ON-LINE-ADAPTED VIBRATION CONTROL

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ABSTRACT

This paper presents a method of vibration compensation for magnetically levitated rotor systems. Using feedforward strategy, compensation signals are produced with an adaptive filter. The rotation speed is the only parameter which has to be known exactly.
In the first part a short introduction into the feedforward vibration compensation is given. The second part discusses the so-called 'method of steepest descent'. This method is based on the gradient of the mean square error. Then the LMS algorithm is presented. It is shown that it behaves 'in the mean' like the method of steepest descent. In the last part simulations and measurements of a high-speed drive system are presented.

INTRODUCTION

At the Electronic Engineering and Design Laboratory of the Swiss Federal Institute of Technology, several projects are targeted on magnetic levitated high-speed spindles up to 120'000 rpm. Because of limited precision of production, unbalance arises which unfortunately cannot be modeled exactly.

For rotationally symmetrical shafts the geometric axis and the axis of inertia are theoretically the same. In this case no unbalance will be measured. The rotor turns around its geometric axis and the orbit will be a single point.

With unbalance one might imagine an additional small mass (Figure 1). Now the rotor turns around the axis of inertia, which is moved away from the geometric axis. The sensors detect a deflection of the shaft, the orbit. If the rotation speed is supercritical, the gyroscopic coupling becomes dominant and the rotor becomes stabilized in a certain orbit. The position controller, assumed to have very high gain, tries to nullify the orbit. The stable orbit could not be corrected because of the limited dynamic bearing forces.

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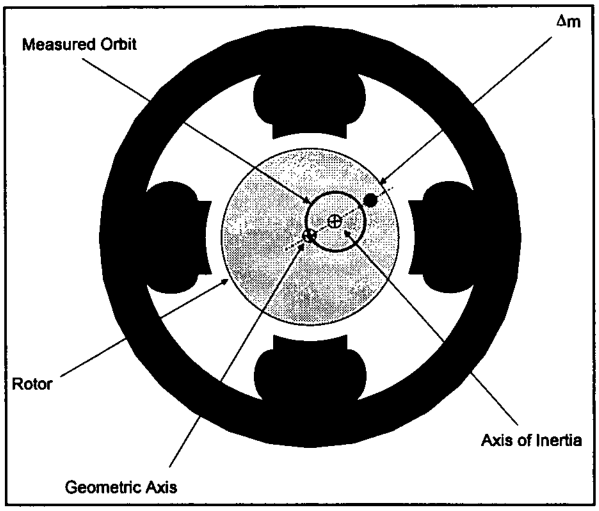

Figure 1: Orbit caused by unbalance

The main idea of vibration compensation is to nullify the 1st harmonic of the orbit. The position controller no longer reacts to a deflection and no harmonic output current is produced. The rotor is turning now force free. This makes it possible to reduce power consumption of the magnetic bearing system.

There are several methods for vibration compensation. They could usually be divided in two families: notch filters in the controllers and open loop control strategies.

In the first family adaptive notch filters are inserted into the feedback of the control loop (Herzog et al.,1996) They affect the phase around the rotation speed so that it is difficult to cross the bending criticals.

Open loop or feedforward strategies produce compensation signals which are added to the sensor signals so that the orbit vanishes. Some of these strategies have the disadvantage that system matrices are needed (Larsonneur, Siegwart and Traxler,1992; Knospe et al. 1993, 1997a, 1997b). These matrices have to be known or estimated. With this knowledge they produce quite good compensation even in the bending criticals of the rotor.

In the following, a new method based on the LMS algorithm (Moschytz, 1995) is presented. It needs only statistical knowledge of the system and does not affect the behavior of the main control loop.

PRINCIPLE OF 'ADAPTIVE FEEDFORWARD CONTROL'

With the help of a reference sine the adaptive filter produces a signal which corresponds to the first harmonic of the desired current in amplitude and phase. In electric drive systems this signal usually can be derived from the motor controller (Figure 2). The only condition is that the first harmonic of the position signal and reference sine must have the same frequency. Knowledge of the phase is not necessary, which is an advantage over other vibration control strategies. The output of the filter is subtracted from the position signal. The controller now „sees“ only some noise around zero.

