High Efficiency Linear Power Amplifier for Active Magnetic Bearings

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Abstract: To improve the overall efficiency of a transconductance amplifier loaded by the high inductance and low resistance typical of the magnetic bearing coils, a modified class G amplifier has been developed. A simple theoretical characterization has been used to provide some diagrams to optimize the power efficiency as a function of a non constant load such as that due to a rotor unbalance. Experimental results are reported showing a more than satisfactory agreement with the developed theory.

Introduction

In comparison with switching amplifiers, linear power amplifiers may still represent a suitable solution for very high speed and low weight rotating machinery on active magnetic bearings (AMB). Moreover, their low emission of electromagnetic noise may be very useful for the use in connection with co-located inductive sensors.

To improve the overall efficiency of a transconductance amplifier loaded by the high inductance and low resistance typical of the magnetic bearing coils, a modified class G amplifier has been developed. A couple of power Darlington transistors is connected on the same load but supplied with two different voltages: the higher to deal with the inductive transient and the lower with the resistive steady state. A third Darlington, connected to the ground, allows for the negative output voltage swing necessary for a current decay with increased efficiency. Particular attention has been paid to assure glitch free commutation between the two transistor pairs.

A theoretical characterization has been developed to assist the design of the modified class G power amplifier according to the specification required by the active magnetic bearings working condition.

Theoretical characterization

Given the standard amplifier connections for each axis of a magnetic bearing shown in figure 1, a so-called class A AMB actuator arrangement [1] is adopted: both amplifiers are always activated simultaneously. Its main advantage is the physical linearization of the current-force action, its major drawback the need to use high bias currents I_b (usually half of the maximum current) in order to eliminate the anisotropy typical of a class B actuator [2] (with only one amplifier activated at a time). As a class A AMB actuator arrangement is adopted throughout the present paper, only one power amplifier will be considered in the following.

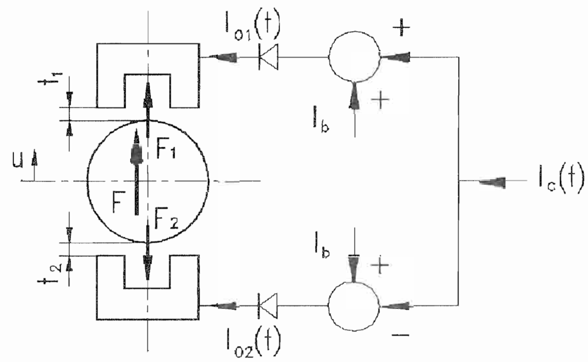

Figure 1: Schematic of an active magnetic bearing.

When the self-centered rotor behavior is desired, the synchronous unbalance forces of the rotor must be counteracted by the magnetic bearing thrust produced by a synchronous sinusoidal control current in the coils

$$I_c(t) = I_c \sin(\omega t) \tag{1}$$

where ω is the angular speed and the control current I_c is proportional to the unbalance force $F_{unbal} = m\varepsilon\omega^2$ with m the rotor mass and ε the eccentricity.

Using the usual linearized expression for the actuating control force, the following expression is obtained

$$I_c = \frac{|F_{unbal}|}{4\dfrac{K}{t_0^2}I_b} = \frac{mt_0^2}{4KI_b}\varepsilon\omega^2 \tag{2}$$

where t_0 is the nominal air gap and K is the electromagnetic constant that takes into account all the constructive parameters. The overall current is then

$$I_o(t) = I_b + I_c \sin(\omega t) \tag{3}$$

In the working frequency range the load, i.e. each electromagnet, may be modeled as a LR series, $Z = R + j\omega L$, and the voltage across it is

$$V_o(t) = |Z| I_c \sin(\omega t + \phi(Z)) + R I_b \tag{4}$$

where

$$\begin{aligned} |Z| &= \sqrt{R^2 + (\omega L)^2} \\ \phi(Z) &= \tan^{-1}(\tfrac{\omega L}{R}) \end{aligned} \tag{5}$$

It may be assumed that load is basically inductive, i.e. $\omega L \gg R$ and thus

$$\phi(Z) \cong 90^\circ \ , \quad |Z| \cong \omega L \ , \quad R I_b \ll |Z| I_c \tag{6}$$

with $I_b \geq \max(I_c)$. The current and voltage signals are then

$$\begin{aligned} I_o(t) &= I_b + I_c \sin(\omega t) \\ V_o(t) &= \omega L \, I_c \cos(\omega t) \end{aligned} \tag{7}$$

These simplifications do not reduce the general applicability because in a well constructed system the resistive part of the impedance is less than the reactive one, up to very low frequency.

