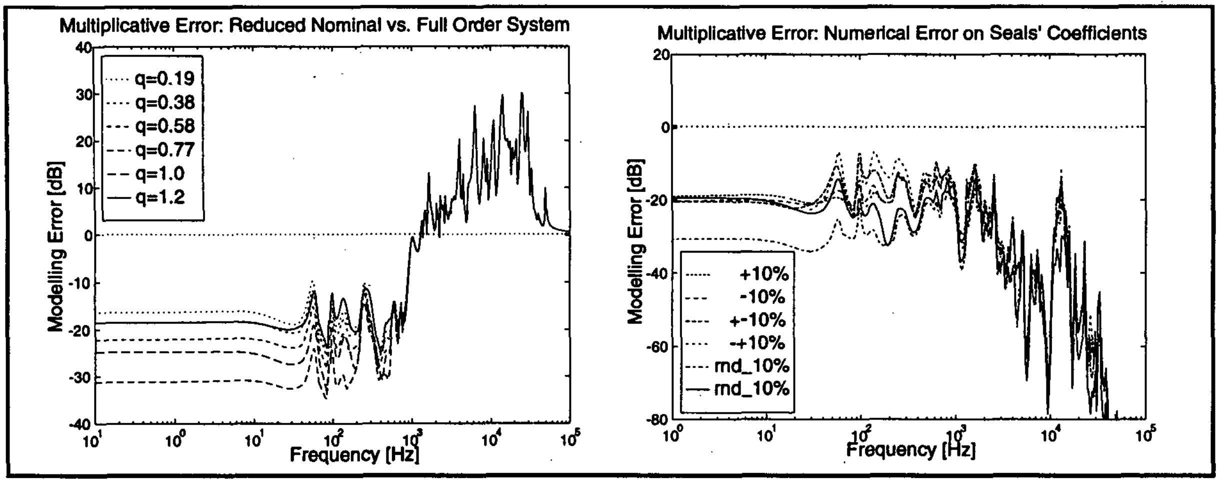


Figure 8 / 9: ***Weighting functions for reduction and throughput variation / seal variation***

be modelled as the sum of the individual errors. To be employable in the design, $\mathbf{W_m}$ is required to be real-rational. Figure 11 shows the maxima of the individual errors along with their sum and a real-rational weighting function that bounds this sum from above. This 3rd order weighting function is our choice for $\mathbf{W_m}$.

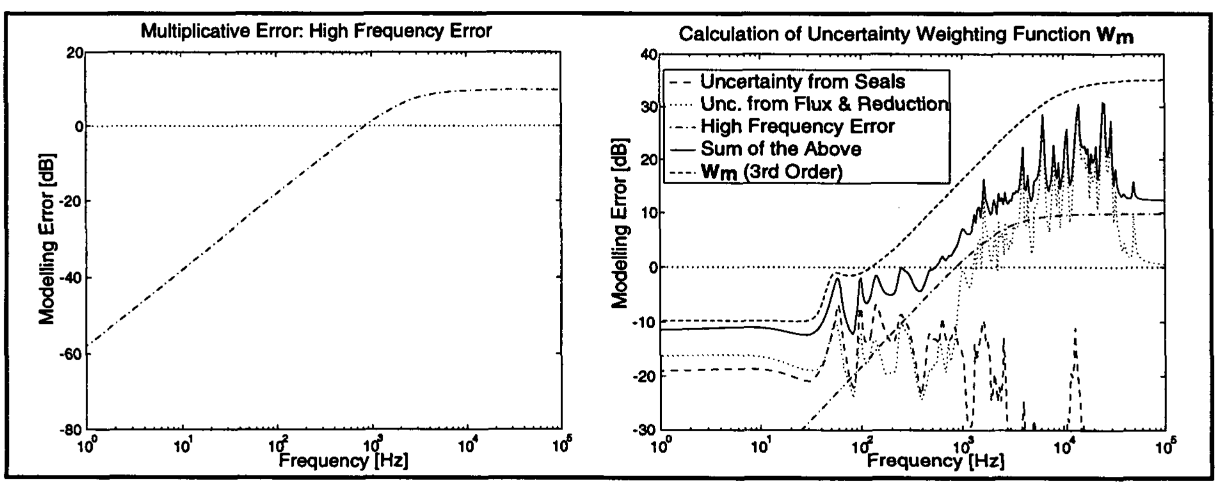
Disturbance Force Shaping Weight $\mathbf{W_d}$