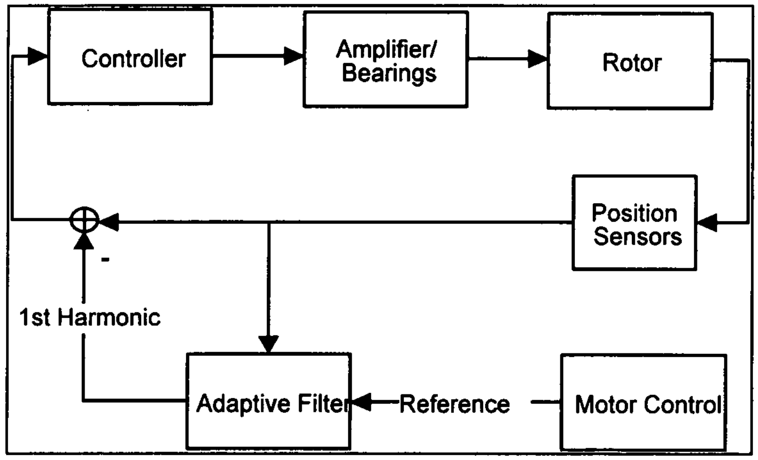

Figure 2: Feedforward Control

THE ADAPTIVE FILTER (AF)

THE FINITE IMPULSE RESPONSE (FIR) FILTER

Figure 3 shows a scheme of an adaptive FIR filter. The basis for an AF that reproduces amplitude and phase of a noisy signal with the help of a reference sine of the same frequency is the finite impulse response filter.

An FIR filter of infinite length could produce theoretically any answer. But we are looking for a harmonic that is equal to our orbit in amplitude and phase. A second order FIR filter creates two out-phased sines $\underline{x}[k]=\begin{bmatrix} x[k] \\ x[k-1] \end{bmatrix}$. With their linear combination, any harmonic signal which is different in amplitude and phase can be produced.

A very simple way for vibration reduction at a fixed frequency is to tune the filter so that the difference between output signal and position signal becomes as small as possible.

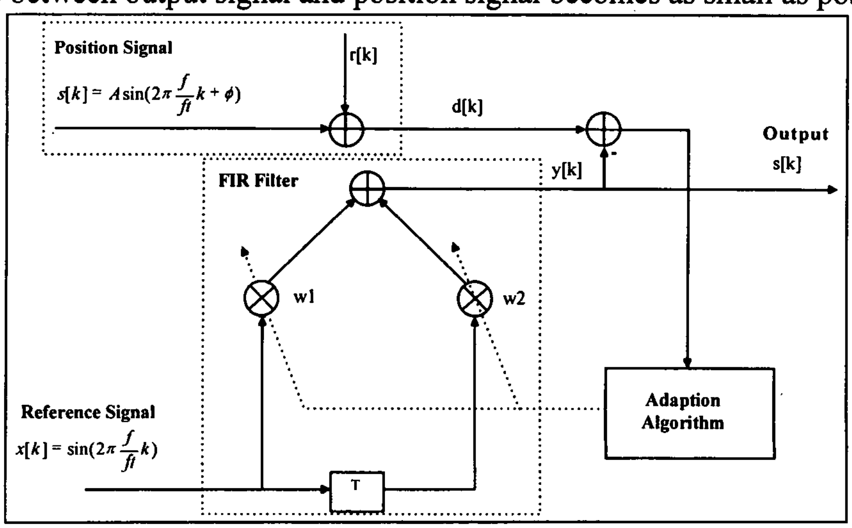

Figure 3: Structure of the Adaptive Filter

THE ERROR FUNCTION

One could assume the position signal to be a sine s[k] superposed by *White Gaussian Noise* r[k]. The orbit can be thought to be a 'desired signal' d[k]=s[k]+r[k] that must be aproximated by the output signal y[k] of the filter. An error e[k] is produced by subtracting the output y[k] of the filter from the position signal d[k].

$$e[k] = d[k] - \underline{w}'[k]\underline{x}[k] \text{ with } \underline{w}[k] = \begin{bmatrix} w_1[k] \\ w_2[k] \end{bmatrix} \qquad (1)$$

