OPTIMIZATION OF THE CONICAL ANGLE OF CONE-SHAPED ACTIVE MAGNETIC BEARINGS

Jussi K. Lähteenmäki,[1] Erkki J. Lantto[1]

ABSTRACT

The conical angle of cone-shaped active magnetic bearings is studied. A model of a conical bearing system and load forces is made. A minimax decision approach is used with the model. The conical angle is optimized so that the control forces required are as small as possible for a given load combination. Also, the conical angle is optimized so that the size of the conical bearing can be made as small as possible. The effect of control strategy on optimization is discussed. As a result, formulae for an optimal conical angle are given. The results also show the effect of using an angle other than the optimum angle, e.g., for manufacturing reasons.

INTRODUCTION

A common implementation of active magnetic bearing (AMB) technique is to use one or two axial and two radial AMBs. Another possibility is to use two conical AMBs. Figure 1 shows that the conical AMBs resemble the radial bearings. The difference is in the shape of the iron core in the stator and in the shape of the iron lamination on the shaft. The conical angle α divides the control force into an axial and a radial component. No axial AMBs are then needed. This reduces the number of components and the space required. It may also become possible to shorten the shaft, which is important, especially in high-speed machines.

Inoue and Shimomura (1989) implemented a conical AMB system on a magnetic momentum wheel for attitude control in a satellite. The value of the conical angle was not reported. Mohamed and Emad (1992) discussed conical AMB approach and a conical angle of 10° was reported. Fukata and Kouya (1992) reported a conical angle of 15° for an experimental rotor construction. Jeong, Kim and Lee (1994) developed a robot joint, using a conical AMB suspension. The conical angle of the AMBs were also 15°. Carabelli et al. (1995) designed a high-speed spindle to simulate a compressor unit in a high performance aircraft. Conical AMBs were used and a conical angle of 15° was reported.

[1]Helsinki University of Technology, P.O. Box 3000, 02015 HUT, Finland.

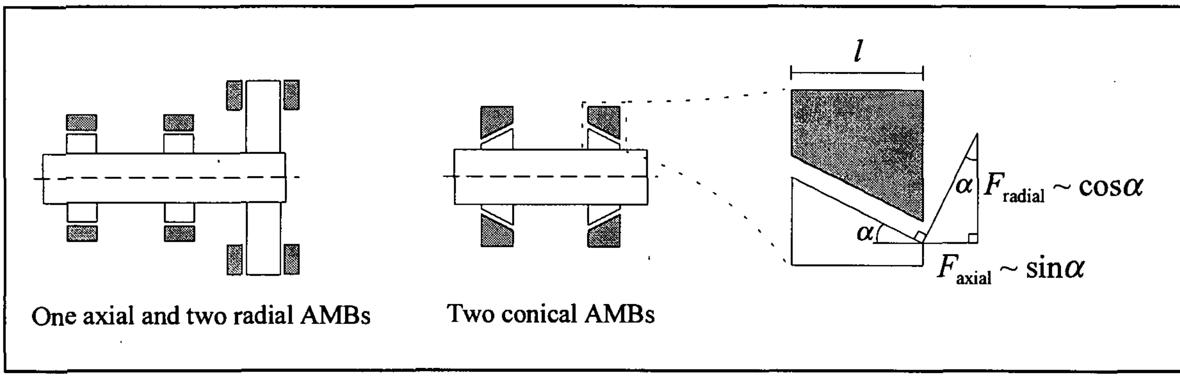

Figure 1. Different implementations of AMB technique and control force components of an conical AMB.

In literature found, the choice of the value for the conical angle was not explained. In this paper, a model to determine the conical angle is developed. The model is constructed for a system of two conical AMBs having four electromagnets each. The optimum value of the conical angle is determined as a function of load forces. The aim of the optimization is either to minimize the control force required or to minimize the size of the electromagnets. Minimax decision approach (worst-case scenario) is used in the optimization.

