Effect of Foundation Stiffness on Active Magnetic Bearing Suspension

Erkki Lantto, Janne Väänänen, Matti Antila

Laboratory of Electromechanics, Helsinki University of Technology
Otakaari 5 A, 02150 Espoo, Finland
tel: +358 0 4514687; fax: +358 0 460224; e-mail: telkka@vakoluku.hut.fi

Abstract: The effect of low foundation stiffness on active magnetic bearing suspension is studied. Criteria for analysing the seriousness of foundation resonances are developed and a measurement procedure is proposed for analysing real machines. The test machine was measured in different mountings to get practical information about foundation behaviour.

1. Introduction

By foundation we mean the mechanical system seen from the active magnetic bearing (AMB) electromagnet surface. So, electromagnets, machine stator, machine bed and maybe the whole building in some cases have their effects on the foundation stiffness (force per displacement). When using high-stiffness bearings, for example ball bearings, it may happen that the effective bearing stiffness is mainly determined by the foundation stiffness, not by the bearing stiffness. However, in ball-bearings or sleeve bearings there is no energy source in the bearings, and low foundation stiffness can not cause instability (at least for a nonrotating rotor).

In the case of AMB the situation with low foundation stiffness is more serious because there is an energy source in the bearings, and actually the bearings do not act as dampers at every frequency and in every vibration mode. Low foundation stiffness does not only affect bearing performance but may cause instability for the suspension even for a nonrotating rotor.

In [1] the effect of low foundation stiffness was studied. It was found that resonances in the foundation may lead to poorly damped vibrations. [1] is the only reference that the authors know, where the effect of the foundation stiffness is seriously studied.

Like in rotor, there are always mechanical resonances in the foundation. In this paper tools are presented for analysing the seriousness of these resonances. When the foundation structure will clearly satisfy the criteria developed in this paper then it may be forgotten in the design of the AMB system (the foundation stiffness may be assumed to be infinite) and its effect on performance is negligible. Of course the foundation system may be modelled and taken into account in the AMB tuning process. However, if we are trying to make robust bearings for series production, then there can not be any individual tuning with every machine. The whole system must be far enough from critical limits so that no tuning is needed even when some parameters are varying. In practice there may be imperfections in the mounting of a machine or different machines may be mounted in different kinds of assemblies. So, the resonances in the foundation may vary from case to case and the system must be robust enough for these variations.

In this paper a measurement procedure is suggested to ensure that the foundation is good enough. The measurement can be compared with the computed limits, the potential risk frequencies and vibration modes can be detected, and the mechanical structure may be modified to avoid problems.

2. Theoretical study

In this theoretical study we consider foundation compliance (force to displacement response). Compliance is a kind of inverse of stiffness and it is better suited for this study. The purpose of the theoretical study is to get qualitative information about how serious high foundation compliance is in different frequency ranges and also to develop useful criteria that can be used in practical problems to analyse real machines. In the analysis, the dynamics of the bearings, rotor and foundation are assumed to be linear.

Consider a radial AMB suspension with two radial bearings. A cartesian coordinate system is fixed so that the X- and Y-axes coincide with the directions of the electromagnet forces and position sensor directions. All the forces and positions have positive direction in the direction of the X- or Y-axis. Combine the bearing forces acting to rotor to a vector $\mathbf{F}_r$ (bearing force acting on rotor). The order of the forces is X_1, X_2, Y_1 and Y_2, where 1 and 2 refer to the two bearings. Forces acting on the electromagnets are combined in vector $\mathbf{F}_b$. Obviously $\mathbf{F}_b=-\mathbf{F}_r$. The electromagnet positions are combined in vector $\mathbf{p}_b$ and the positions of the displacement sensors are combined in vector $\mathbf{p}_s$. First we suppose that the displacement sensors are tightly connected to electromagnets and move with the electromagnets $\mathbf{p}_s=\mathbf{p}_b$. By foundation compliance we mean the transfer function from bearing forces $\mathbf{F}_b$ to bearing displacements $\mathbf{p}_b$. Actually in this presentation we only need the frequency response of the compliance transfer function.

