Linear Oscillatory Actuator with Magnetic Bearing

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Abstract: This paper describes the control of a magnetic bearing system used in a linear oscillatory actuator. The actuator consists of a stator inductor, a piston and two magnetic bearings. The piston is levitated by the two magnetic bearings in the lateral direction and driven by the magnetic force between the three magnetic coils of the stator inductor and the two permanent magnets of the piston in the axial direction. Forward and reverse motions of the piston are available by alternating the direction the exciting current in the coils. The dynamic characteristics of the linear oscillatory actuator are analyzed theoretically and the magnetic bearings are controlled by PID and H_∞ control methods.

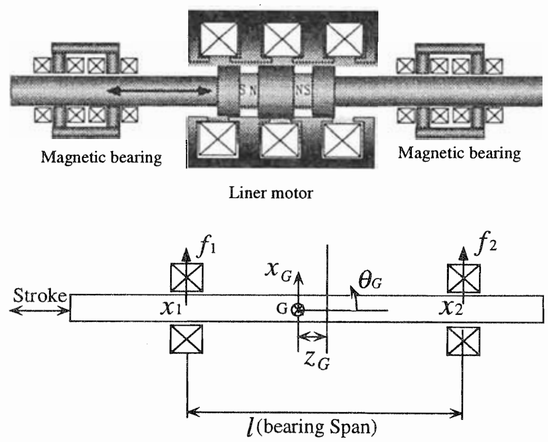

Fig. 1 Model and Mathematical Model

1 Introduction

Magnetic bearings have been applied in many machinery and equipment recently due to their advantage of the non-contact drive. According to the motion of shafts and direction of generated forces, magnetic bearings can be divided into three type: thrust magnetic bearings to limit axial motion of shafts, radial bearings to support shafts with rotary motion and linear bearings to support shafts with axial linear motion. Control is necessary for magnetic bearings since they are intrinsically unstable systems. A lot of studies have been carried on the control of thrust magnetic bearings and high-speed radial bearings[1]-[4]. This paper presents a study on the control of linear magnetic bearings used in a linear oscillatory actuator by using the conventional PID control and H_∞ robust control. It is obvious that the bearing system is a time variant due to the linear oscillatory motion of the piston in the axial direction. The performances and robustness of the PID controller and H_∞ controller are investigated and compared by changing the speed and position of the piston.

2 Modeling

The schematic drawing of the linear oscillatory actuator system is shown in Fig.1. The system consists of a stator inductor, a piston and two linear magnetic bearings. The stator inductor contains three coils and the piston contains two permanent magnets to generate strong thrust force and long stroke in the axial direction. The piston is supported by the two magnetic bearings in the lateral direction and driven by the electromagnetic force between the coils and permanent magnets in the axial direction. The motion of the piston in the axial direction was analyzed in the previous paper[5]. The equation of motion of the piston in the lateral direction shown in Fig.1 can be written in the following form[6]:

$$m\ddot{x}_G = f_1 + f_2 - mg \tag{1}$$

$$x_G = \left[x_1(l/2+z_G) + x_2(l/2-z_G)\right]/l \tag{2}$$

$$J\ddot{\theta}_G = f_2(l/2+z_G) - f_1(l/2-z_G) \tag{3}$$

$$\theta_G = (x_2 - x_1)/l \tag{4}$$

where m and J are mass and moment of inertia of the piston, l the span between magnetic bearings 1 and 2, f_1 and f_2 the attractive forces of bearings 1 and 2, x_G and z_G the displacements of mass center in x and z direction, and θ_G the angle of shaft. Since this system is uncoupled between x and y directions, we need only to consider the control of vibration in x direction. Even though eddy currents may be generated on the attractive surface due to the oscillatory motion and have some influence on the attractive force, the control input force is supposed to be a linear function of the current and can be expressed as follows[7]:

$$f_1=k_{x1}x_1+k_{i1}i_1+F_1 \quad (5)$$

$$f_2=k_{x2}x_2+k_{i2}i_2+F_2 \quad (6)$$

$$F_1=F_2=mg/2 \quad (7)$$

where F_1 and F_2 are constant forces generated by the bias currents in the two magnetic bearing, i_1 and i_2 the input currents of the two magnetic bearings, k_{x1} and k_{x2} are stiffness of the two bearings, and k_{i1} and k_{i2} are constants. In this system the two magnetic bearings are supposed to have same performances so that $k_{x1}=k_{x2}, k_{i1}=k_{i2}$ and the equations of motion can be expressed in the following matrix form:

$$M\ddot{x}_B+C\dot{x}_B+Kx_B=Fi+F_d \quad (8)$$

$$x_B=\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \quad i=\begin{bmatrix} i_1 \\ i_2 \end{bmatrix} \quad (9)$$

$$M=\begin{bmatrix} m(1/2-z_G/l) & m(1/2+z_G/l) \\ J/l & J/l \end{bmatrix} \quad (10)$$

$$C=\begin{bmatrix} -2\dot{z}_G m/l & 2\dot{z}_G m/l \\ 0 & 0 \end{bmatrix} \quad (11)$$

$$K=\begin{bmatrix} -m\ddot{z}_G/l-k_{x1} & m\ddot{z}_G/l-k_{x2} \\ -k_{x1}(l/2+z_G) & k_{x2}(l/2-z_G) \end{bmatrix} \quad (12)$$

$$F=\begin{bmatrix} k_{i1} & k_{i2} \\ k_{i1}(l/2+z_G) & -k_{i2}(l/2-z_G) \end{bmatrix} \quad (13)$$

$$F_d=\begin{bmatrix} 0 \\ mgz_G \end{bmatrix} \quad (14)$$

3 PID control and Result

In this section the PID control method is used in the control of the magnetic bearing system. First the equation of motion (8) of the dynamic system is expressed in the form of state equation as follows:

$$\dot{X}=AX+BU+DW \quad (15)$$

$$Y=X \quad (16)$$

where $X=\begin{bmatrix} x_B^T & \dot{x}_B^T \end{bmatrix}^T$ is the state vector, U the control

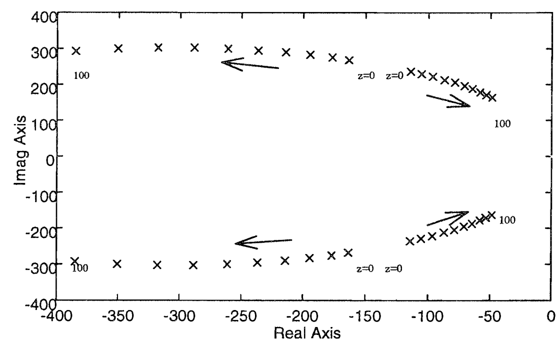

Fig. 2 Root locus of closed loop system with z change

input, W the external disturbances, and $A\in R^{4\times4}$ and $B\in R^{4\times2}$ matrices which are defined in the following forms:

$$A=\begin{bmatrix} 0 & I \\ -M^{-1}K & -M^{-1}C \end{bmatrix} \quad (17)$$

$$B=\begin{bmatrix} 0 \\ M^{-1}F \end{bmatrix} \quad (18)$$

Since system matrices A and B are functions of z_G and z_G varies with time, the system is time variant. To obtain good control performances, gain scheduling is necessary for different values of system matrices A and B. However, for convenience the variation of system matrices is ignored and the system is considered to be a linear time invariant (LTI). The system matrices A and B for $z_G=0, \dot{z}_G=0, \ddot{z}_G=0$ are used as the normal system matrices and the controller is designed on the basis of the normal system. For the LTI system, the input of PID control can be written in the following form:

$$U=-K_pX-K_dX-K_i\int XdX \quad (19)$$

where K_p K_d and K_i equal to 4, 5, 0.5, respectively.

Figure 2 shows the root locus plot of the closed loop system. It is found that all the poles is on the left side of the complex plane when z_G changes from 0 to 100mm. This means that the system remains stable in the whole range of piston stroke. However the influence of $\dot{z}_G$ and $\ddot{z}_G$ on system matrices is not considered.

Figures 3 and 4 show the simulation results of step response induced by the initial displacement at the bearing 1 when z_G=0mm and z_G=20mm, respectively. It is found that the displacement of the piston can easily be reduced to 0 in both cases though the convergence for z_G=0mm is a little faster than that for z_G=20mm. Figure 5 shows the response of piston when it moves in a constant speed of 200mm/s in the axial direction. It is found that a steady-state error is produced in this case.