Class A, class G and modified class G power amplifiers

The schematic of a conventional class A power amplifier is reported in figure 2a). As an inductance cannot dissipates any power (it can only store energy) when an almost purely inductive load as the coils of magnetic bearings has to be driven, the situation in which the power flows back from the load to the amplifier has to be taken into account.

The istantaneous output power P_o on the inductive load and the supplied one P_s are respectively

$$P_o = V_o I_o, \ P_s = V_s I_s \cong V_s I_o \tag{8}$$

in the assumption of negligible power absorbed by the driving electronic circuits, then the dissipated power is

$$P_d = P_s - P_o \cong (V_s - V_o) I_o \geq 0 \tag{9}$$

and the drop-out term $(V_s - V_o)$ that causes the thermal dissipation is univocally defined by the design goals (V_s is the upper limit of the dynamic of the power amplifier output voltage as imposed by the maximum specified force slew rate).

The output power $P_o = V_o I_o = L \frac{dI_o}{dt} I_o$ actually flows from the amplifier for any rise of I_o while it returns into the amplifier for any current reduction. Due to the intrinsic unidirectionality of I_o, for any amplifier configuration there is a power flux from the supply to the amplifier. For a class A power amplifier with an inductive load, when the current I_o decreases, i.e. the

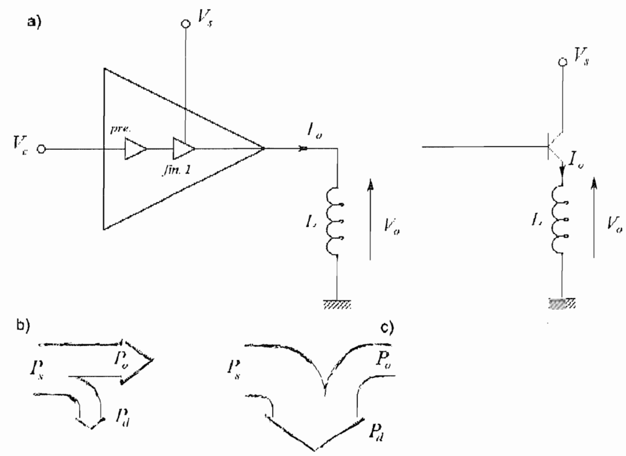

Figure 2: Class A power amplifier. a) Scheme with purely inductive load; b) I_o increasing, V_o positive: the supply power P_s is partly given to the load and partly dissipated as $P_d = P_s - P_o$; c) I_o decreasing, V_o negative: the output power P_o is transferred from the load to the amplifier while the supply power P_s is still absorbed from the supply: the amplifier then dissipates $P_d = P_s + P_o$.

load voltage reverses its polarity, the amplifier dissipates both the power P_o returned from the load and the power $P_s = V_s I_o$ absorbed from the supply.

In order to reduce the dissipated power P_d on amplifiers loaded by high inductance and low resistance typical of magnetic bearing coils, a class G power amplifier can be used [3, 4, 5]. In this amplifier class a couple of power transistors is connected on the same load but supplied with different voltages: the higher (V_s) to deal with the inductive transient and the lower (αV_s, $0 < \alpha < 1$) with the resistive steady state. The schematic of a class G power amplifier is reported in figure 3a) and its behavior can be so described: until $V_o < \alpha V_s$ only the final stage *fin.1* is on and the final stage *fin.2* is off, as soon as $\alpha V_s < V_o < V_s$ the *fin.2* and *fin.1* are respectively switched on and off.