The basic idea is simple. When a electromagnet is moved, the position of the rotor relative to electromagnet and displacement sensor changes and the AMB tries to compensate this change by applying force to rotor. The same force, direction reversed, is acting also on the electromagnet in the stator side. If the compliance of the foundation is not zero, the stator side will move, which will cause a change in the bearing force and so on. This unpleasant feedback through the nonzero foundation compliance is illustrated in Figure 1.

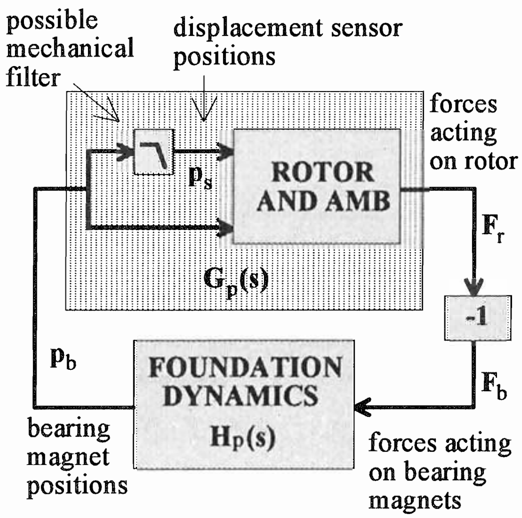

Figure 1. Feedback through nonzero foundation compliance.

The transfer function $\mathbf{G}_p$ (rotor bearing transfer function from bearing magnet position to force) is quite easy to compute when bearing parameters, rotordynamic model and magnetic bearing controller are known. Usually these are known quite accurately. The dynamic model of the rotor and active magnetic bearings is given in the Appendix. The "unknown" system $\mathbf{H}_p$ (transfer function from bearing forces to bearing positions), called here foundation compliance, is not usually known well but the frequency response of this can be measured. Let us define also a transfer function matrix from bearing speed to bearing force (acting on rotor) $\mathbf{G}_v$ and a transfer function from bearing force (acting on bearing) to bearing speed $\mathbf{H}_v$ (velocity transfer functions). Transfer functions from bearing magnet acceleration to bearing force $\mathbf{G}_a$ and from bearing force to bearing acceleration $\mathbf{H}_a$ (acceleration transfer functions) are also defined. These transfer function matrixes are interrelated:

$$\begin{aligned} \mathbf{H}_v(s) &= s\mathbf{H}_p(s), \mathbf{G}_v(s) = \frac{1}{s}\mathbf{G}_p(s) \\ \mathbf{H}_a(s) &= s^2\mathbf{H}_p(s), \mathbf{G}_a(s) = \frac{1}{s^2}\mathbf{G}_p(s) \end{aligned} \tag{1}$$

where s is the Laplace s ($i\omega$ on the imaginary axis).

$\mathbf{G}_p$ is a stable transfer function matrix because the bearings are designed to work well with zero foundation compliance. The transfer function matrix $\mathbf{H}_p$ is also stable, because it describes mechanical system consisting of masses, springs and dampers. Actually we know more about the foundation. Because the foundation has no energy source, it can only dissipate energy in any frequency. Of course there may be for example a vibrating machine near the machine in consideration that may transfer mechanical power through bearing forces to the rotor bearing system. However, because of the assumption of linear dynamics, this external noise source does not affect the stability of the suspension and it may be left out of the consideration in the stability analysis. So, the total mechanical power going to foundation must be positive at every frequency. This means mathematically

$$\text{real}\left[\mathbf{x}^H\mathbf{H}_v(i\omega)\mathbf{x}\right] > 0 \quad \forall\ 0 < \omega < \infty \tag{2}$$

In other words, the transfer function matrix $\mathbf{H}_v$ is positive definite on the imaginary axis.