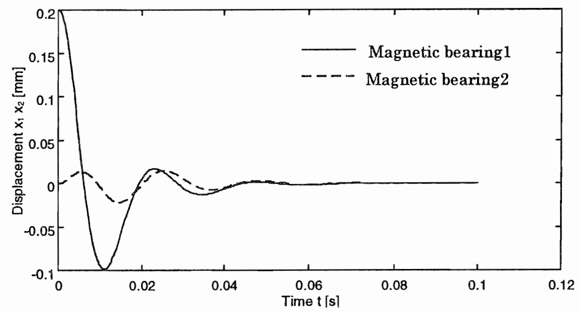

Fig. 3 Step response z_G=0mm (PID control)

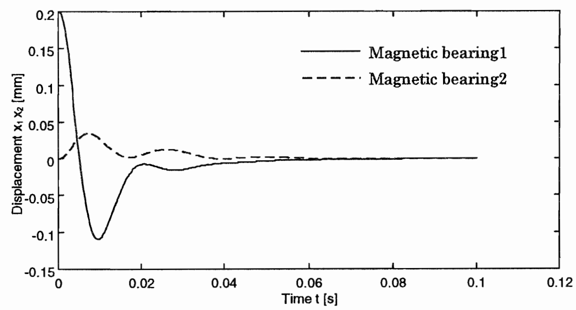

Fig. 4 Step response z_G=20mm (PID control)

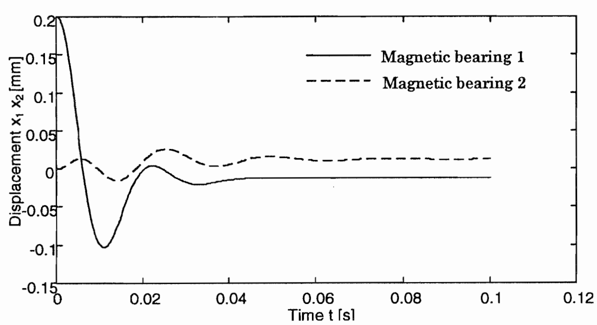

Fig. 5 Step response z_G=200mm/s (PID control)

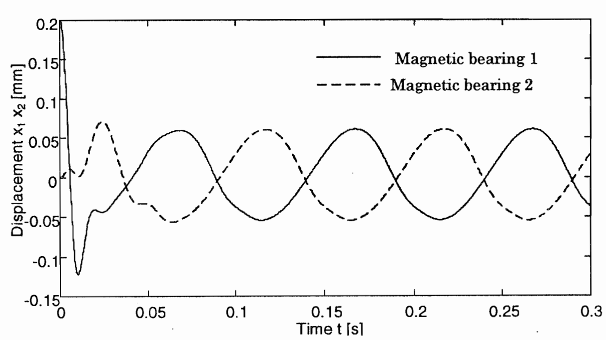

Fig. 6 Step response z_G:10Hz (PID control)

Moreover stable lateral vibration of amplitude 0.07mm is induced in the piston when it moves reciprocally in the axial direction as shown in Fig.6. The vibration is induced by the periodical variation of weight load acting on each magnetic bearing due to the reciprocal movement of the mass center of the piston.

4 H_∞ Control and Result

As discussed in the previous section, the real system is time variant. However it is considered as the LTI system in the design of controller. It is necessary that the designed controller should have good robust stability so that the closed system remains stable even if the control object deviates from the normal plant. In this section H_∞ robust control theory is applied in the design of controller. The variation of system matrices is considered as the model error of the nominal plant and its frequency domain property is described by a weight function W_{del} in the design of H_∞ controller. The desired control performance in the frequency domain is described by a weight function W_p. H_∞ controller is designed to satisfy the following inequality[7]:

$$\left\| \begin{matrix} W_p(1+PK)^{-1} \\ W_{del}PK(1+PK)^{-1} \end{matrix} \right\|_\infty < 1 \qquad (18)$$

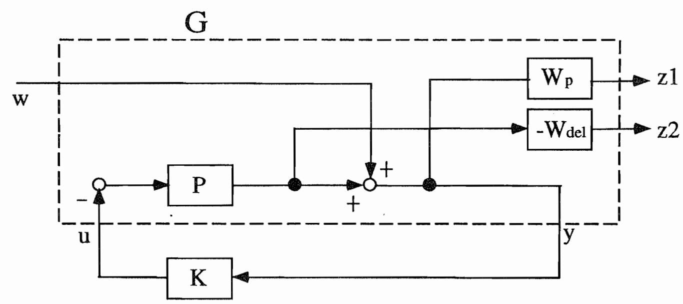

Fig. 7 Structure of system

Figures 8 and 9 show the simulation results of step response of H_∞ control under the same conditions as Figs.3 and 4, respectively. Compared with the result of PID control, the natural frequency of the closed loop system is higher and the damping ratio is smaller. However the total decaying times of H_∞ control are almost the same as those of PID control. Under the condition that the piston moves at a constant speed 200mm/s, the response of the H_∞ control is shown in Fig.10. It is found that the steady-state error of H_∞ control is much smaller than that of PID. Figure 11 shows the response when the piston makes oscillatory motion. The amplitude of residual vibration under H_∞ control is smaller than that under PID control. These results means that H_∞ control has better robust performance than PID control.