We are looking now for a suitable error measure which helps to adapt the filter coefficients. Because only the statistical characteristics of the signals are known, the *Mean Square Error* (MSE) is chosen. Assuming that the signals $\underline{x}[k]$ and d[k] are weakly stationary the MSE could be calculated as follows:

$$MSE = J(\underline{w}) = E\{e^2[k]\} = \qquad (2)$$

$$= E\{d^2[k]\} + \underline{w}'E\{\underline{x}[k]\underline{x}'[k]\}\underline{w} - 2E\{d[k]\underline{x}[k]\}\underline{w}$$

$$R = E\{\underline{x}[k]\underline{x}'[k]\} = \frac{1}{2}\begin{pmatrix} 1 & \cos(2\pi\frac{f}{ft}) \\ \cos(2\pi\frac{f}{ft}) & 1 \end{pmatrix}$$ is the *Autocorrelation Matrix* and

$$\underline{p} = E\{d[k]\underline{x}[k]\} = \frac{A}{2}\begin{pmatrix} \cos(\Phi) \\ \cos(2\pi\frac{f}{ft}) \end{pmatrix}$$ the *Cross-Correlation Vector.*

R and $\underline{p}$ can be inserted in (2).

$$J(\underline{w}) = E\{d^2[k]\} + \underline{w}'R\underline{w} - 2\underline{p}'\underline{w} \qquad (3)$$

This error function can be graphically demonstrated very easily for a second order FIR-Filter:

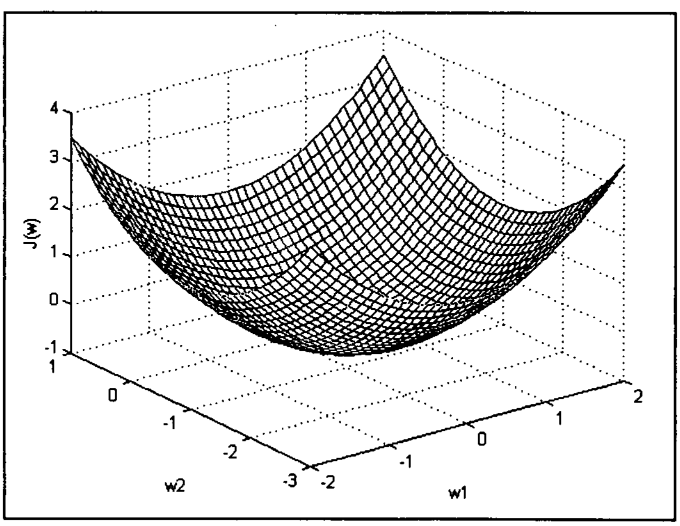

Figure 4: Error plane

We are looking now for the optimal coefficient vector $\underline{w}^o = \begin{bmatrix} w_1^o \\ w_2^o \end{bmatrix}$, for which the error plane is negligible:

$$J(\underline{w}^o) = \min \tag{4}$$

Theoretically it can be calculated exactly by setting the gradient equal to zero provided that the system is well known:

$$\underline{w}^o = R^{-1}\underline{p} = \frac{A}{4(1-\cos^2(2\pi\frac{f}{ft})}\begin{pmatrix} \cos(\phi) - \cos(4\pi\frac{f}{ft}+\phi) \\ \cos(2\pi\frac{f}{ft}+\phi) - \cos(2\pi\frac{f}{ft}-\phi) \end{pmatrix} \tag{5}$$

But there are various iterative search algorithms which do not require this knowledge.

THE METHOD OF STEEPEST DESCENT

The Method of Steepest Descent serves as basis for the LMS algorithm, which is better known under ‘Procedure of Newton’. Assume any starting vector $\underline{w}[k]$. With a small step in the direction of the negative gradient vector we reach the next coefficient vector $\underline{w}[k+1]$. The MSE decreases with this action.

$$\underline{w}[k+1] = \underline{w}[k] - c\nabla_{\underline{w}}\{J(\underline{w})\} \tag{6}$$

The gradient vector can be derived from (3).

$$\nabla_{\underline{w}}\{J(\underline{w})\} = -2\underline{p} + 2R\underline{w} \tag{7}$$