MODEL FOR CONICAL AMB SYSTEM

Figure 1 showed how a rotating shaft having five degrees of freedom is supported by a conical AMB at both ends of the shaft. In this study, we examine an AMB of eight poles that form four electromagnets. The poles are symmetrically spaced and on the same plane perpendicular to the shaft (Figure 2). The opposing electromagnets pull the shaft in opposite directions and the total force affecting the shaft is the sum of the two forces.

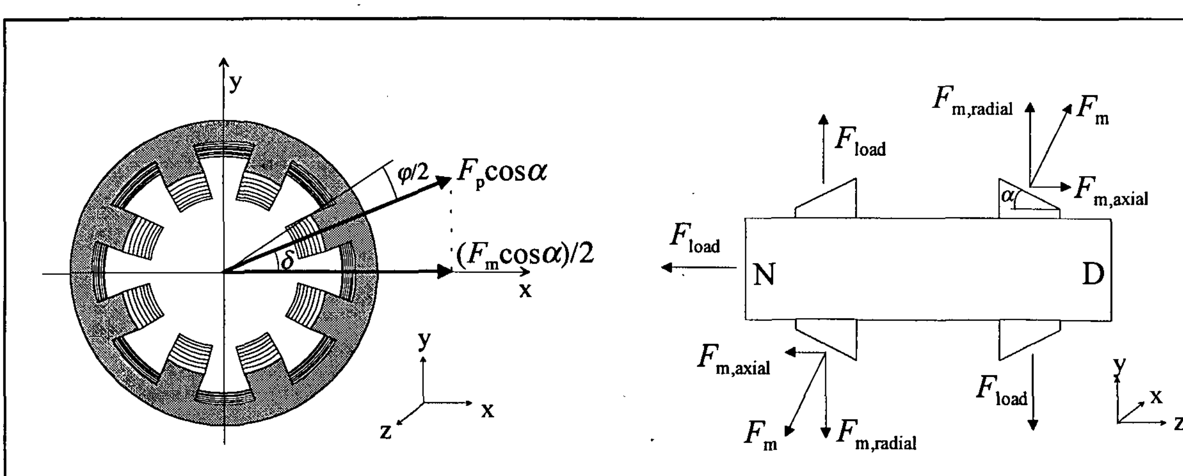

Figure 2. AMB model: geometry, control and load forces.

Each pole of the conical AMB affects the shaft with its pulling force. It is assumed that the magnetic flux is perpendicular to the air gap surface and the flux density is constant at the air gap. Thus, a vector of magnetic pole force $\Delta \boldsymbol{F}_p$ per differential area $\Delta \boldsymbol{A}_a$ is perpendicular to the air gap and its magnitude is constant. The total radial force of the pole reduced to its symmetry axis is proportional to $2\sin(\varphi/2)$, while the total axial force is reduced to the z-axis and is proportional to the pole angle φ. In most constructions of conical AMB, the value of $(\varphi/2)$ is small and the sine term can be simplified: $2\sin(\varphi/2)\approx\varphi$. Thus, the division of control force into axial and radial components depends only on the conical angle α:

$$F_{p,radial}=F_p\cos\alpha \qquad F_{p,axial}=F_p\sin\alpha \tag{1}$$

In one electromagnet, the magnitudes of F_p at both poles are considered to be the same, i.e., leakage fluxes are omitted and shaft/rotor is considered to be symmetrically positioned. For an electromagnet we get:

$$F_{m,radial}=2k_gF_p\cos\alpha \qquad F_{m,axial}=2F_p\sin\alpha \tag{2}$$

where k_g is a geometric factor. The poles of a conical AMB are evenly spaced and in the case of an eight pole AMB, the value of the geometric factor is (Figure 2):

$$k_g=\cos\delta=\cos(2\pi/16)\approx 0.924 \tag{3}$$

WORST-CASE COMBINATION OF LOAD FORCES

Active magnetic bearing systems are designed to handle certain static and dynamic loads. Static loads are the mass of the rotor together with loads of an electric motor or connected load machinery. Dynamic loads are, for example, base vibrations, accelerations and dynamic disturbance forces from the electric motor and load machinery. The direction and magnitude of the total load force in an axial and in a radial direction may vary with time.