During operation, disturbance forces are acting on the pump's impellers. These include:

- static forces (radial thrust)
- forces circulating at a low frequency (0-10 Hz)
- forces circulating at rotational freqency times # of running blades/impeller (700 Hz)

Unbalance is compensated using feedforward compensation (Herzog, 1996), such that the rotor is allowed to rotate about its main axis of inertia. Therefore, unbalance forces are not further considered here.

Figure 10 / 11: ***Weighting function for modeling high frequency error / total error, weight*** $\mathbf{W_m}$



Disturbance forces are modelled as disturbances acting on the pump's output (i.e. displacements) instead of forces on the impellers. Therefore, the physical forces expected have to be transformed into corresponding displacements on the pump's outputs. The size of these forces was computed to be a maximum of 300N on each of the impellers. Again, the worst case with all forces pointing in the same direction was assumed. Figure 12 shows the displacements at the bearings over the rotational frequency of the circulating forces. The shaded area is of no interest since in this frequency band no forces are expected to occur. It is outside this area only where the real-rational weighting function $\mathbf{W_d}$ must be larger than the displacements corresponding to the expected disturbance forces.

It is worth noting that here actually different weighting functions are used for the channels of the high and the low pressure AMB, whereas all other weighting functions considered in this design are of the form $\mathbf{w}*\mathbf{I_4}$, with $\mathbf{w}$ being a scalar weighting function and $\mathbf{I_4}$ the unit matrix of order 4.

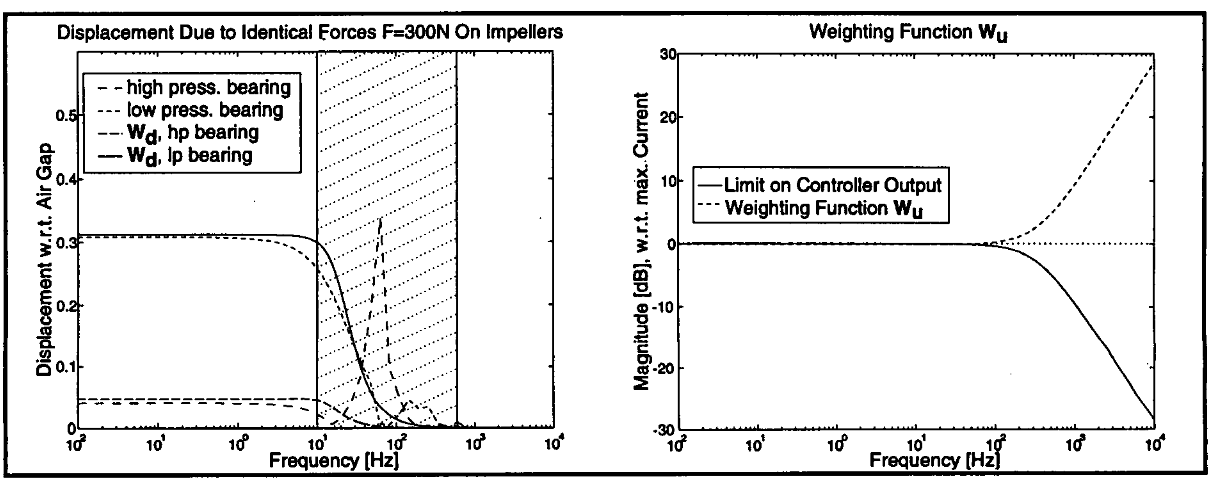
Uncertainty Weight $\mathbf{W_n}$

The sensor noise was assumed to be 0.1% of the air gap for all frequencies. The weight $\mathbf{W_n}$ was therefore chosen to be a constant 0.001.