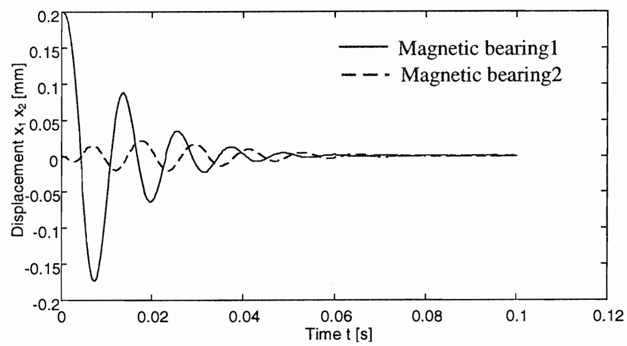

Fig. 8 Step response z_G=0mm (H_∞control)

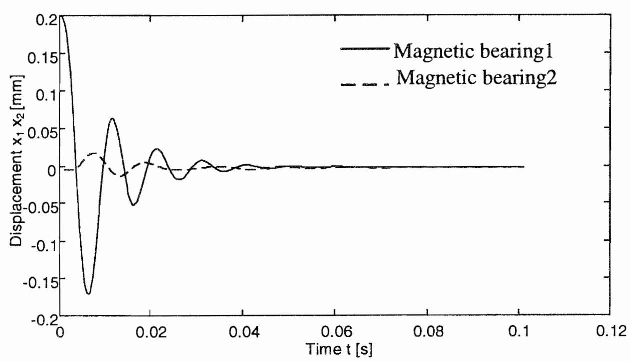

Fig. 9 Step response z_G=20mm (H_∞control)

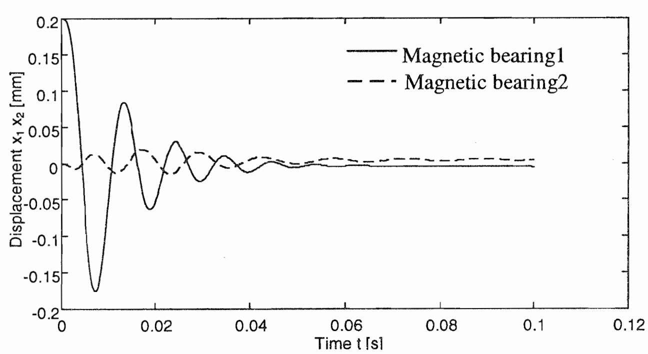

Fig. 10 Step response z_G=200mm/s (H_∞control)

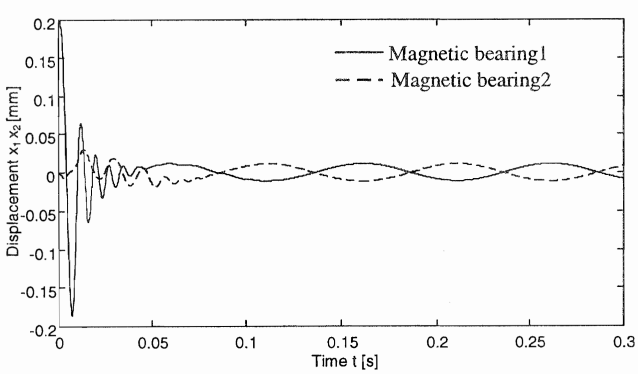

Fig. 11 Step response z_G:10Hz (H_∞control)

5 Conclusion

PID and H_∞ controls are applied to the control of a linear magnetic bearing system used in a liner oscillatory actuator. From simulation results, the following conclusions can be obtained:

(1) Both PID and H_∞ controls have enough robust stability to endure the variation of the system parameters due to the motion of piston.

(2) The H_∞ control gives better robust performance than PID when the piston moves at a constant speed or reciprocally in the axial direction

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