Fifth International Symposium on Magnetic Bearings
Kanazawa, Japan, August 1996

A Two Active-axes Suspension for Maglev Vehicles

S. Carabelli*, F. Cerruti*, C. Delprete**, F. Di Majo***, G. Genta**, S. Zanolli**

* Dipartimento di Automatica e Informatica - Politecnico di Torino
** Dipartimento di Meccanica - Politecnico di Torino
Corso Duca degli Abruzzi 24, 10129 Torino - Italia
*** CNR, Progetto Finalizzato Trasporti 2, Roma - Italia

Abstract: A maglev three passive-axes levitation layout solution, in which the forces supporting the vehicle are supplied by the repulsion acting between permanent magnets located under the vehicle and on the guideway, is shortly described. The control design is aimed to stabilize the plant and to achieve a so-called zero power (control) consumption strategy with a two active-axes configuration of electromagnets.

Introduction

When studying maglev vehicles two solutions are usually considered: active electromagnetic suspension and electrodynamic levitation. The first one, developped mainly in Germany, was applied in the *transrapid* system. The vehicle is suspended under ferromagnetic rails by the magnetic field generated by electromagnets while other electromagnets perform the guiding task. The stability is assured by the active control of the magnetic fields, so the system can be considered as a *five active-axes magnetic suspension*. The main disadvantage of the system lays in the bulk and mass of the electromagnets, which add up to two thirds of the mass of the vehicles.

The second solution, developped mainly in Japan, is based on the repulsive forces acting between the magnetic field generated on the moving vehicle by very powerful superconducting coils, and that generated in stationary coils located in the guideway by the eddy currents due to the first one. This levitation mechanism has the advantage of being passively stable but has also a number of disadvantages, as the need of superconductors (low-temperature, liquid helium superconductors are used in the applications), the very strong magnetic fields, which need shielding, the higher electromagnetic drag and finally the high cost of the whole system.

The solution here tested, sometimes referred to as the *synthesis solution*, is a three passive-axes levitation layout, in which the forces supporting the vehicle are supplied by the repulsion acting between permanent magnets located under the vehicle and on the guideway [1]. This solution is passively stable in heave, roll and pitch while is unstable in lateral direction and yaw. An active system, based on the attration forces between *guiding* electromagnets and ferromagnetic rails is used to control the unstable modes. Note that the active system is used only to render the system stable, while transversal forces are supplied directly by the suspension permanent magnets whose repulsive force can have a component in the horizontal plane. The advantages are mainly in the low mass of the onboard magnets and electromagnets and the possibility of supporting the vehicle on its whole length, resulting in a much lower structural weight.

The control design is aimed to stabilize the plant and to achieve a so-called zero power consumption configuration. The error with a zero current reference is integrated to give the command that allows to achieve zero power consumption under static force load on the levitating system. In practice, the regulated system finds a new equilibrium position that allows to produce the magnetic force to counterbalance the external load without augmenting the required current but diminuishing the magnetic gap. To be noted that the negative stiffness due to the active magnetic actuator is augmented by the lateral component of the permanent magnet.

The present paper deals with the construction and operation of a test rig aimed to verify the stability of the system in stationary levitation conditions. A platform, which constitutes a reduced scale model of the actual vehicle and has a mass of about 200 kg, is levitated on a section of guideway with a length of about 1 m by a set of ferrite permanent magnets.

The design and construction of the electro-mechanical elements and of the control system of the test rig are described together with the optimization of the configuration of the permanent magnets and the results of the tests conducted on it are described.

The test rig constitutes the first step towards a reduced scale demonstration model, in which the levitating platform will be free to translate on a short track to study the dynamic behaviour of the system when moving along the sixth, uncontrolled, axis.