The basics of the multivariable feedback theory and the use of singular values can be found for example in [2]. According to generalized Nyquist criteria an open-loop stable multivariable feedback system (negative feedback) is stable if the sum of the eigenvalue encirclements of the open loop transfer function matrix around point -1 in the complex plane is zero when s encircles the right half plane. If λ is an eigenvalue of the open loop transfer function matrix $\mathbf{GH}$, then the following holds for some nonzero eigenvector $\mathbf{x}$

$$\mathbf{G}(i\omega)\mathbf{H}(i\omega)\mathbf{x} = \lambda\mathbf{x} \tag{3}$$

The maximum gain (output vector 2-norm per input vector 2-norm) of $\mathbf{GH}$ is limited by $\bar{\sigma}(\mathbf{GH})$ where $\bar{\sigma}$ means the greatest singular value. The greatest singular value of a matrix $\mathbf{A}$ is the square root of the largest eigenvalue of $\mathbf{AA}^H$. Because $\bar{\sigma}(\mathbf{GH}) \le \bar{\sigma}(\mathbf{G})\bar{\sigma}(\mathbf{H})$, none of the absolute values of the eigenvalues can be >1, if

$$\bar{\sigma}\left[\mathbf{H}_p(i\omega)\right] < \frac{1}{\bar{\sigma}\left[\mathbf{G}_p(i\omega)\right]} = g_p(\omega) \tag{4}$$

where g_p is called here the gain limit for the position transfer function. In (4), velocity or acceleration functions could have been used as well, and the corresponding gain limits would have been g_v and g_a respectively.

Eigenvalue equation (3) can also be written with the velocity functions

$$\begin{aligned} &\mathbf{G}_v(i\omega)\mathbf{H}_v(i\omega)\mathbf{x} = \lambda\mathbf{x} \Rightarrow \\ &\lambda = \frac{\mathbf{x}^H\mathbf{H}_v(i\omega)\mathbf{x}}{\mathbf{x}^H\mathbf{G}_v^{-1}(i\omega)\mathbf{x}} \end{aligned} \tag{5}$$

Because the real part of the numerator (5) is known to be positive (2), there cannot be an eigenvalue on the negative real axis if $\mathbf{G}_v^{-1}$ or $\mathbf{G}_v$ is positive definite (if a matrix is positive definite then its inverse is also positive definite). It is easy to find the frequency intervals where $\mathbf{G}_v$ is positive definite using the bearing model and rotordynamic model. This, however, is an on-off criterion and in practice it would be nice to get some kind of measure of how positive definite the system is. Let us define an angle α as follows

$$\alpha(\omega) = 90^\circ - \max_{\mathbf{x}} \left\{ \left| \arg_{-180^\circ \cdots 180^\circ} \left(\mathbf{x}^{\mathrm{H}} \mathbf{G}_{\mathrm{v}}(i\omega) \mathbf{x} \right) \right| \right\} \tag{6}$$

To compute the maximum, first separate $\mathbf{G}_v$ as follows

$$\begin{aligned} \mathbf{G}_{\mathrm{v}} &= \mathbf{G}_{\mathrm{vr}} + i\mathbf{G}_{\mathrm{vi}} \\ \mathbf{G}_{\mathrm{vr}} &= \frac{1}{2}\left(\mathbf{G}_{\mathrm{v}} + \mathbf{G}_{\mathrm{v}}^{\mathrm{H}}\right) \\ \mathbf{G}_{\mathrm{vi}} &= \frac{1}{2i}\left(\mathbf{G}_{\mathrm{v}} - \mathbf{G}_{\mathrm{v}}^{\mathrm{H}}\right) \end{aligned} \tag{7}$$

Now the real part of $\mathbf{x}^{\mathrm{H}}\mathbf{G}_{\mathrm{v}}\mathbf{x}$ is $\mathbf{x}^{\mathrm{H}}\mathbf{G}_{\mathrm{vr}}\mathbf{x}$ and the imaginary part is $\mathbf{x}^{\mathrm{H}}\mathbf{G}_{\mathrm{vi}}\mathbf{x}$. The extremum values for the ratio between real real and imaginary parts are achieved as eigenvalues for $\mathbf{G}_{\mathrm{vi}}^{-1}\mathbf{G}_{\mathrm{vr}}$. By studying these eigenvalues it is possible to find the maximum of the phase angle.

Now if α is positive then $\mathbf{G}_v$ is positive definite and it can be multiplied by $e^{i\alpha}$ or $e^{-i\alpha}$ and it will still remain positive definite. So this is a kind of "phase margin". There can be phase errors in the model and still the known system will remain positive definite.

