OPTIMAL DESIGN OF RADIAL MAGNETIC BEARINGS

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ABSTRACT

The geometric design of radial magnetic bearings is considered using an optimal design procedure. General expressions are derived for the performance variables of magnetic bearings of common geometry. An optimization algorithm is used to arrive at a solution that maximizes the load capacity subject to thermal and other mechanical constraints. The optimal solution is discussed with respect to the effect of various geometric parameters on the load capacity.

INTRODUCTION

Magnetic bearings for the purpose of supporting rotating machinery are attracting increased research interest in recent years as evidenced by the number of publications (Allaire, 1994), (Schweitzer et al., 1994). They have several advantages over conventional bearings including reduced friction; moreover, the fact that they can be controlled in an active control loop is especially advantageous since proper design could accomplish reduction of vibration and noise.

Although a number of papers have appeared on the science and application of magnetic bearings, most of them do not deal with the mechanical design aspects but instead deal with other issues such as controllability. In fact, the thermal aspects of magnetic bearings have received very little attention with the exception of (Jones and Nataraj, 1997).

In an attempt to design radial magnetic bearings for a heavy shaft, we found that the problem of a high load capacity conflicted with having to satisfy thermal requirements. Conventional attempts to select the geometric parameters of the magnetic bearing led to either an insufficient load capacity or a bearing that was too hot to meet the insulation specifications. This led to the need for an optimal design procedure that is presented in this paper.

MATHEMATICAL DEVELOPMENT

ELECTROMAGNETICS

Figure 1 shows a typical radial bearing with some of the nomenclature used in the analysis presented below. In the following treatment, we assume that the radial bearing with N_p poles can be modeled accurately by considering each pole-pair separately, and then by adding the force contributions. This is the most common approach in magnetic bearing literature.

Considering each pole-pair, we assume that the magnetic field is uniform. Using Ampere's Law (Woodson and Melcher, 1968), the magnetic flux density follows to be (for the kth pole-pair),

$$B = \frac{\mu_0 N i}{\ell_k} \tag{1}$$

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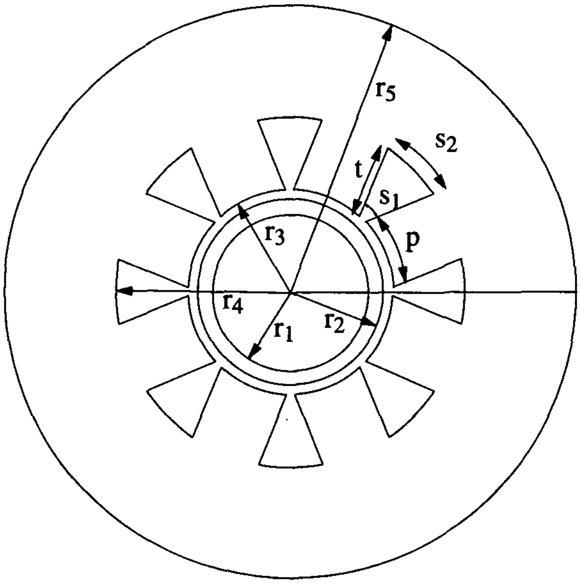


Figure 1: Radial Bearing Nomenclature

and the magnetic path length, ℓ_k is given by

$$\ell_k = \frac{\ell_{\mathrm{Fe}}}{\mu_r} + 2g_k \tag{2}$$

where,

$$\ell_{\mathrm{Fe}} = 2t + \frac{2\pi}{N_p}(r_2 + r_4) \tag{3}$$

The gap between the pole faces and the rotor is a function of the rotor displacement (v, w), and is given by

$$g_k = g_0 - v\cos\phi_k - w\sin\phi_k. \tag{4}$$

g_0 is the nominal gap, ϕ_k is the angle of the centerline of the pole-pair with respect to the y-axis.