Model of the vehicle

The vehicle can be initially modelled as a rigid body, suspended on a number of elastic and damping elements

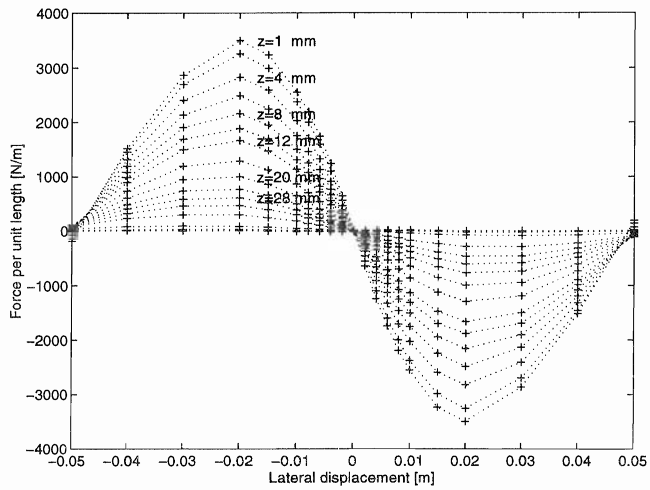

Figure 1: Measured lateral component of permanent magnets force.

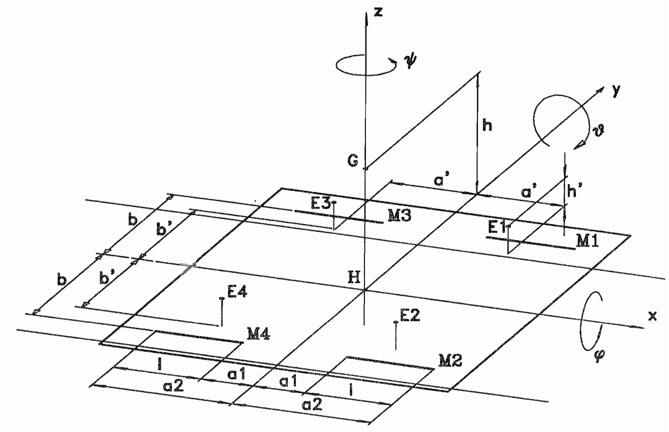

Figure 2: Vehicle sketch.

which simulate the passive electromagnetic suspension. The force-displacement characteristics of such elements is intrinsically nonlinear, as it is possible to desume from the experimental plots reported in figure 1 but a linearization for the study "in the small" is feasible. From the plots it is clear that the lateral force F_y can be considered linear with the lateral displacement y in a range of ± 10 mm around the central position, except for very small vertical distances (below 4 mm) when larger deviations from the linearity can be expected. Also the normal force F_z shows a linear dependence for variations of the vertical displacement z in a range of some millimiters around any equilibrium position. A more rough approximation is the assumption the the lateral behaviour is uncoupled from the normal one, as the derivatives $\partial F_z/\partial y$ and $\partial F_y/\partial z$ vanish only for very small movements. In particular, while $\partial F_z/\partial y$ can be neglected for a lateral displacement of about ± 4 mm around the centered position, except when the magnets are very close to each other, the variation of the lateral force with normal displacements is not so small even for equilibrium positions with a distance between the magnets as large as 10 mm.

In the following analysis the force per unit length of "magnetic" rail will be assumed to follow the law

$$\left\{ \begin{array}{c} f_z \\ f_y \end{array} \right\} = \left[\begin{array}{cc} k_z & 0 \\ 0 & k_y \end{array} \right] \left\{ \begin{array}{c} w \\ v \end{array} \right\} \quad (1)$$

where displacements w and v in z and y direction are referred to the static equilibrium position.

The stiffnesses per unit length k_z and k_y are measured in N/m^2 and the latter is negative, as the passive suspension is intrinsically unstable. Note that coupling term k_{yz} must be set to zero within the frame of a linearized model as its dependence from the lateral displacement y is too strong to be neglected (when $y = 0$, $k_{yz} = 0$). The coupling between normal and laterlal forces will then be considered in the following analysis as a disturbance.