Now let us study the situation when α is zero or slightly negative and there is a sharp mechanical resonance. This situation is typical at low frequencies. Suppose that the foundation structure is a linear mechanical system with viscous damping. Suppose further that the damping has such a form that there is no modal coupling through material damping. If the N first eigenmodes of the foundation structure are taken into account, the foundation frequency response will be

$$\begin{aligned} \mathbf{H}_{\mathrm{v}}(i\omega) &= \mathbf{T}\begin{bmatrix} d_1 & 0 & \cdots & 0 \\ 0 & d_2 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & d_N \end{bmatrix}\mathbf{T}^{\mathrm{T}} \\ d_{\mathrm{n}} &= \frac{i\omega\omega_n^2}{\left(\omega_n^2 - \omega^2\right) + i2\zeta_n\omega_n\omega} \end{aligned} \tag{8}$$

where $\mathbf{T}$ is a constant matrix.

If there is a sharp mechanical resonance (low modal damping ζ_n) then the corresponding diagonal element d_n has a very large value near the resonance frequency ω_n. This means that when the absolute value of the numerator $\mathbf{x}^{\mathrm{H}}\mathbf{H}_{\mathrm{v}}\mathbf{x}$ (5) is high (relative to denominator), its phase angle is practically determined by the diagonal element d_n. In the resonance, the diagonal element is real and positive. Therefore, the phase angle of $\mathbf{x}^{\mathrm{H}}\mathbf{H}_{\mathrm{v}}\mathbf{x}$ is near zero (when the absolute value is high). The phase angle of $\mathbf{x}^{\mathrm{H}}\mathbf{G}_{\mathrm{v}}^{-1}\mathbf{x}$ is located in the sector ±(90°-α). It is easy to prove that if values of $\mathbf{x}^{\mathrm{H}}\mathbf{G}_{\mathrm{v}}\mathbf{x}$ will stay in the sector ±(90°-α) then also the values of $\mathbf{x}^{\mathrm{H}}\mathbf{G}_{\mathrm{v}}^{-1}\mathbf{x}$ will stay in the same sector. This means that an eigenvalue having a great absolute value (>1) cannot be located on the negative real axis, i.e. there is margin on the phase angles. Therefore, we are allowed to violate the gain limit condition (4) without losing stability. When we move away from the resonance frequency ω_n, the margin in the phase angles decreases. On the other hand, the gain limit condition (4) is met when we are far enough from the resonance. In order to maintain stability, the gain limit must be met before the margin in the phase angles is lost.

In the neigbourhood of a sharp resonance, the stability is preserved if the ratio of the resonance peak and the gain limit is less than

$$k = \frac{1}{\sin(-\alpha)}. \tag{9}$$

Even though mechanical resonances in the foundation would not cause instability, they may considerably alter bearing behaviour, for example the unbalance response or the response to disturbance forces. Consider the frequency response from some disturbance to the bearing force. This frequency response function is $\mathbf{G}_{\mathrm{NF}}(i\omega)$ when the foundation compliance is zero. When the foundation compliance is not zero the frequency response from this noise input to the bearing force is $\left(\mathbf{1}+\mathbf{G}(i\omega)\mathbf{H}(i\omega)\right)^{-1}\mathbf{G}_{\mathrm{NF}}(i\omega)$. If the response to some harmonic signal $\mathbf{u}e^{i\omega t}$ is $\mathbf{F}_1e^{i\omega t}$ with zero foundation compliance and $\mathbf{F}_2e^{i\omega t}$ with nonzero foundation compliance, then

$$\Delta\mathbf{F} = \mathbf{F}_1 - \mathbf{F}_2 = \left\{\left[\mathbf{1}+\mathbf{G}(i\omega)\mathbf{H}(i\omega)\right]^{-1}\mathbf{G}(i\omega)\mathbf{H}(i\omega)\right\}\mathbf{F}_1. \tag{10}$$

The greatest singular value of the frequency response function in brackets { } is limited as follows [2]

$$\bar{\sigma}\{\ \} \le \frac{\bar{\sigma}\left[\mathbf{G}(i\omega)\mathbf{H}(i\omega)\right]}{1-\bar{\sigma}\left[\mathbf{G}(i\omega)\mathbf{H}(i\omega)\right]} \tag{11}$$

assuming $\bar{\sigma}\left[\mathbf{G}(i\omega)\mathbf{H}(i\omega)\right] < 1$. In other words, if the gain limit criterion (4) is clearly satisfied then the foundation has a negligible effect on the bearing forces and on physical quantities related to force (like AMB control voltage).