By inserting (7) in (6) and with $\mu = 2c$ (the step size of the algorithm) we get:

$$\underline{w}[k+1] = (I - \mu R)\underline{w}[k] + \mu\underline{p} \tag{8}$$

Note that $\underline{w}[k]$ only converges if

$$\|(I-\mu R)\underline{v}\| < \|\underline{v}\|, \text{ for any vector } \underline{v}. \tag{9}$$

SELECTION OF μ

In chapter 0 it was shown that convergence is not garanteed for any step size. An upper limit can be derived from (9).

$$0 < \mu < \frac{2}{\lambda_{max}} = \mu_{max} \tag{10}$$

Here, λ_{max} is the largest eigenvalue of the Autocorrelation matrix R. In practice one would select the step size 10 to 100 times smaller than μ_{max} to get a monotonic descending convergence behaviour.

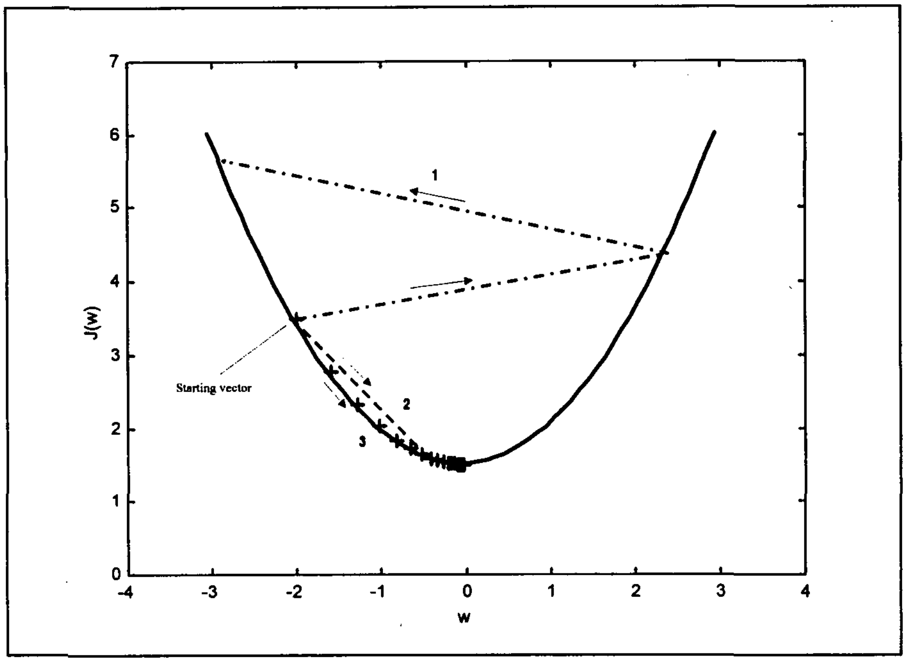

Figure 5: Learning curve of the AF

Figure 5 illustrates the behaviour of the AF for different μ:

1: $\mu > \mu_{max}$

2: $\mu < \mu_{max}$

3: (+) $\mu << \mu_{max}$

INFLUENCE OF THE STEP SIZE μ ON THE CONVERGENCE TIME

The convergence time of a filter coefficient is defined as the time during which w[k]-w° falls short of $\frac{1}{e}(w[0]-w^o)$. Therefore the maximum convergence time depends on the slowest coefficient. The following equations can serve as estimations:

$$\tau_{max} = \frac{-1}{\ln(1-\mu\lambda_{min})} \quad (11)$$

and for $\mu << \mu_{max}$

$$\tau_{max} \cong \frac{1}{\mu\lambda_{min}} \quad (12)$$

THE LEAST MEAN SQUARE (LMS) ALGORITHM

The big advantage of the LMS algorithm, which is discussed in here, is its very simple structure. That is the reason why this algorithm is one of the most widespread algorithms for FIR- based adaptive filters.