Note that αV_s is a supply source completely independent from V_s; its representation as a fraction of V_s is only due to computational simplification. It can be considered that the class G power amplifier is supplied by an equivalent voltage V_{eq} function of V_o

$$V_{eq} = \begin{cases} \alpha V_s & \text{if } V_o < \alpha V_s \\ V_s & \text{if } \alpha V_s < V_o < V_s \end{cases} \tag{10}$$

then

$$P_d = (V_s - V_o) I_o = \begin{cases} (\alpha V_s - V_o) I_o & \text{if } V_o < \alpha V_s \\ (V_s - V_o) I_o & \text{if } \alpha V_s < V_o < V_s \end{cases} \tag{11}$$

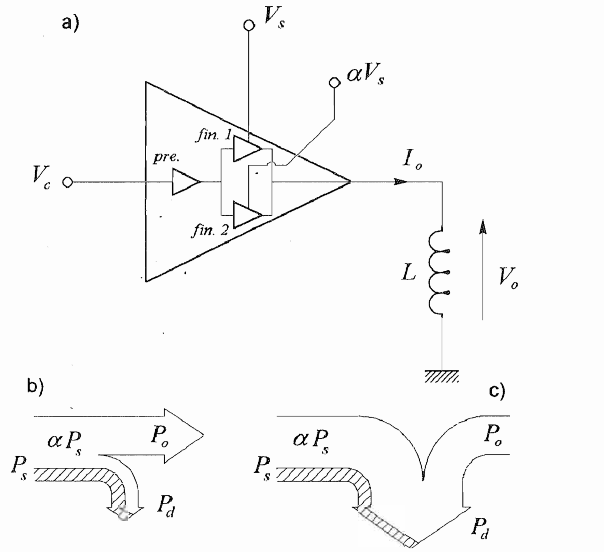

Figure 3: Class G power amplifier. a) Scheme with purely inductive load. b) I_o increasing, V_o positive: power P_o is delivered to the load; c) I_o decreasing, V_o negative: power P_o is transferred from the load to the amplifier. The dashed areas show the power saving of the class G with respect to the class A: case b) $P_d^A = P_s - P_o$ and $P_d^G = \alpha P_s - P_o$, case c) $P_d^A = P_s + P_o$ and $P_d^G = \alpha P_s + P_o$.

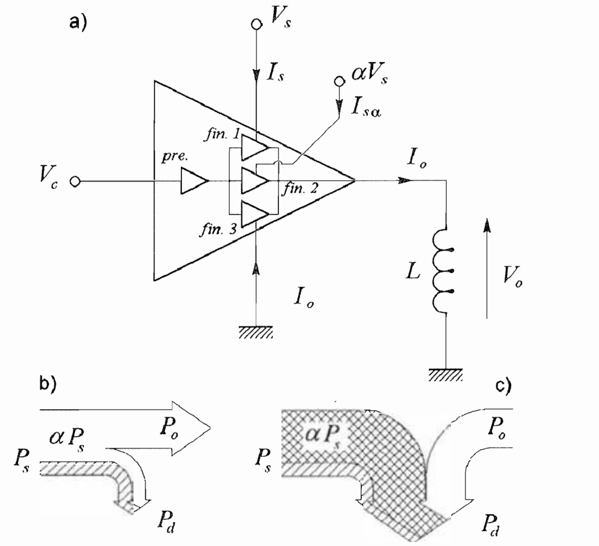

Figure 4: Modified class G power amplifier. a) Scheme with purely inductive load. b) I_o increasing, V_o positive: power P_o is delivered to the load; c) I_o decreasing, V_o negative: power P_o is transferred from the load to the amplifier. The different dashed areas show the power saving of the modified class G with respect to the standard class G and the class A: case b) $P_d^A = P_s - P_o$ and $P_d^G = P_d^{Gmod} = \alpha P_s - P_o$, case c) $P_d^A = P_s + P_o$, $P_d^G = \alpha P_s + P_o$ and $P_d^{Gmod} = P_o$.

In this way, for $V_o < \alpha V_s$, P_d results lower than that related to the situation without the second voltage supply

$$(\alpha V_s - V_o)I_o < (V_s - V_o)I_o \tag{12}$$

while for $\alpha V_s < V_o < V_s$ the P_d term is limited by the reduced value of drop-out voltage.