The gain limit criterion (4) or the criterion $\alpha>0$ are sufficient conditions for the system to be stable. Of course in some circumstances they may be too conservative, ie. a foundation that does not fulfill these criteria may work well. However, because of the nature of this problem little conservativeness does not matter. We are trying to build a robust AMB suspension that will work even if the mounting of the machine will change slightly.

3. Simulation results

In simulations a model of a real high-speed motor was used. The machine parameters are listed in the Appendix. In Figure 2 the acceleration function gain limit g_a (greatest allowed singular value of the matrix $\mathbf{H}_a$) and the phase margin α are plotted. In the lower subfigure a horizontal line marked by k=10 is plotted. When α is above this line a sharp resonance peak is allowed to violate the gain limit by factor 10, see Equation (9).

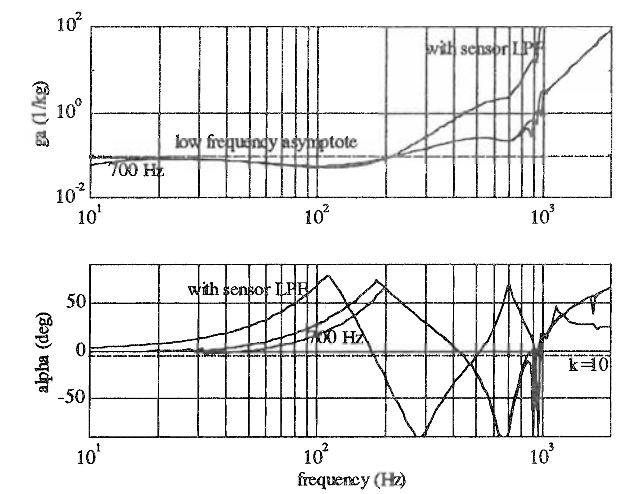

Figure 2. Gain limit (greatest allowed singular value) of the acceleration function $\mathbf{H}_a$ and the phase margin α for the test machine.

At low frequencies the gain limit approaches a low-frequency asymptote defined by the inertia properties of the rotor. To ensure that the gain limit is fulfilled at low frequencies one has to have essentially more mass in the stator than in the rotor. This is usually the case, and typically the stator is not floating in free space but is connected to something very massive, so the greatest singular value of the foundation acceleration matrix will decrease at low frequencies proportionally to ω^2. The frequency response $\mathbf{G}_v$ is also positive definite (or almost positive definite) up to 400 Hz, which means that even very high resonances in this frequency range should not cause instability. Despite of this it is possible that a resonance at these frequencies causes instability if the electromagnet and the displacement sensor are not moving in the same phase. For example, if the sensor and magnet are far apart, it may happen that in the stator bending mode these will be in the other sides of the vibration node and are moving 180° phase shifted. However in practice the sensor and the magnet are quite near each other. Also in the frequency range where the phase margin α has a high value the system will tolerate a slight phase shift between the magnet and the sensor movement.

In the frequency range 400 Hz to 850 Hz there is a potential risk of instability if there are mechanical resonances in the foundation. In this machine the bearings are not acting as dampers at these frequencies, because there is a sharp phase drop in about 700 Hz in the AMB controller (caused by 2nd order low-pass filter). The purpose of this filter is to drop the phase of the bearing stiffness below -180° before the first rotor bending mode. Of course it could be possible to make the bearings act as dampers up to the first rotor bending mode, then the frequency range up to 1 kHz could be made resonance free. However, this would lead to high bearing stiffness (and low gain limit) at high frequencies, and the hazardous frequency range would be transferred to higher frequencies, where the foundation resonances are much more unpredictable (see Figure 4). Also the power amplifier voltage is limited in a practical AMB (so the current rise rate is limited) which means that even if the bearings may be dampers with small perturbations the bearings are not dampers in so high signal amplitudes when the power amplifier becomes saturated. To conclude, there is always a hazardous frequency range (400 Hz to 850 Hz in this case) where foundation resonances may cause problems.