In a worst-case situation, the amount of control force required from a single electromagnet is greatest, and using that value the size of the electromagnets and their poles can be determined. For the calculation let us choose one of the electromagnets in the drive end to have the greatest load.

The radial load at the drive end calls for a balancing control force. Axial load force pulls the shaft towards the non-drive end and increases the need of the control force in the drive end. The radial control of the non-drive end generates an axial component that is in the same direction as the axial load. This component further increases the control capacity needed at the drive end (Figure 2).

The model uses the Cartesian coordinate system. The z-axis is parallel to the shaft and it is the axis of axial forces. The xy-plane is the plane of radial forces and it is fitted so that a symmetry axis of each electromagnet lies in an axis of xy-plane.

WORST-CASE COMBINATION OF THE NON-DRIVE END

The axial force component due to radial control in the non-drive end has the same direction as the axial load force and thus it is harmful. In a worst-case situation the axial component is at its maximum. The value of the component depends on the magnitude and direction of the radial control. In Figure 3a, we see the radial control force that has a magnitude F_{radial} and a direction determined by angle β_N. Due to symmetry of the AMB, it is enough to restrict the value of angle β_N between 0° and 45°. From the figure 3a we get the following equation:

$$F_{m,radial,y} = F_{radial} \sin \beta_N \qquad F_{m,radial,x} = F_{radial} \cos \beta_N \qquad (4)$$

Using equations 2 and 4, the axial component is determined:

$$F_{axial,N} = \frac{F_{radial}(\sin\beta_N + \cos\beta_N)}{k_g} \tan\alpha \qquad (5)$$

This component has its maximum value when the direction of the radial control force (and the load force) is symmetrically between two electromagnets, i.e. the angle β_N is 45°. The value of the axial component in the worst-case situation is

$$F_{axial,N} = \frac{\sqrt{2}F_{radial}}{k_g} \tan\alpha \qquad (6)$$

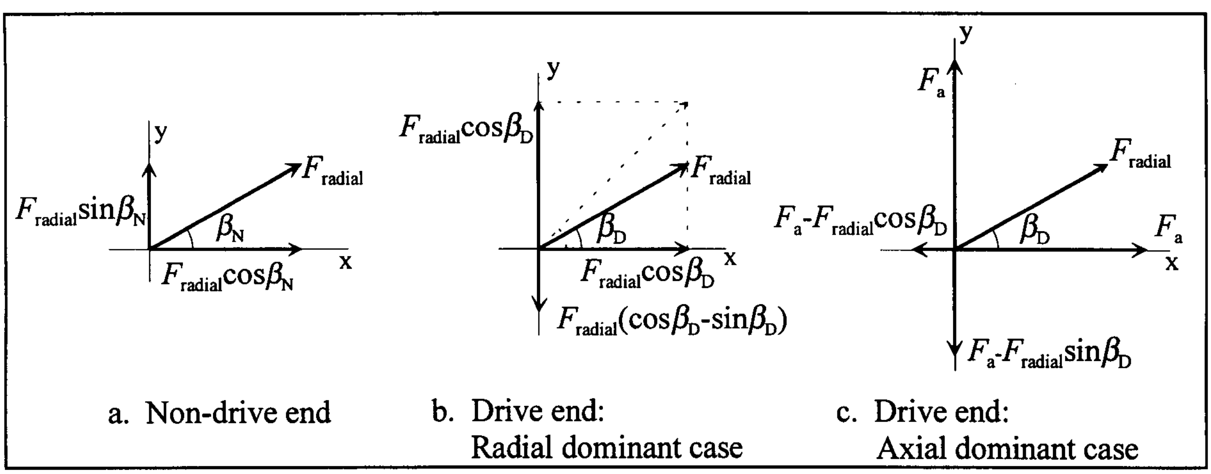

Figure 3. Control forces of the conical AMBs in the non-drive and drive end. Forces in the drive end are divided into two possible cases, radial or axial force dominant case.