The reduction of the average dissipated power is reflected on the choice of an adequate value of the parameter α so that the reduced voltage supply αV_s becomes sufficient to cover the request of the most probable values of V_o. When the current I_o rises the absorbed power P_s can be equal to $V_s I_o$ or to $\alpha V_s I_o$ according to the value of V_o while it will be equal to $\alpha V_s I_o$ for any reduction of I_o.

To further improve the overall efficiency a modified class G amplifier has been developed: to the couple of existing power transistors a third one is added, connected to the ground (to the authors' knowledge, this method has never been used in power amplifier, the only found marginal reference is in [6]), that allows for the negative output voltage swing necessary for a current decay with increased efficiency. Particular attention has been devolved to assure glitch free commutation between the transistors. The schematic of a modified class G power amplifier is reported in figure 4a).

The third stage *fin.3* works only when the load gives back power to the amplifier. This stage is not powered and so it does not absorb any energy from the supply section; its only aim is to dissipate into heat the power given back from the load allowing the re-circulation of the current I_o. The instantaneous dissipated power is then

$$P_d = \begin{cases} (V_s - V_o)I_o & \text{if } \alpha V_s < V_o < V_s \\ (\alpha V_s - V_o)I_o & \text{if } 0 < V_o < \alpha V_s \\ -V_o I_o & \text{if } V_o < 0 \end{cases} \tag{13}$$

The progressive reduction of the dissipated power part is qualitatively evidenced by the dashed areas in figure 3 and figure 4 part b) and c) respectively. The reduction of the dissipated power is important in term of energy consumption, supply devices and heat sinks dimension with reduction of overall dimension of power amplifier and related cost. Moreover the lower the operating temperature, the higher the reliability.

Computation of the average dissipated power

To minimize the average dissipated power an adequate choice of the parameter α with respect to the signals V_o and I_o has to be done. The current (and the voltage) depends on the unbalance of the rotor as pointed out in equations (7) thus the choice of α depends on the allowed maximum degree of unbalance requested for the specific application. In the following, the average dissipated power will be analytically evaluated for a sinusoidal signal (an idealization of actual situations as defined by (3)) to evidentiate the way to minimize P_d and to give some useful hints for the design of the power amplifier.

Due to the unavoidable voltage drop on electronic devices the presence of commutation thresholds are offset from their ideal values. The commutations of the output towards one of the three final stages *fin.1*, *fin.2* and *fin.3* then occur in correspondence of values lower than $V_o = \alpha V_s$ and $V_o = 0$ and mainly related to the saturation voltage V_{sat} typical of the used power device. It is useful to express the commutation thresholds as function of V_s using a threshold for the ground equal to

$$-V_{sat} = -\beta_o V_s \tag{14}$$

and a threshold for the reduced voltage αV_s equal to

$$\alpha V_s - V_{sat} = (\alpha - \beta_o) V_s = \beta V_s \tag{15}$$

with $0 < \beta_o < 1$ and consequently $0 < \beta < 1$.

With the hypothesis of purely inductive load, the average power absorbed by the load is equal to zero. The average power dissipated by the amplifier is then equal to the average power supplied by the supply stages: $\langle P_d \rangle \equiv \langle P_s \rangle$. Note that these quantities are averaged in the time domain. As periodic signals have been assumed, the power P_s is equal to the value of the instantaneous supplied power averaged over a period T

$$\langle P_s \rangle = \frac{1}{T} \int_0^T P_s(t) dt \tag{16}$$

As the instantaneous supplied power is strongly influenced by the value of the voltage V_o, the comparison between V_o and the commutation thresholds gives information on the supply stage from which it is better to take the power.