At high frequencies well beoynd the AMB bandwidth the gain limit increases rapidly, so the frequency range above 1 kHz is safe.

In Figure 2 there is also plotted the limit curve at rotational speed 700 Hz. Rotational speed will cause the gain limit to drop proportionally to ω in very low frequencies. This is not a big problem because the greatest singular value of the foundation structure will drop proportionally to ω^2 in small frequencies and the rotational speed has no significant effect on the phase margin α. Of course the situation may be worse when the rotor is more gyroscopic.

There is often an integrator in the AMB controller to achieve high static stiffness. This integrator has practically no effect on curves in Figure 2. This can be explained so that at low frequencies the transfer function $\mathbf{G}$ depends essentially on the inertial properties of the rotor, not on the bearing properties.

If very high stiffness is required of the bearings, the gain limit will stay at low level up to high frequencies. In this case the situation is very serious considering foundation resonances. From the bearing model (see Appendix) it is found that the main problem is not that the bearing magnets move but that the displacement sensors move. A straightforward solution to this problem is to install a mechanical low-pass filter between the bearing magnets and displacement sensors. This can be done, for example, by connecting the metal piece where the displacement sensors are installed by rubber o-ring to the machine stator. In the simulation the low-pass filter 3-dB frequency was set at 200 Hz and damping factor 0.5 (2nd order low-pass filter). This filter will clearly raise the gain limit at high frequencies. This may be a solution to foundation resonances in the case of very high stiffness AMBs.

4. Experimental results

The experimental setup is shown in Figure 3.

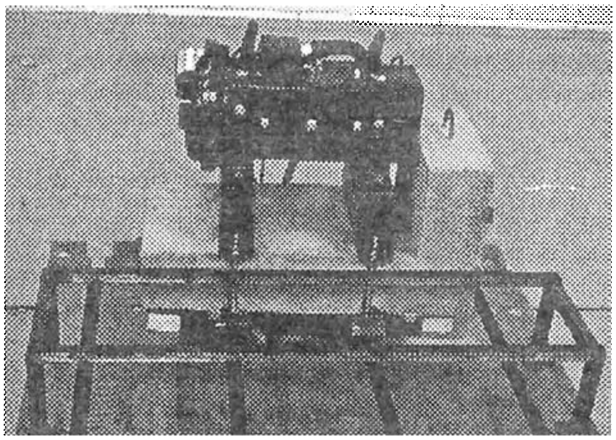

Figure 3. Experimental setup. Very elastic steel rack in the front. Machine mounted with heavy legs into the concrete bed in the background.

Before the tests the linearized parameters of the bearing were measured accurately. By tuning the AMB controller we could demonstrate various kinds of limit cycle oscillations and unstable behavior especially in axial direction. However the oscillations were in the wrong frequency range and the instabilities occurred as well in very rigid mountings as very elastic mountings. The reasons for these "foundation stiffness effects" turned out to be anything else but low foundation stiffness: For example bearing hysteresis and eddy currents. When all these phenomena were carefully analyzed it was found that we could not make the test machine unstable by inproper mounting.

We mounted the machine with very soft rubbers (poorly damped resonances in the frequency range 1 Hz to 5 Hz). No problems. Then we mounted the machine to a very elastic steel rack (in front in Figure 3). Now there were very poorly damped foundation resonances in the frequency range 5 Hz to 30 Hz. Again no problems, even though the gain limit was clearly not satisfied. These tests confirmed, however, that foundation resonances at low frequencies are not very dangerous considering the stability.

Then we tried to find out why the resonances in the frequency range 400 Hz to 850 Hz did not cause stability problems. We measured the test machine in three different mountings. The rotor was not inside the machine in the measurements. First, the stator was in soft rubbers. Then we connected heavy legs (about 30 kg) to the bearings and mounted these legs with soft rubbers to the ground. In the third mounting the stator was with heavy legs which were tightly connected to concrete bed (200 kg). The concrete bed was mounted with rubbers to ground. See Figure 3.