WORST-CASE COMBINATION OF THE DRIVE END

In addition to a radial control the drive end AMB has to counter the sum of the axial load force and the axial force caused by the non-drive end radial control. The forces are defined in the same way as with the non-drive end, but this time the situation falls into two possible cases, radial force dominant and axial force dominant (Figures 3b and 3c). An electromagnet that lies on the positive x-axis is chosen to handle the maximum force.

In the radial force dominant case, the maximum force required from the electromagnet is defined by radial load (Figure 3b). An adequate axial force is adjusted with the electromagnets on the y-axis. Because the magnet on the positive x-axis was chosen to have the maximum load, the amount of axial control force this model can produce is restricted (this situation is drawn in Figure 3b):

$$F_{\text{axial}} \leq F_{\text{radial}}(3\cos\beta_{\text{D}} - \sin\beta_{\text{D}})\frac{\tan\alpha}{\text{k}_{\text{g}}} - \frac{\sqrt{2}F_{\text{radial}}\tan\alpha}{\text{k}_{\text{g}}} \tag{7}$$

The first term on the right side is the maximum axial control force that the radial force dominant case can produce in the drive end. The second term is the axial component of the non-drive end radial control (Equation 6). The maximum load in the radial force dominant case for one pole is:

$$F_{\text{p}} = \frac{F_{\text{radial}}\cos\beta_{\text{D}}}{2\text{k}_{\text{g}}\cos\alpha} \tag{8}$$

In the worst-case situation the direction of the radial control force (and the load force) coincides with the x-axis, i.e., the angle β_{D} is 0°. Equations 7 and 8 can now be rewritten as:

$$F_{\text{p}} = \frac{F_{\text{radial}}}{2\text{k}_{\text{g}}\cos\alpha}, \qquad \tan\alpha \geq \frac{F_{\text{axial}}}{F_{\text{radial}}}\frac{\text{k}_{\text{g}}}{(3-\sqrt{2})} \tag{9}$$

In the axial force dominant case the maximum force required from the electromagnet is defined by the axial and radial load (Figure 3c). Both electromagnets on the positive x-axis and y-axis have the maximum load and the other electromagnets adjust the magnitude and direction of the radial control force. This model sets a minimum requirement for the axial control force (compare with Equation 7):

$$F_{\text{axial}} \geq F_{\text{radial}}(3\cos\beta_{\text{D}} - \sin\beta_{\text{D}})\frac{\tan\alpha}{\text{k}_{\text{g}}} - \frac{\sqrt{2}F_{\text{radial}}\tan\alpha}{\text{k}_{\text{g}}} \tag{10}$$

The maximum load in the axial force dominant case for one pole is:

$$F_{\text{p}} = \frac{F_{\text{radial}}(\sqrt{2} + \sin\beta_{\text{D}} + \cos\beta_{\text{D}})}{8\text{k}_{\text{g}}\cos\alpha} + \frac{F_{\text{axial}}}{8\sin\alpha} \tag{11}$$

In the worst-case situation, the direction of the radial control force is symmetrically between two electromagnets, i.e. the angle β_D is 45°. Equations 10 and 11 can now be rewritten as:

$$F_p = \frac{2\sqrt{2}F_{radial}}{8k_g\cos\alpha} + \frac{F_{axial}}{8\sin\alpha}, \qquad \tan\alpha \leq \infty \tag{12}$$