Actuator Limitation $\mathbf{W_u}$

The force the AMBs can exert on the rotor is limited by the maximum current of the amplifiers (4A). For higher frequencies it is further diminished by amplifier voltage limitations. This has to be taken into account in the controller design by limiting the feasible controller output. Figure 13 shows both the admissible controller output and the weighting function $\mathbf{W_u}$.

Figure 12 / 13: *Disturbance forces' effect on plant outputs, weight* $\mathbf{W_d}$ / *actuator limitation and weight* $\mathbf{W_u}$



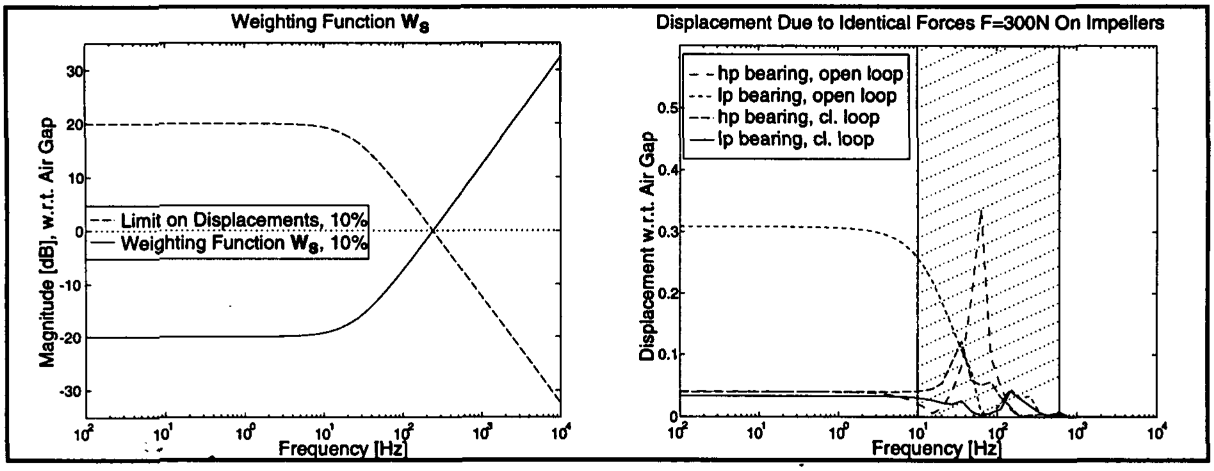


Figure 14 / 15: Displacement limitation weight $\mathbf{W_s}$ ***/ performance of reduced controller***

Displacement Limit Weight $\mathbf{W_s}$

The last weighting function that needs to be choosen is $\mathbf{W_s}$. Its purpose is to define an upper limit on the tolerable displacement at the bearings that has to be maintained despite all uncertainties and disturbances expected. In the framework chosen, this weighting function is the only true design parameter in this problem. All other weighting functions are defined by the physical properties of the plant. Figure 14 shows a weighting function $\mathbf{W_s}$ designed to limit the maximum acceptable output to 10% of the air gap.

GENERALIZED PLANT AND CONTROLLER DESIGN

With the framework from above the order of the generalized plant was 62. For this plant, a controller has been designed using the MATLAB ***D-K***-iteration script dkit (Balas et al., 1995). The corresponding uncertainty structure for the ***D-K***-Iteration according to the main loop theorem (figure 5) is block diagonal and has two blocks. The first block adresses robustess and has dimension 4 by 4, and the second block adresses performance and has dimension 8 by 8. The order of the resulting controller was 90. The increase in order results from the weighting functions added in the ***D***-step of the iteration.

A second design attempt with a displacement limitation to 5% of the air gap failed.

CONTROLLER REDUCTION, DISCRETISATION & PERFORMANCE ANALYSIS

The resulting controller achieved the performance and robustness specifications. However, 90 states are far too many for implementation. Hence, a controller reduction was carried out. The maximum implementable controller size was computed to be 32 states. Several controller reduction techniques were compared to achieve a controller of this size. The best results were achieved using a simple balanced truncation. The reduced controller still met all specifications.

To indicate the *reduced* controllers performance, the (nominal) pump's response to the expected disturbance forces is shown in figure 15. A comparison with figure 8 shows that the displacements due to disturbances have been reduced well below 5% of the air gap. For the AMB on the low pressure side of the pump, this is a reduction of about 85%.