The force exerted by the kth pole-pair on the rotor is then given by

$$\vec{F}_k = \frac{\mu_0 N^2 A_a}{\ell_k^2} i_k^2 \cos\left(\frac{\pi}{N_p}\right)\left(\cos\phi_k\vec{\jmath} + \sin\phi_k\vec{k}\right) \tag{5}$$

where,

$$\phi_k = \left(\frac{4(2k-1)}{2N_p} - \frac{1}{4}\right)\pi. \tag{6}$$

With a differential driving mode (Schweitzer et al., 1994) the currents supplied to the four quadrants are as follows.

$$i_1 = i_b + i_{cy0} + i_{cy} \tag{7}$$
$$i_2 = i_b + i_{cz0} + i_{cz} \tag{8}$$
$$i_3 = i_b - i_{cy0} - i_{cy} \tag{9}$$
$$i_2 = i_b - i_{cz0} - i_{cz} \tag{10}$$

Here, i_b is the bias current, (i_{cy0}, i_{cz0}) are the currents supplied to offset the static load in the (y, z) directions, and $(i_{cy}(t), i_{cy}(t))$ are the control currents in the (y, z) directions that are determined by the control circuit in response to the dynamic motion of the rotor.

We next expand the forces in a Taylor series about the undisturbed equilibrium position $(v = 0, w = 0, i = i_0)$. For example,

$$F_y = F_{y0} + \left.\frac{\partial F_y}{\partial v}\right|_0 v + \left.\frac{\partial F_y}{\partial w}\right|_0 w + \left.\frac{\partial F_y}{\partial i_{cy}}\right|_0 i_{cy} + \left.\frac{\partial F_y}{\partial i_{cz}}\right|_0 i_{cz} \tag{11}$$

Some involved algebra and use of trigonometric identities (Gradshteyn and Ryzhik, 1980) such as

$$\sum_{k=1}^{n} \sin(2k-1)x = [\sin(nx)]^2 \operatorname{cosec} x \tag{12}$$

$$\sum_{k=1}^{n} \cos(2k-1)x = \frac{1}{2}\sin(2nx)\operatorname{cosec} x \tag{13}$$

leads to the following.

$$F_{y0} = \frac{\sqrt{2}}{\sin(\pi/N_p)} \frac{\mu_0 N^2 A_a}{\left(\frac{\ell_{Fe}}{\mu_r} + 2g_0\right)^2} i_b i_{y0} \tag{14}$$

where,

$$K_m = \frac{\mu_0 N^2 A_a}{\ell_0^2} \tag{15}$$

Similarly,

$$F_{z0} = \frac{\sqrt{2}}{\sin(\pi/N_p)} \frac{\mu_0 N^2 A_a}{\left(\frac{\ell_{Fe}}{\mu_r} + 2g_0\right)^2} i_b i_{z0} \tag{16}$$

In this paper, we assume that a static load exists only in the z-direction, and since our system has a large static load which we would like the bearing to support, we define the load capacity of the bearing to be F_{z0}. If a large side-load exists, or if a large dynamic load were expected, then the load capacity would have to be defined differently.

The position stiffnesses are evaluated from the following expressions.

$$K_{pyy} = -\frac{\partial F_y}{\partial v}, \qquad K_{pyz} = -\frac{\partial F_y}{\partial w} \tag{17a}$$

$$K_{pzy} = -\frac{\partial F_z}{\partial v}, \qquad K_{pzz} = -\frac{\partial F_z}{\partial w} \tag{17b}$$

After some manipulations we get the following expressions.

$$\begin{aligned} K_{pyy} &= -\frac{4K_m}{\ell_0}\cos\left(\frac{\pi}{N_p}\right)\left[(i_b^2 + i_{y0}^2)\left(\frac{N_p}{16} + \frac{1}{2\sin(4\pi/N_p)}\right)\right. \\ &+ \left.(i_b^2 + i_{z0}^2)\left(\frac{N_p}{16} - \frac{1}{2\sin(4\pi/N_p)}\right)\right] \end{aligned} \tag{18}$$

$$K_{pyz} = K_{pzy} = 0 \tag{19}$$

$$\begin{aligned} K_{pzz} &= -\frac{4K_m}{\ell_0}\cos\left(\frac{\pi}{N_p}\right)\left[(i_b^2 + i_{z0}^2)\left(\frac{N_p}{16} + \frac{1}{2\sin(4\pi/N_p)}\right)\right. \\ &+ \left.(i_b^2 + i_{y0}^2)\left(\frac{N_p}{16} - \frac{1}{2\sin(4\pi/N_p)}\right)\right] \end{aligned} \tag{20}$$

Note that the position stiffness terms are negative which indicates that the magnetic bearings are inherently unstable. The controller gains would have to be sufficiently large to overcome this negative stiffness. Also, a control circuit failure would result in catastrophic damage to the bearings (and possibly the shaft); hence a set of suitably designed backup bearings is an absolute necessity. Also, the cross-coupling terms are zero because of certain simplifying assumptions we make about the magnetic field.