Another assumption is that all cross sections of the magnets with planes parallel to yz plane behave exactly in the same way, i.e. that no end effect on the levitation magnets is present. This is clearly more realistic in the case of the actual vehicle, with two continuous magnets running for the whole length, than for the platform supported by four relatively short magnets

The vehicle is sketched in figure 2: the levitation magnets are modelled as four straight lines of length l (M_1 to M_4), placed symmetrically with respect to the projection H of the center of gravity of the vehicle on the horizontal plane containing them. The electromagnets supplying the stabilization forces are assumed to act in points E_1 to E_4, located with the same symmetry but at a distance h' from such plane. Note that if $a_1 = 0$ a continuous magnetic support of the vehicle can be modelled.

The six generalized coordinates for the dynamic study of the vehicle are the x', y' and z' coordinates of point H with respect to the $x'y'z'$ inertial reference frame and the yaw, pitch and roll angles ψ, θ and φ taken in order as it is usual in vehicle dynamics [2].

The rotation matrix allowing to transform a vector from the vehicle-fixed frame $Hxyz$ to the inertial frame $x'y'z'$ is

$$[R] = [R_1][R_2][R_3] \quad (2)$$

where

$$[R_1] = \left[\begin{array}{ccc} \cos(\psi) & -\sin(\psi) & 0 \\ \sin(\psi) & \cos(\psi) & 0 \\ 0 & 0 & 1 \end{array} \right] \quad (3)$$

$$[R_2] = \left[\begin{array}{ccc} \cos(\theta) & 0 & \sin(\theta) \\ 0 & 1 & 0 \\ -\sin(\theta) & 0 & \cos(\theta) \end{array} \right] \quad (4)$$

$$[R_3] = \left[\begin{array}{ccc} 1 & 0 & 0 \\ 0 & \cos(\varphi) & -\sin(\varphi) \\ 0 & \sin(\varphi) & \cos(\varphi) \end{array} \right] \quad (5)$$

In the following all angles will be assumed to be small, which is clearly acceptable only in the case of straight rails, and the rotation matrix reduces to

$$[R] \approx \begin{bmatrix} 1 & -\psi & \theta \\ \psi & 1 & -\varphi \\ -\theta & \varphi & 1 \end{bmatrix}. \tag{6}$$

If the vehicle has to manage a bend with large radius the model can still be applied provided that inertia forces due to the curvature of the trajectory are added only if the characteristic times of the yaw dynamics are far shorter than those involved in the manouver. In this case angle ψ is the angle between x-axis and the direction of the tangent to the trajectory imposed by the rails. A deeper study of the effect of the lateral displacement due to the curved shape of the magnetic rails while the moving magnets retain their straight configuration is still needed.

Assuming that xz plane is a plane of symmetry for the vehicle, moments of inertia J_{xy} and J_{zy} vanish and the kinetic energy of the vehicle can be expressed as

$$\begin{aligned} \mathcal{T} &= \frac{1}{2}m\left(\dot{x}'^2 + \dot{y}'^2 + \dot{z}'^2\right) + \frac{1}{2}(J_x + mh^2)\dot{\varphi}^2 \\ &+ \frac{1}{2}(J_y + mh^2)\dot{\theta}^2 + \frac{1}{2}J_z\dot{\psi}^2 - J_{xz}\dot{\psi}\dot{\varphi} \\ &+ \dot{x}'mh\left(\dot{\theta} + \varphi\dot{\psi} + \psi\dot{\varphi}\right) + mh\dot{y}'\dot{\varphi} \end{aligned} \tag{7}$$

Note that terms $\dot{x}'\varphi\dot{\psi}$ e $\dot{x}'\psi\dot{\varphi}$ can be large enough to need to be taken into consideration even in a linearized model, as $\dot{x}'$ is the forward velocity of the vehicle.

The gravitational potential energy of the vehicle can be written as

$$\mathcal{U}_g = mg\left[z' + h\left(1 - \frac{\theta^2}{2} - \frac{\varphi^2}{2}\right)\right] \tag{8}$$

Note that the term containing the square of small angles in the series of the cosine must be retained in this case, as it yields linear terms in the equations of motion.