THE STRUCTURE OF THE LMS ALGORITHM

The basis for the LMS algorithm is the momentary quadratic error. Its momentary gradient is calculated by:

$$\nabla_{\underline{w}}\{e^2[k]\} = 2e[k]\nabla_{\underline{w}}\{d[k] - \underline{w}'\underline{x}[k]\} = -2e[k]\underline{x}[k] \qquad (13)$$

By replacing the gradient in the Method of Steepest Descent by this momentary gradient we obtain the LMS algorithm:

1. Calculate the output of the filter with the actual coefficients.
   $y[k] = \underline{w}'[k]\underline{x}[k]$
2. Calculate the error.
   $e[k] = d[k] - y[k]$
3. Adapt the coefficient vector by using the momentary gradient.

$$\underline{w}[k+1] = \underline{w}[k] + \mu e[k]\underline{x}[k] \qquad (14)$$

COMPARISON BETWEEN LMS ALGORITHM AND METHOD OF STEEPEST DESCENT

Now we want to show that the LMS algorithm behaves on average the same as the Method of Steepest Descent. The following depend on a few suppositions that are known in the literature as fundamental assumptions.
If we build the expectation of $\underline{w}[k+1]$ we get:

$$E\{\underline{w}[k+1]\} = (I - \mu R)E\{\underline{w}[k]\} + \mu \underline{p} \qquad (15)$$

Which is the same as the expectation of (8). We see that the same rules are valid for convergency as in the Method of Steepest Descent.

RESULTS

As test rig serves the spindle shown in Figure 6. The rotor has a mass of about 0.680 kg and a length of 18 cm. The average diameter is 3cm. It is designed to operate at 120'000 rpm with the first free-free bending mode at 100'000 rpm. The rotor is supported by a radial bearing and a combined radial and axial bearing, both permanent magnet biased. The control system is a TMS320C50 based 5-channel electronic with a maximum of 48V output voltage and 3A output current.

The simulation (Figure 7) and measurements (Figure 8) are made at a speed of about 50'000 rpm below the first bending mode. Simulation and measurements showed that with the adaptive filter strategy the output current can be reduced by a factor of about ten. The reference signal derived from the motor control makes a continuous adaption and also a continuous vibration control possible at different speeds of rotation. Thus a run-up with the vibration control turned on is possible even through the first bending mode.