The current stiffnesses are evaluated from (for example)

$$K_{iyy} = \frac{\partial F_y}{\partial i_y} \tag{21}$$

After some algebraic manipulations, we get the following expressions.

$$K_{iyy} = K_{izz} = K_m i_b \cos\left(\frac{\pi}{N_p}\right)\frac{\sqrt{2}}{\sin(\pi/N_p)} \tag{22}$$

$$K_{iyz} = K_{izy} = 0 \tag{23}$$

THERMAL ANALYSIS

A nodal network lumped parameter method was used (Palm, 1983) to estimate the temperature rise in the bearings. Although not as accurate as a detailed distributed parameter model such as in (Jones and Nataraj, 1997), this method does give us a reasonable estimate. In addition, since it formed a part of the optimization code it had to be computed several hundred times, and speed of computation was an issue that could not be ignored.

Essentially, we assume that there are three thermal subsystems at uniform temperatures: the copper coils, the insulation, and the iron core. The air is assumed to be at a certain temperature, and the heat transfer coefficient (for convection) is estimated from handbooks (Baumeister et al., 1978). It should be noted that it is very difficult to accurately estimate these numbers from theory without an experimental corroboration. The heat generation is from the electrical current in the coils (which would be a maximum of twice the bias current). Then, the model reduces to the following linear algebraic equations.

$$\begin{bmatrix} \frac{1}{R_1}+\frac{1}{R_4} & -\frac{1}{R_1} \\ -\frac{1}{R_4} & \frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{R_3} \end{bmatrix} \begin{bmatrix} T_{Cu} \\ T_{Fe} \end{bmatrix} = \begin{bmatrix} P+\frac{1}{R_4}T_a \\ \left(\frac{1}{R_2}+\frac{1}{R_3}\right)T_a \end{bmatrix} \tag{24}$$

where, T_{Cu} is the copper coil temperature, T_{Fe} is the temperature in the iron core, T_a is the ambient temperature, P is the power generated, and the R_i are the thermal resistances computed from conduction or convection coefficients and the areas available for heat transfer. Note that in an optimal design situation, each new feasible design will result in different heat transfer areas which will affect the heat transfer rates.

The thermal resistances as functions of the system parameters are as follows.

$$\begin{array}{ll} R_1 = t_i/(A_{i1}k_i) & R_2 = 1/(h_1 A_{Fe1}) \\ R_3 = 1/(h_2 A_{Fe2}) & R_4 = 1/(h_3 A_{i2}) \end{array} \tag{25}$$

where, the areas for heat transfer are given by

$$\begin{array}{ll} A_{i1} = 2tl & A_{i2} = A_{i1} \\ A_{Fe1} = 2(pt + (r5 - r4)\phi) & A_{Fe2} = s_2 l + pl \end{array} \tag{26}$$

Here, k_i is the conduction coefficient for the insulation material, and h_i are the various convection coefficients.

OPTIMAL DESIGN

The design of the bearing was cast into the following problem.

$$\text{Minimize } (-F_{zo})$$

subject to the following constraints.

- Insulation temperature, T_{Cu} should not exceed a stated maximum (T_{max}).
- The magnetic field, B should be below the saturation limit, B_{sat} of the magnetic material.
- The number of turns of wire, N should be less than the maximum, N_{max} that can be wound on the pole.
- The outer radial ring of the stator, r_5 should be greater than a minimum value ($r_{5,min}$).

A detailed look at the geometric parameters of the magnetic bearing and the equations presented earlier reveals that only a handful of them are independent. The other parameters are determined as a function of these independent parameters, which are called design variables in the context of optimization.