For a class A power amplifier, having a single supply voltage, one has

$$\langle P_s \rangle = V_s I_b \tag{17}$$

In figure 5 the two cases that can occur during the service of a traditional class G power amplifier with a unique commutation threshold βV_s are reported. The analytical expression of the average supply power are respectively

$$\begin{array}{ll} \text{a)} & \langle P_s \rangle = V_s I_b \left[\alpha + (1-\alpha) \frac{1}{\pi} \cos^{-1} \left(\frac{\beta V_s}{|Z| I_c} \right) \right] \\ & \text{if } \beta V_s < |Z| I_c < V_s \\ \text{b)} & \langle P_s \rangle = \alpha V_s I_b \\ & \text{if } 0 \le |Z| I_c < \beta V_s \end{array} \tag{18}$$

In figure 6 the three cases that can occur during the service of the modified class G power amplifier with two commutation thresholds are reported: one at βV_s and the other at $-\beta_o V_s$. The analytical expressions of the average supply power are respectively

$$\begin{array}{ll} \text{a)} & \langle P_s \rangle = V_s I_b \left\{ \alpha \left[1 - \frac{1}{\pi} \cos^{-1} \left(\frac{\beta_o V_s}{|Z| I_c} \right) \right] \right. \\ & \left. \quad + (1-\alpha) \frac{1}{\pi} \cos^{-1} \left(\frac{\beta V_s}{|Z| I_c} \right) \right\} \\ & \text{if } |Z| I_c > \beta V_s > \beta_o V_s \\ \text{b)} & \langle P_s \rangle = \alpha V_s I_b \left[1 - \frac{1}{\pi} \cos^{-1} \left(\frac{\beta_o V_s}{|Z| I_c} \right) \right] \\ & \text{if } \beta_o V_s < |Z| I_c < \beta V_s \\ \text{c)} & \langle P_s \rangle = \alpha V_s I_b \\ & \text{if } |Z| I_c < \beta_o V_s \end{array} \tag{19}$$

Note that equation (19a)) is a decreasing function of β_o that reaches the minimum value for $\beta_o = 0$ (ideal situation).

In the following, a graphical representation of the average supply power $\langle P_s \rangle$ as a function of the current amplitude I_c for various α (for a fixed ω) is presented

$$\langle P_s \rangle = f(I_c; \alpha) \tag{20}$$

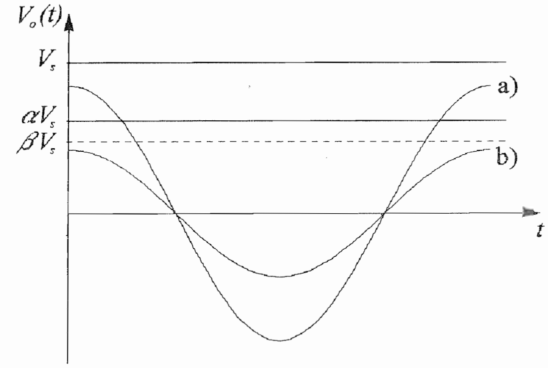

Figure 5: Output voltage V_o vs. time for the class G power amplifier; the commutation thresholds are qualitatively reported. Two cases are present: a) V_o crosses the threshold βV_s, b) V_o does not cross any threshold.

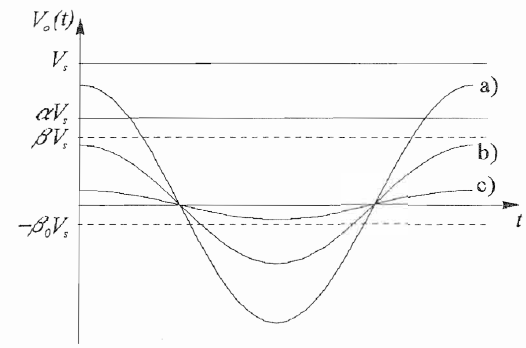

Figure 6: Output voltage V_o vs. time for the modified class G power amplifier; the commutation thresholds are qualitatively indicated. Three cases are shown: a) V_o crosses both thresholds βV_s and $-\beta_o V_s$, b) V_o crosses only the threshold $-\beta_o V_s$, c) V_o does not cross any threshold.