In the measurement the machine stator was hit by an impulse hammer many times (to get more accurate results) in the positions of the electromagnets to the directions of the electromagnet. The acceleration responses were measured in the directions of every electromagnets by four piezoelectric acceleration sensors. From this measurement we got the 4*4 frequency response matrix $\mathbf{H}_a(i\omega)$ in discrete frequency points. Then the greatest singular value of this matrix was computed. The results are plotted in Figure 4 with the computed gain limit and phase margin.

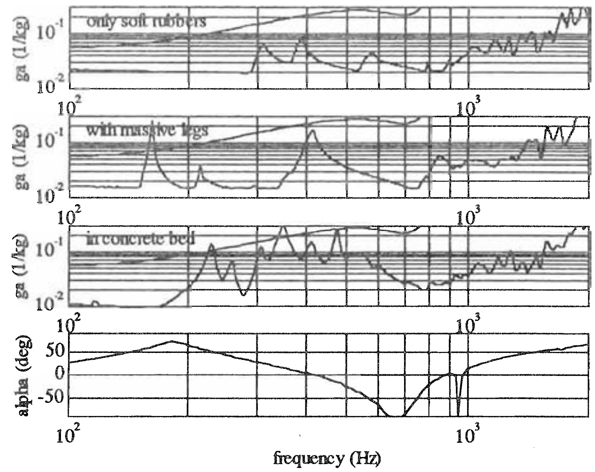

Figure 4. Measured singular values in different mountings.

The results were quite surprising. The worst resonances were achieved when the machine was mounted in the concrete bed. When mounted with heavy legs there is one resonance near 400 Hz which may cause problems. In the case of the concrete bed there are many hazardous resonances in the range 300 Hz to 500 Hz. Even though these resonances did not cause instability, they caused very large vibration amplitudes, when the AMB was disturbed by an signal generator. From that it can be predicted that high vibrations could occur, if a rotor with high unbalance would be driven over these frequencies. The test machine had not high unbalance.

The best mounting for this machine is to connect the machine with rubbers to ground. The resonances at 300 Hz and 400 Hz are so well damped ($\zeta \approx 0.04$ for the corresponding mode) that they will remain clearly below the gain limit as can be seen from Figure 4.

At high frequencies above 1 kHz there is a lot of resonance in all the mountings. However they are well above the AMB bandwidth, so they will not cause problems. Also in this machine the stator is relatively massive compared with the rotor. So, no extra mass is needed to the bearings.

5. Conclusions

In this paper the effect of the foundation stiffness was studied. It was found that the resonances at low frequencies do not (easily) cause suspension instability but may alter suspension performance. In the case of AMB there is always a hazardous frequency range where AMB stiffness is quite high and the foundation resonances may cause instability if the resonances are poorly damped. At high frequencies clearly above the AMB bandwidth the foundation resonances are not dangerous. From this study some advice can be stated on how to avoid problems with foundation resonances:

- Do not make AMB unnecessarily stiff
- Make sufficiently massive bearings to make the basic level of foundation stiffness high enough
- Try to achieve damping to the foundation elastic resonances to limit the resonance peaks

In this paper a gain limit and a phase marging were introduced. These are computed from the rotor and AMB models and they will state the safe limit for foundation compliance and the hazardous frequency ranges. A measurement procedure was proposed for the foundation frequency response matrix. The measured greatest singular value can be compared to the computed gain limit and the potential problems can be detected. This information can be used to modify the foundation structure, so that the problems are avoided.

References

[1] R. B. Zmood, D. K. Anand, J. A. Kirk, E. Zivi, "The Effect of Structural Vibrations on Magnetic Bearing Operation", Proceedings of the 24th Intersociety Energy Conversion Engineering Conference, Vol. 3, pp. 1499-1503, 1989

[2] J. M. Maciejowski, "Multivariable Feedback Design", Addison-Wesley Publishing Company, Inc. England, 423 p. 1989.