Hence, the design variables were chosen to be the bias current (i_b), the nominal gap (g_0), the ratio of the outer radius of the stator (r_5) to the shaft radius (r_1), the ratio of pole width to its depth (p/t), the ratio of pole width to the slot width (p/s_1), and the number of poles (N_p). Each of these design variables is also subject to lower and upper limits.

The above problem is a constrained minimization problem. Although the problem falls into the category of what is called shape design which is best solved using special solution algorithms (Barba et al., 1994), the present study used a standard nonlinear optimization algorithm called the Sequential Quadratic Programing (SQP) method (Branch and Grace, 1996). The gradients of the objective function and the constraints with respect to the design variables were computed by finite differences, and an estimate of the Hessian of the Lagrangian was updated at each iteration using the Boyden-Fletcher-Goldfarb-Shanno (BFGS) formula. For details about the optimization algorithms the reader is referred to the references cited in (Branch and Grace, 1996).

NUMERICAL RESULTS

The constraints were as follows.

1. $T_i < 150^0$C
2. $\frac{r_5}{r_4} > 1.1$
3. $2i_b < i_{\text{sat}}$
4. $N < N_{\text{max}}$

The first constraint specifies that the insulation temperature should be less than 150^0C, which is a fairly standard limit on most commercial insulation. The second constraint ensures that the outer ring has sufficient structural strength. The third constraint states that the total current in any of the coils should not exceed the level at which magnetic saturation can occur since that would lead to a severe performance impairment of the bearing. The last condition ensures that the number of turns of coil is not so large that it can not be wound on the poles. Both i_{sat} and N_{max} are determined dynamically in the optimal process since they will change when the shape of the bearing changes.

In addition, the following lower and upper limits were used for the design variables.

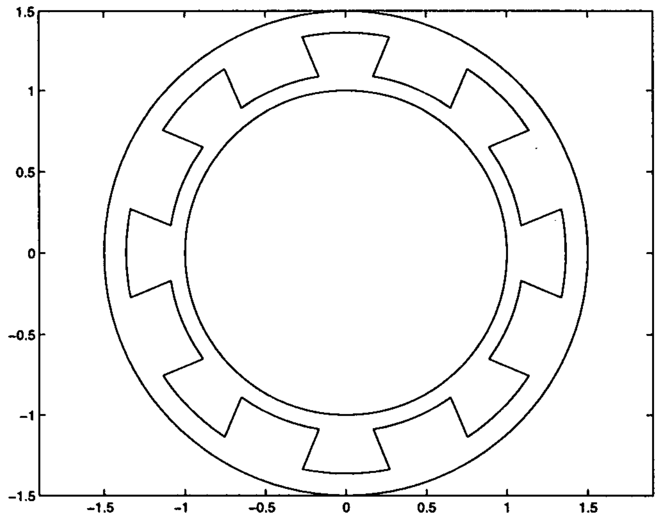
1. $4 \leq i_b \leq 10$
2. $0.1 \leq \frac{p}{t} \leq 2.0$
3. $0.1 \leq \frac{s_1}{p} \leq 0.9$
4. $10\,\text{mil} \leq g_0 \leq 125\,\text{mil}$
5. $1.1 \leq \frac{r_5}{r_1} \leq 1.5$

Some of the thermal and magnetic properties assumed for the analysis are listed in Table 1. They were mostly taken from standard sources such as (Baumeister et al., 1978).

Convection coefficient for face area	h_1	4 W/m^2 K
Convection coefficient for enclosed area	h_2	4 W/m^2 K
Conduction coefficient for insulation	k_i	0.16 W/ m K
Ambient temperature	T_a	20^0C
Magnetic permeability in space	μ_0	$4\pi \times 10^{-7}$
Relative permeability	μ_r	3000
Derating factor		0.9

Table 1: Some parameter values used in the analysis

The optimization code was run for several values of N_p in multiples of 8. Note that if the number of poles is too large the air gap may need to be too small for practical applications. The small gap is required to assure reasonable flux linkage to the rotor and to avoid flux linkage between adjacent bearing poles. In any case, irrespective of the number of poles, it was assumed that the same current was supplied to an entire quadrant using a differential driving mode. An individualized current supply, although complicating the control system, might be more advantageous and could well change the design configuration.