OPTIMIZATION OF THE CONICAL ANGLE

Equations 9 and 12 tell us how much control force each pole must be capable of exerting. Using these equations, we can solve the optimum angle for given axial and radial loads F_{axial} and F_{radial} so that the required control force is at its minimum. The required load ratio is then

$$\gamma = \frac{F_{axial}}{F_{radial}} \tag{13}$$

The result is derived using minimax decision approach. For each load ratio γ the conical angle α is chosen so that the greatest possible requirement of control force Fp is minimized as the load angle β_D varies. In the worst-case situation β_D is either 0° or 45°. This setup may be considered as a two-player game. Player B controls the value of β_D and player A controls the value of α. The outcome of the game, in this case, is the control force requirement. Player B tries to maximize the result and Player A tries to minimize the result. If player A uses minimax decision criteria, he will calculate a worst possible outcome for each value of α. Then he selects the value of α that has the minimal outcome calculated.

The optimum angle is always the smaller of the two angles obtained from:

$$\alpha = \operatorname{atan}(\gamma\frac{k_g}{4-2\sqrt{2}}) \qquad\qquad \alpha = \operatorname{atan}\sqrt[3]{\gamma\frac{k_g}{2\sqrt{2}}} \tag{14}$$

The first equation for α corresponds the intersection of Fp curves given by equations 9 and 12. The second equation corresponds the minimum value of Fp given by equation 12. The choice of the smaller angle is based on the properties of the equations 9 and 12.

Figure 4 shows the optimum conical angle for a minimum pole force as a function of the load ratio. The nodge in the curve shows the place, where the control strategy changes from the radial dominant control to the axial dominant control. The optimal value of the conical angle lies between 0° and 90° for all γ.

For a reference, an angle for maximum axial and radial control force is plotted. It is determined by the maximum radial force in any direction and the maximum axial force that the conical AMB can produce, but not at the same time. This angle is achieved from the equation

$$\alpha_{ref} = \operatorname{atan}(\gamma\frac{k_g}{4}) \tag{15}$$

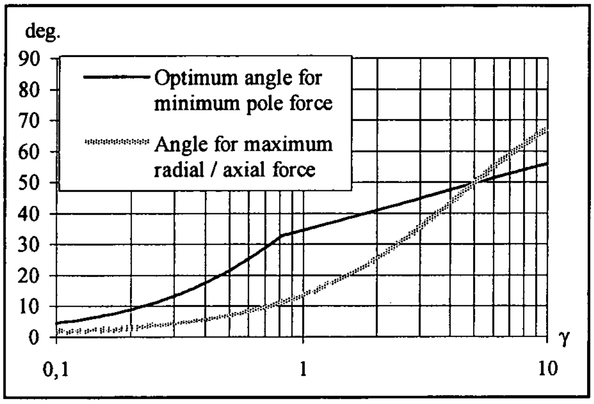

Figure 4. Optimum conical angle for a minimum pole force.

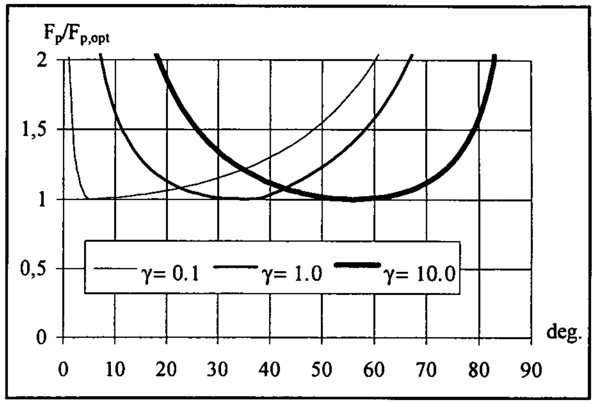

Figure 5. Sensitivity of pole force required.

Figure 5 shows the relative pole force required, when the conical angle is different from the optimum. The change is plotted for three different load ratios. It can be seen that the pole force required is not too sensitive to the conical angle. The result indicates that other than optimum angle can be used without heavy penalties. Other angle may then be chosen for manufacturing reasons, for example.