Appendix

The model of an elastic symmetric rotating shaft is

$$\begin{bmatrix}\mathbf{M} & \mathbf{0}\\ \mathbf{0} & \mathbf{M}\end{bmatrix}\ddot{\mathbf{q}}+\Omega\begin{bmatrix}\mathbf{0} & \mathbf{G}\\ -\mathbf{G} & \mathbf{0}\end{bmatrix}\dot{\mathbf{q}}+\begin{bmatrix}\mathbf{K} & \mathbf{0}\\ \mathbf{0} & \mathbf{K}\end{bmatrix}\mathbf{q}=\begin{bmatrix}\mathbf{B}_b & \mathbf{0}\\ \mathbf{0} & \mathbf{B}_b\end{bmatrix}\mathbf{F}_r$$
$$\mathbf{p}_{rb}=\begin{bmatrix}\mathbf{B}_b & \mathbf{0}\\ \mathbf{0} & \mathbf{B}_b\end{bmatrix}^T\mathbf{q} \qquad \text{(A1)}$$
$$\mathbf{p}_{rs}=\begin{bmatrix}\mathbf{C}_s & \mathbf{0}\\ \mathbf{0} & \mathbf{C}_s\end{bmatrix}^T\mathbf{q}$$

where $\mathbf{M}$, $\mathbf{G}$ and $\mathbf{K}$ are the mass, gyroscopic and stiffness matrixes respectively, $\mathbf{B}_b$ is a transformation matrix from real forces to generalized forces, $\mathbf{C}_s$ is a transformation matrix from generalized coordinates $\mathbf{q}$ to real displacements in the sensor positions. $\mathbf{p}_{rb}$ is the vector of rotor positions in the bearing locations, $\mathbf{p}_{rs}$ is the vector of rotor positions in the displacement sensor positions. Ω is rotational speed and $\mathbf{F}_r$ is the vector of bearing forces acting on rotor.

The electromechanical model of the bearings is

$$L\dot{\mathbf{I}}+h(\dot{\mathbf{p}}_{rb}-\dot{\mathbf{p}}_b)+(r+k_i)\mathbf{I}=k_i\mathbf{I}_{ref}$$
$$\mathbf{F}_r=c(\mathbf{p}_{rb}-\mathbf{p}_b)+2h\mathbf{I} \qquad \text{(A2)}$$

where L is the inductance, h is the current-force factor, r is the coil resistance and k_i is the current feedback coefficient. $\mathbf{I}$ is the vector of current differences in opposite coils (actually half of the difference) and $\mathbf{I}_{ref}$ is the current reference vector computed by the position controller. $\mathbf{p}_b$ is the vector of bearing magnet positions.

The position controller is given by a transfer function matrix $\mathbf{G}_{PC}(s)$

$$\mathbf{I}_{ref}=\mathbf{G}_{PC}(s)(\mathbf{p}_s-\mathbf{p}_{rs}) \qquad \text{(A3)}$$

where $\mathbf{p}_s$ is the vector of displacement sensor positions.

The test machine had the following parameters

$$\mathbf{M}=\text{diag}([21.5 \quad 0.44 \quad 0.69 \quad 0.80])$$
$$\mathbf{K}=10^6\,\text{diag}([0 \quad 0 \quad 25 \quad 93])$$
$$\mathbf{G}=0.001\begin{bmatrix}0 & 0 & 0 & 0\\ 0 & 20 & -1.2 & 0.27\\ 0 & -1.2 & 89 & -43\\ 0 & 0.27 & -43 & 134\end{bmatrix} \qquad \text{(A4)}$$
$$\mathbf{B}_b=\begin{bmatrix}1 & 1\\ 0.20 & -0.22\\ 0 & 0.18\\ 0.23 & 0.13\end{bmatrix},\quad \mathbf{C}_s=\begin{bmatrix}1 & 0.24 & 0.23 & 0.08\\ 1 & -0.26 & 0.30 & 0.54\end{bmatrix}$$
$$L=27\text{ mH},\quad h=82\frac{\text{N}}{\text{A}},\quad c=0.67E6\frac{\text{N}}{\text{m}},\quad r=1\Omega$$

All the radial channels had similar position regulators. The transfer function of the requlator was

$$G_{PC}(s)=\frac{4444^2}{s^2+888s+4444^2}\cdot\left[17000\left(1+\frac{1}{0.15s}\right)+\frac{38s}{0.00015s+1}\right] \qquad \text{(A5)}$$

and the current feedback coefficient k_i was 75 V/A.