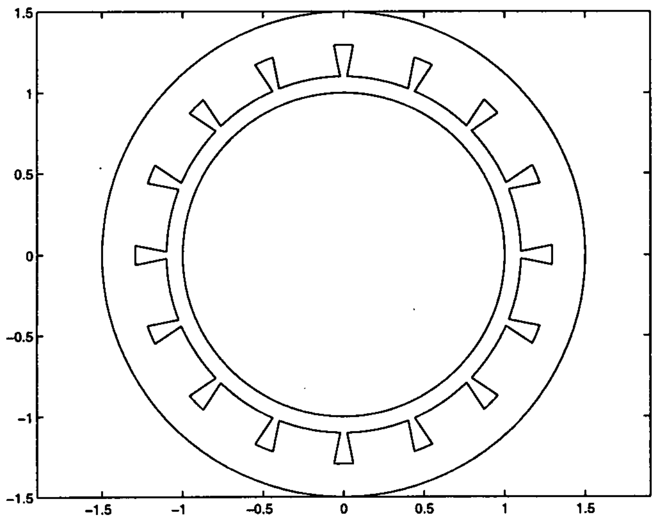
p/t	2
sl/p	0.64046
r2/rl	1.1
r3/rl	1.1003
r4/rl	1.3636
r5/rl	1.5
Number of poles	8
Width of the bearing	0.5 m
Bias current	5.7105 A
Nominal gap	10 mil
Number of turns per pole	57
Temperature of winding	146.3435 C
Static current (z)	7.778 A
Nominal inductance	2.1168 H
Saturation load	0.73419
Current Stiffness	0.12857
Position Stiffness (y)	-0.55262
Position Stiffness (z)	-1.5778

Figure 2: Optimally designed bearing with 8 poles

Three optimized configurations are shown in Figs. 2-4. Some of the key parameters that correspond to each of them are also shown in the adjoining tables. It should be mentioned that the parameter values are all shown in non-dimensional form in the figures and tables. Hence, a casual look at the figures might be misleading; for instance, the slot widths might seem very small, although in fact they are not. Note that the load capacity, stiffness and other similar quantities have also been scaled by the static load and are non-dimensional.

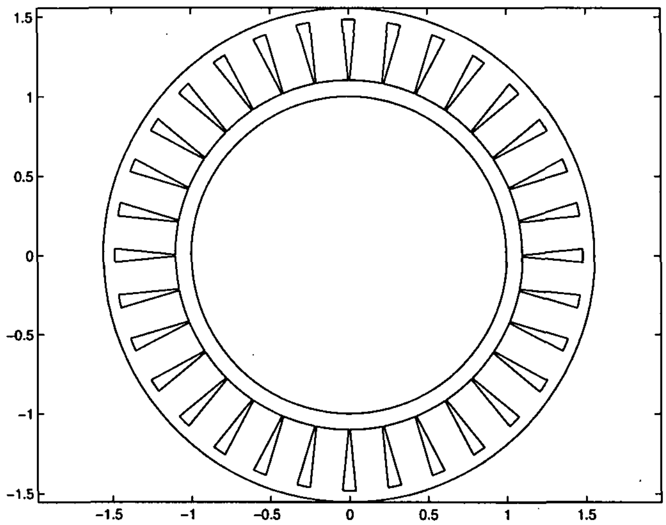
Although it is especially difficult to get optimal solutions for larger values of N_p, such solutions may lead to larger load capacities and hence may be desirable. Note that the last case presented ($N_p = 32$) is not really feasible since the temperature of the insulation exceeds the stated maximum. Still, it is presented as a possible starting point for an improved design iteration.