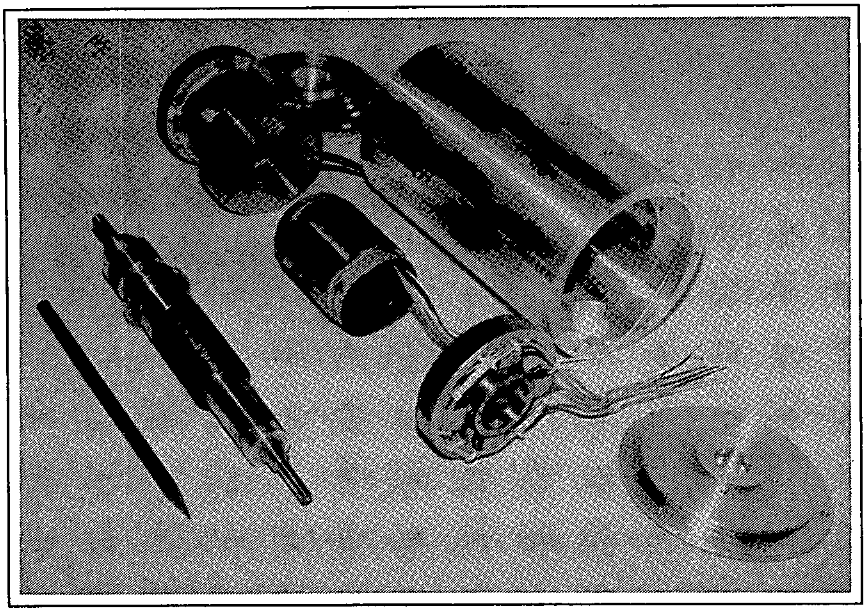

Figure 6: Magnetic bearing system

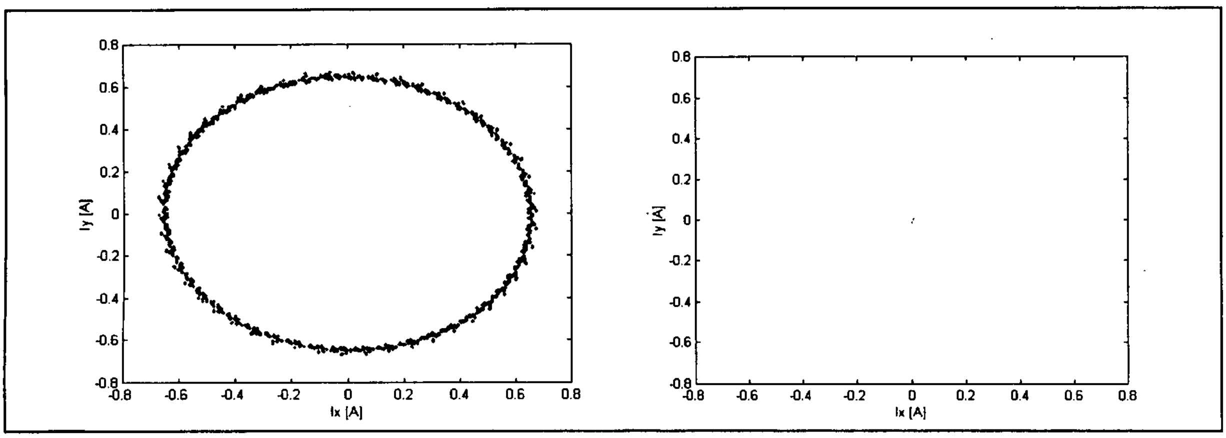

Figure 7:Simulated current orbits; left VC turned off, right VC turned on

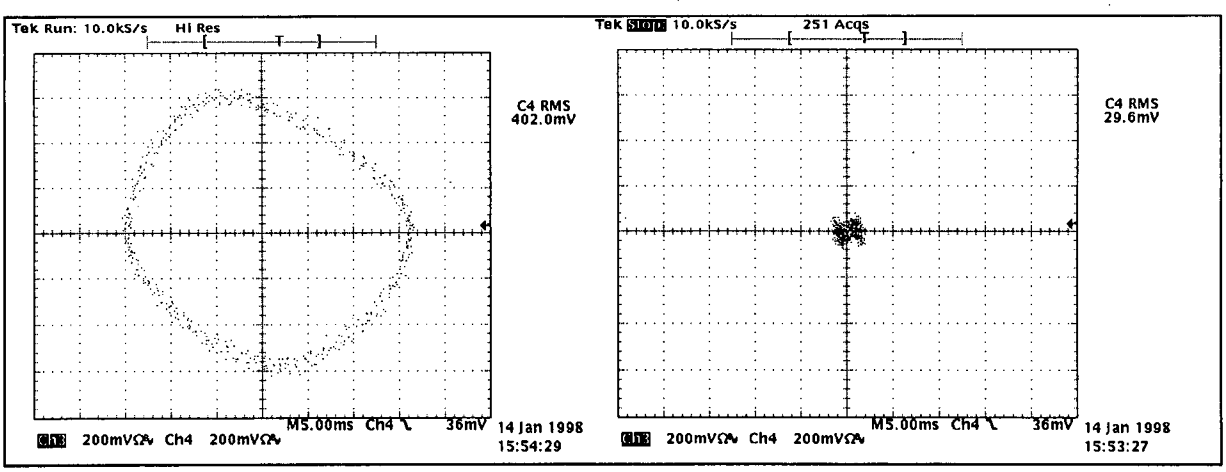

Figure 8: Measured current orbits; left VC turned off, right: VC turned on
(1V corresponds to 1A)

SUMMARY AND CONCLUSION

The only parameter that influences the convergence is the step size. However the estimation of this parameter is relatively easy. Because of its inherent stability the algorithm adapts to parameter changes in the system like changes in rotation speed or unbalance.
Because of its simple structure the LMS algorithm is suited for time-critical magnetic bearing control. Nevertheless it allows very good vibration compensation without applying additional disturbance signals and without exact knowledge of the system parameters.

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SYMBOLS AND ABBREVIATIONS USED

AF	Adaptive filter
j	$\sqrt{-1}$
MSE	Mean square error
R	Autocorrelation matrix
$\underline{p}$	Cross-correlation vector
E	Expectation
∇	Gradient
μ	Step size of AF
d	Desired signal
r	White gaussian noise
s	1st Harmonic of the orbit
x	Reference signal
y	Output of AF
λ	Eigenvalue
f	Rotating frequency
ft	Sampling frequency

LITERATURE

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