To get these equations we have assumed that the conical angle does not affect the force the poles of certain size can produce. In the worst-case situation this may not be true, however. If the magnetic circuit saturates in the worst-case situation, the maximum magnet flux density at the air gap decreases when the conical angle is increased. The force one pole of certain size can produce becomes approximately inversely proportional to the cosine of the conical angle. It is then reasonable to optimize the conical angle so that the size of the magnet, i.e., the area of the magnet core perpendicular to magnet flux, can be made as small as possible. Thus equations 9 and 12 are rewritten to yield the relative area required, respectively:

$$A_p \propto \frac{F_{\mathrm{radial}}}{2\mathrm{k_g}\cos^2\alpha}, \qquad \tan\alpha \ge \frac{F_{\mathrm{axial}}}{F_{\mathrm{radial}}}\frac{\mathrm{k_g}}{(3-\sqrt{2})} \tag{16}$$

$$A_p \propto \frac{2\sqrt{2}F_{\mathrm{radial}}}{8\mathrm{k_g}\cos^2\alpha} + \frac{F_{\mathrm{axial}}}{8\sin\alpha\cos\alpha}, \qquad \tan\alpha \le \infty \tag{17}$$

The optimum angle, optimized for the pole size needed, is always smaller of the two angles obtained from:

$$\alpha = \mathrm{atan}(\gamma\frac{\mathrm{k_g}}{4-2\sqrt{2}}) \qquad \tan^3\alpha + \frac{\mathrm{k_g}\gamma}{4\sqrt{2}}\tan^2\alpha - \frac{\mathrm{k_g}\gamma}{4\sqrt{2}} = 0 \tag{18}$$

Figure 6 shows the optimum conical angle for a minimum pole area as a function of the load ratio. In this case, the optimal value of the angle lies between 0° and 45° for all γ.

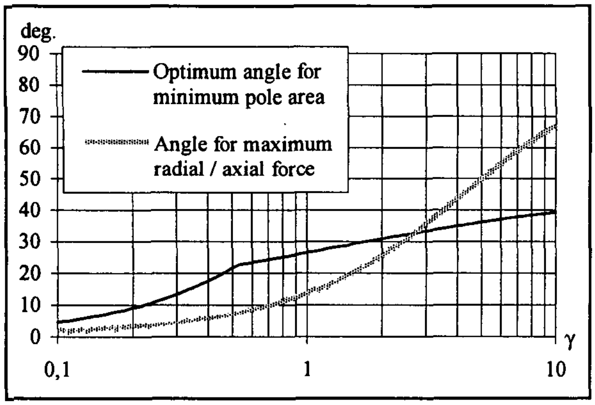

Figure 6. Optimum conical angle for a minimum pole area.

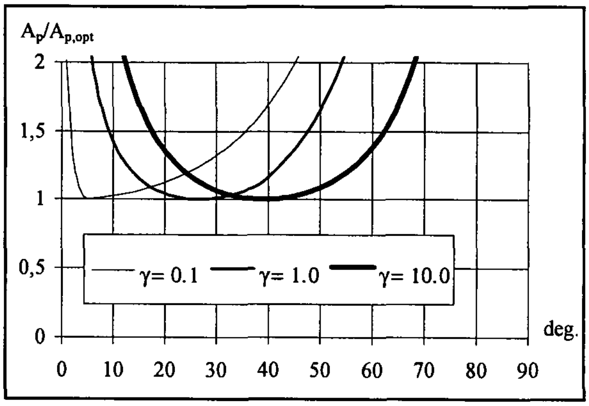

Figure 7. Sensitivity of pole area required.

As we see from Figure 7, the required size of an electromagnet increases, if the angle is not optimal, but the increase is only gradual if the angle is near the optimum. This means that small deviations from the optimum angle are acceptable.