The optimization process was highly sensitive to initial guesses and the results presented here are a result of an extensive numerical investigation. Still, it should also be noted that the problem has a number of local minima and it is almost impossible to guarantee that the solution obtained is a true global minimum; in other words, it may be possible to get an even higher load capacity starting with the final converged solution and using some engineering judgement along with more tinkering with the optimization algorithms.



p/t	2
sl/p	0.11111
r2/rl	1.1
r3/rl	1.1003
r4/rl	1.2947
r5/rl	1.5
Number of poles	16
Width of the bearing	0.5 m
Bias current	4 A
Nominal gap	10 mil
Number of turns per pole	81
Temperature of winding	140.7606 C
Static current (z)	1.946 A
Nominal inductance	3.1556 H
Saturation load	2.0554
Current Stiffness	0.51386
Position Stiffness (y)	-1.9967
Position Stiffness (z)	-2.4693

Figure 3: Optimally designed bearing with 16 poles



p/t	0.53948
sl/p	0.04593
r2/rl	1.1
r3/rl	1.1002
r4/rl	1.4831
r5/rl	1.5561
Number of poles	32
Width of the bearing	0.5 m
Bias current	3.9439 A
Nominal gap	9.7792 mil
Number of turns per pole	80
Temperature of winding	166.662 C
Static current (z)	1.7951 A
Nominal inductance	1.6719 H
Saturation load	2.1971
Current Stiffness	0.55708
Position Stiffness (y)	-2.1761
Position Stiffness (z)	-2.6269

Figure 4: Optimally designed bearing with 32 poles

The following observations result from the numerical investigations.

- In general, as N_p, the number of poles increases, it gets more difficult to get a feasible solution that satisfies all the constraints.
- The nominal gap tends to the lower limit value for all of the investigated cases.
- In general, as the load capacity goes up, so does the current stiffness, K_i.
- As the load capacity goes up, the negative position stiffnesses (which are destabilizing) also go up. It may be noted that since we have a correction current to account for the static load in the z direction, $|K_{pzz}| > |K_{pyy}|$.
- The optimal slot width tends to a small quantity in general. Moreover, as the number of poles is increased, this tendency gets intensified; i.e., the slot widths (in relation to the pole width) get to be smaller with larger number of poles.

In the above analysis, the width of the bearing was held to a constant value since the thermal as well as the electromagnetic model are simplified and limited to two dimensions. A more detailed design procedure should include the width as a design variable. In addition, a lumped parameter steady-state thermal model such as the one used in the current study has its limitations. For example, it will not predict the temperature variations in the iron core, which can be substantial. A detailed thermal model on the other hand can add enormous computational cost and complexity to the problem since the optimization process typically involves hundreds of iterations.

The current and position stiffnesses were not a part of the objective function or the constraints in the current study. In our physical system, the load capacity (and not the stiffnesses) was a critical issue and hence was used as the objective function. However, for other situations, they may well be required to be in certain ranges in which case they would have to be integrated either into the objective function or the constraints. Clearly, whether the solution is optimal depends upon our specification on what we want to be a maximum or a minimum. In any case, it should be clear that a procedure such as the one outlined here would be able to provide an optimal solution when design constraints and objectives other than the ones considered here become important.

CONCLUSION

This paper dealt with the geometric design of radial magnetic bearings. General expressions were derived for the performance variables of magnetic bearings of standard geometry. Several key independent parameters were identified as design variables. An optimization algorithm was used to arrive at a solution that maximized the load capacity subject to thermal and other mechanical constraints.

The problem is a difficult one with several local minima and no clear global minimum. It is by no means clear that what we presented in this paper are the globally best solutions. Since the constraints were quite stringent, even a feasible solution is not very trivial. It should be borne in mind that it is also possible to pose a problem that has no mathematical optimum and whether an optimal or even a feasible solution exists depends strongly on the constraints that have been specified. In fact the current study was motivated by the fact that the techniques in the literature were not adequate to design the bearings for our objective. What this paper does present is a general procedure that makes it possible to use optimization techniques to obtain the best possible solutions when even a feasible solution would not be obtained by conventional techniques.

Improved thermal (as well as possibly electromagnetic) models when integrated into the optimal design process outlined in this paper would considerably enhance the confidence in the results presented here. An experimental corroboration for the thermal model is being planned for the future and will be used to validate and correct the analytical model used here.

Finally, as mentioned earlier, the design objective and the constraints may well be very different in other situations. In such a case, it is expected that a procedure such as the one outlined here would be able to provide an optimal solution (which could be quite different from the ones presented here).

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