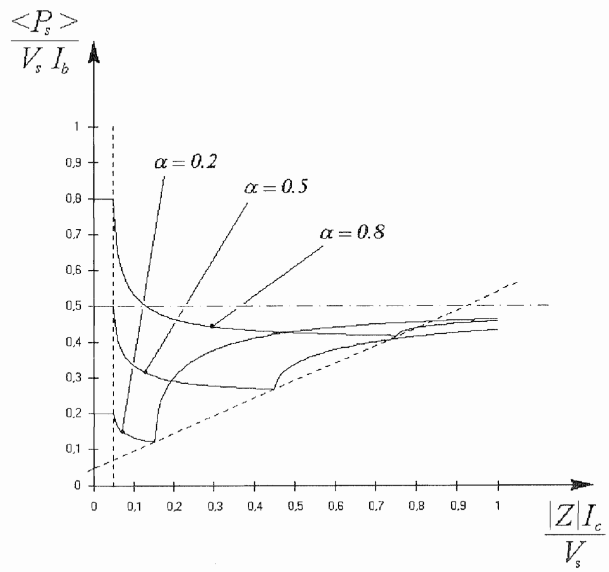

Figure 7: Normalized average supplied power as a function of normalized load current.

The resulting family of curves with varying α may be conveniently used to choose an appropriate reduced supply voltage for the unbalance force range of the particular application under study.

In order to generalize such a family of curves, they are normalized as

$$\frac{\langle P_s\rangle}{V_s I_b} = f\left(\frac{|Z|I_c}{V_s};\alpha\right) \tag{21}$$

where the left hand term approaches one as the amplifier moves toward the class A and the independent variable approaches one when the load voltage reaches saturation. The curves are obtained using equations (19) and are shown in figure 7. The following considerations may be pinpointed:

- the horizontal asymptotic behavior is given by

$$\lim_{\frac{|Z|I_c}{V_s}\to+\infty} \frac{\langle P_s\rangle}{V_s I_b} = 0.5 \tag{22}$$

  and the curves are already close to it for $\frac{|Z|I_c}{V_s} = 1$;

- the first switching condition does not depend on α but only on the saturation voltage V_{sat} and the resulting locus is a vertical line of equation

$$\frac{|Z|I_c}{V_s} = \beta_0 \tag{23}$$

  Before reaching this point, the normalized power supplied is constant and equal to

$$\frac{\langle P_s\rangle}{V_s I_b} = \alpha \tag{24}$$

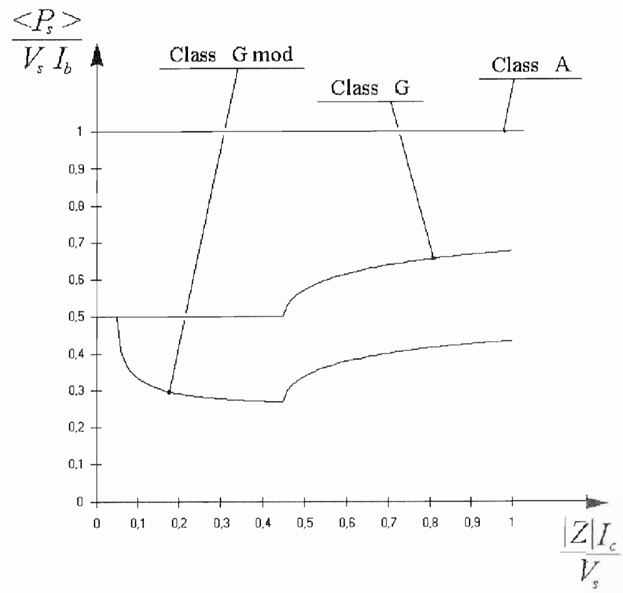

Figure 8: Comparison between the normalized average supply power for the class A ($\alpha = 1$), the class G and the modified class G power amplifiers (both with $\alpha = 0.5$).

- the second switching condition is given by

$$\frac{|Z|I_c}{V_s} = \beta \tag{25}$$

  and represents the point with minimum average dissipated power for a particular value of α.

With reference to a conventional class G amplifier (eqs. 18) it can be observed the lack of the first switching point and the horizontal asymptote is given by

$$\lim_{\frac{|Z|I_c}{V_s}\to+\infty} \frac{\langle P_s\rangle}{V_s I_b} = 0.5 + \frac{\alpha}{2} \tag{26}$$

and thus always greater than that of the modified class G amplifier (eq. 22). Moreover for $\alpha = 1$ the class A power amplifier curve is obtained. In figure 8 the comparison of the different class amplifiers is reported.