Finally, the reduced controller was discretized using the z-transform. Again, there was no noticeable degradation in both performance and robustness.

CONCLUSION

A systematic, formalized way for derivation of the controller design parameters (weighting functions) from requirements on control variables, system limitations and uncertainties in physical model parameters has been presented. The applicability of the method has been demonstrated on a real-world system with many sources of uncertainties and large disturbance forces. Furthermore, it has been shown that μ-Synthesis can be used to obtain discrete controllers of implementable order for high order plants.

APPENDIX A

Proof of Theorem 1: For simplicity of the proof, assume that $\mathbf{G}(j\omega) - \tilde{\mathbf{G}}(j\omega) \neq 0 \forall \omega$. We have to show that there is a $\Delta_\mathbf{m}$ with $\|\Delta_\mathbf{m}\|_\infty \leq 1$ s.t. $\mathbf{G} = \tilde{\mathbf{G}}(\mathbf{I_n} + \Delta_\mathbf{m}\mathbf{W_m})$. By solving for $\Delta_\mathbf{m}$, we get $\Delta_\mathbf{m} = (\tilde{\mathbf{G}}^{-1}\mathbf{G} - \mathbf{I_n})\mathbf{W}_\mathbf{m}^{-1}$. With $\mathbf{w_m}(j\omega) = \bar{\sigma}(\tilde{\mathbf{G}}^{-1}(j\omega)(\mathbf{G}(j\omega) - \tilde{\mathbf{G}}(j\omega)))$, we get for $\Delta_\mathbf{m}$:

$$\begin{aligned}\bar{\sigma}(\Delta_\mathbf{m}(j\omega)) &= \bar{\sigma}((\tilde{\mathbf{G}}^{-1}(j\omega)\mathbf{G}(j\omega) - \mathbf{I_n})\mathbf{W}_\mathbf{m}^{-1}(j\omega)) \\ &= \frac{1}{\mathbf{w_m}(j\omega)}\bar{\sigma}(\tilde{\mathbf{G}}^{-1}(j\omega)\mathbf{G}(j\omega) - \mathbf{I_n}) = 1\end{aligned}$$

Thus, for this choice of $\mathbf{W_m}$, $\mathbf{G}$ lies in $\mathbf{S}(\tilde{\mathbf{G}}, \mathbf{W_m})$. The minimality is shown by contradiction.

REFERENCES

Balas, G. J., J.C. Doyle, K. Glover, A. Packard, and R. Smith: *MATLAB μ-Analysis and Synthesis Toolbox.* The Mathworks, Natick, MA, USA, 1995.

Diewald, W.: *Das Biegeschwingungsverhalten von Kreiselpumpen unter Berücksichtigung der Koppelwirkungen mit dem Fluid.* Fortschrittsberichte VDI Reihe 11 Nr.121, VDI-Verlag, Düsseldorf, 1989.

Green, M. and D.J.N. Limebeer: *Linear Robust Control,* Prentice Hall, Englewood Cliffs, 1995.

Herzog, R., Ph. Bühler, C.Gähler, R. Larsonneur: *Unbalance Compensation Using Generalized Notch Filters in the Multivariable Feedback of Magnetic Bearings.* IEEE Transactions on Control Systems Technology, Sept.1996.

Lösch, F.: *Modellierung und H_∞-Reglerentwurf für eine Kesselspeisepumpe mit aktiver magnetischer Lagerung.* Diploma thesis, University of Kaiserslautern (Germany) and ETH Zurich (Switzerland), 1997.

Ramb, Th.: *Konstruktion einer Speisepumpe mit aktiven magnetischen Lagern,* Diploma thesis, Fachhochschule Mannheim, 1995.

Schweitzer, G., H. Bleuler and A. Traxler: *Active Magnetic Bearings.* Verlag der Fachvereine, Zurich, Switzerland, 1994.

Zhou, K., J.C.Doyle, K.Glover: *Robust and Optimal Control,* Prentice Hall, Upper Saddle River, NJ, 1996.