FEASIBILITY OF CONICAL BEARING DESIGN

The comparison between conical bearing design and axial/radial bearing design is not a straightforward problem. While a bundle of attributes must be considered, some thoughts are presented, using the model above.

One criterion for the comparison can be the size of the conical bearing compared to the size of the corresponding radial bearing. Both bearings must produce the same maximum radial control force. If the size of the conical bearing becomes much larger than that of the radial bearing, it is better to use radial and axial bearings.

The comparison is made using equations 16, 17 and 18. For the radial bearing, the conical angle α is kept at 0°. For the conical bearing, the optimal angle is chosen and the size is calculated accordingly. Figure 8 shows that for higher values of load ratio the feasibility of the conical bearing design becomes questionable.

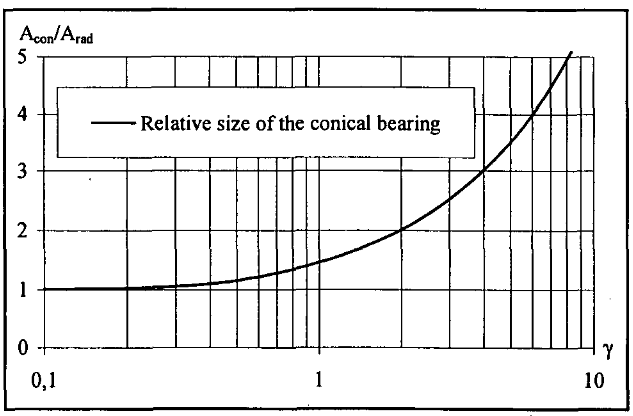

Figure 8. Comparison of the required size of the conical bearing to the radial bearing.

WORST-CASE MODEL AND CONTROL OF THE CONICAL AMB

In the worst-case model used, the conical AMBs were ideally controlled. No unnecessary control forces were developed to hold the rotating shaft in its proper place.

Conical AMBs are controlled with linearized control techniques (Mohamed and Emad, 1992; Fukata and Kouya, 1992; Jeong, Kim and Lee, 1994). Linearized control uses bias currents that generate unnecessary force components. In the worst-case situation, however, the electromagnets most heavily loaded begin to saturate and if the bias current is properly chosen the unnecessary force components will vanish.

The control system should keep the suspension stable at all the possible load conditions, including the worst-case situation. Finding such a controller is not trivial, because the bearing parameters change a lot as the load combinations change. In addition, there will be considerable cross-connection between the axial control and radial force (and vice versa) in the highly loaded conditions.

In this paper the loads were assumed to be maximum instantaneous forces and no difference was made whether they are dynamic or static. If the dynamic force is below the force bandwidth (the frequency at where the bearing is able to produce the maximum force), there should not be big difference whether it is static or dynamic, considering the optimum conical angle. Note that the load ratio γ is computed based on real bearing forces. For example high speed "unbalance force" (if the unbalance is interpreted as external force) is not considered as radial load, because the bearings are not producing this force (the rotor is allowed to vibrate).

CONCLUSIONS

A model of a conical bearing suspension and load forces is made for a five degrees of freedom rotating shaft. The model includes two conical active magnetic bearings of eight poles and four electromagnets. Minimax decision approach is used in order to get the combination of load forces most difficult to control.

The value of the conical angle is optimized to minimize the control force required. Also, the conical angle is optimized to minimize the pole size required. The optimum angles are calculated as a function of load force ratio, axial load per radial load. The results indicate that the pole force and the pole size required are insensitive to small deviations from the optimum angles. The insensitivity can be utilized if there exists some other preferences to the right value of the conical angle.

As far as the size of the electromagnets are concerned, the conical bearing design is worth considering under small load force ratios.

The optimum angles were calculated using an ideal bearing control model. This is no restriction, however, because an ordinary linear control with bias currents behaves in an optimal way in the worst-case situation.

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