Experimental results

An experimental validation of the proposed characterization can be obtained with a measure of the currents flowing from power supply to power amplifier, respectively I_s and $I_{s\alpha}$ (see fig. 4). The measure is obtained simulating the unbalance forces acting on the rotor by imposition of a sinusoidal current I_0 in the coils with the rotor blocked. In order to verify the power consumption of the circuit alone, the currents I_s and $I_{s\alpha}$, are also measured under the $I_0 = 0$ condition. The analytical expression for the computation of the average supply power is then the following

$$< P_s >= V_s\cdot(< I_s > -I_{sserv}) + \alpha Vs\cdot(< I_{s\alpha} > -I_{s\alpha serv}) \tag{27}$$

$\alpha = 0.2$	$\alpha = 0.3$	$\alpha = 0.5$
$\beta_0 = 0.098$	$\beta_0 = 0.098$	$\beta_0 = 0.098$
$\beta = 0.102$	$\beta = 0.202$	$\beta = 0.402$
$\beta_{meas} = 0.108$	$\beta_{meas} = 0.218$	$\beta_{meas} = 0.5$

Table 1: Experimental measures for switching threshold with $V_s = 32$ V.

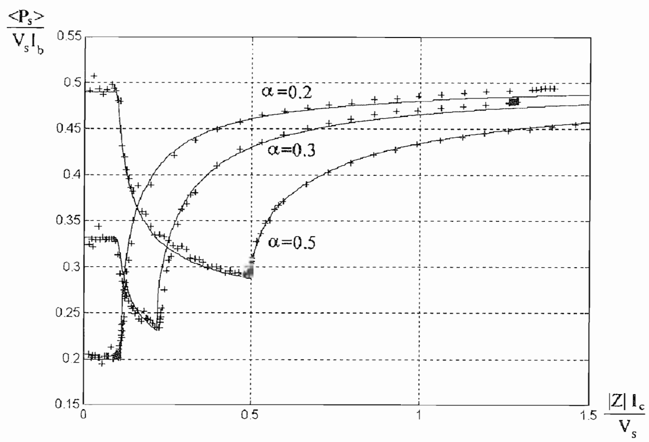

Figure 9: Comparison between experimental results (crosses) and theoretical results (continuous line) for α =0.2, 0.3, 0.5

where I_{sserv} and $I_{s\alpha serv}$ are respectively the values of I_s and $I_{s\alpha}$ when $I_0 = 0$.

The values of β_0 and β are obtained from the observation of voltage V_0 deformation due to the switching thresholds. To be noted the difference with expected values of β, mostly due to the non instantaneous commutation between final stages (tab. 1).

To be noted that in the reported experiment the $\frac{|Z|I_c}{V_s}$ axis is swept imposing different frequencies to the control voltage V_c. The resulting experimental curves are shown in figure 9. In figure 9, for $\alpha = 0.2$ the absence of the curve from β_0 to β must be related to the presence of a low-pass filter on the supply voltage: for small values of α the effect is relatively important and can not be neglected. To be noted the strong similarity of the curve in figure 9, for $\alpha = 0.2$, with the class G curve in figure 8 as expected for small values of α when the basic difference between class G and modified class G amplifier disapperars.

The theoretical value of β does not take into account the saturation of pre-final stage of final amplifier 2, this saturation taking place before the actual switching of final amplifier 2. Another difference between theoretical curves and experimental curves is also due to the power dissipation on the resistive component of the load, neglected in the starting hypotheses.

Conclusion

A modified class G power amplifier for magnetic bearing has been presented. Major differences of this circuit, compared with other class G designs are the unipolarity of the output current, the almost purely inductive load and the continuous biasing current I_b which preloads the suspension. These constraints have determined the development of a new type of class G amplifier including three power transistors, two supplied by different voltages, as in common class G circuits, and a third connected to the ground, in order to decrease the dissipated power during the phase of negative output voltage.

The dissipated power has been analytically evaluated and compared against experimental results for different values of α parameter, and comprehensive diagrams have been reported for the optimum design choices.

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