The displacement from the equilibrium position of a generic point belonging to line M_i at distance ξ from H can be written in the form

$$\left\{ \begin{array}{c} w(\xi) \\ v(\xi) \end{array} \right\}_i = [N(\xi)]_i \{x\}, \tag{9}$$

where matrix $[N(\xi)]$ and vector $\{x\}$ are respectively

$$[N(\xi)]_i = \begin{bmatrix} 0 & 0 & 1 & 0 & \pm\xi & \pm b \\ 0 & 1 & 0 & \pm\xi & 0 & 0 \end{bmatrix}$$

$$\{x\} = \{\ x' \quad y' \quad z' \quad \psi \quad \theta \quad \varphi\ \}^T$$

and the double signs are respectively $(-,+,+)$ for $i=1$, $(-,-,+)$ for $i=2$, $(+,+,-)$ for $i=3$ and $(+,-,-)$ for $i=4$.

The suspension magnets are modelled as linear distributed springs, whose potential energy referred to that characterizing the static equilibrium position is

$$\mathcal{U}_m = \frac{1}{2}\{x\}^T \sum_{i=1}^{4} \int_{a_1}^{a_2} [N(\xi)]_i^T \begin{bmatrix} k_z & 0 \\ 0 & k_y \end{bmatrix} [N(\xi)]_i d\xi \{x\} \tag{10}$$

By performing the integration, it is easy to verify that owing to the symmetry of the system, after adding the contributions of the four magnets, the stiffness matrix reduces to a diagonal. Introducing the stiffnesses $K_z = k_z l$ and $K_y = k_y l$, the stiffness matrix of the system reduces then to

$$[K] = \text{diag}\left(\ 0 \quad 4K_y \quad 4K_z \quad 4K_y\overline{d}^2 \quad 4K_z\overline{d}^2 \quad 4K_z b^2\ \right) \tag{11}$$

where $\overline{d}^2 = (a_1^2 + a_2^2 + a_1 a_2)/3$.

If the damping of the suspension, for instance due to eddy currents, which is however very small, is considered and modelled as viscous dampig, a Rayleigh dissipation function can be introduced

$$\mathcal{D}_m = \frac{1}{2}\{\dot{x}\}^T [C] \{\dot{x}\} \tag{12}$$

where the damping matrix C is the same as the stiffness matrix, provided that the damping coefficients C_z and C_y are substituted for the stiffnesses K_z and K_y.

Equations of motion

As it is usual for the linearized behaviour of vehicles possessing a plane of symmetry, the equations of motion uncouple in two distinct sets, one for "handling", involving the generalized coordinates for lateral motion y, yaw ψ and roll φ, and one for "comfort", involving the generalized coordinates for longitudinal motion x, vertical motion z and pitch θ. In the present case, as only straight running is considered at present, coordinates x' and y' can be used instead of x and y.

Comfort behaviour

By considering also a vector $\{F_{e1}\}$ of external forces and moments acting on the system (disturbances) and a set of control forces $\{F_{1c1}\}$ due to the control electromagnets, the equations governing the comfort behaviour of the vehicle are

$$[M_1]\{\ddot{q}_1\} + [C_1]\{\dot{q}_1\} + [K_1]\{q_1\} = \{F_{e1}\} + \{F_{c1}\} \tag{13}$$

where

$$\{q_1\} = \{\ x' \quad z' \quad \theta\ \}^T$$

$$[M_1] = \begin{bmatrix} m & 0 & mh \\ 0 & m & 0 \\ mh & 0 & J_y + mh^2 \end{bmatrix}$$

$$[C_1] = 4C_z \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & \overline{d}^2 \end{bmatrix}$$

$$[K_1] = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 4K_z & 0 \\ 0 & 0 & 4K_z\overline{d}^2 - mgh \end{bmatrix}$$

$$\{F_{e1}\} = \{\ F_{x_e} \quad F_{z_e} \quad M_{y_e} + hF_{x_e}\ \}^T$$

$$\{F_{c1}\} = \{\ F_{x_c} \quad F_{z_c} \quad M_{y_c}\ \}^T$$

Note that the external forces are applied in the center of gravity, as it is the case of forces due to the curvature of the trajectory, if an approximate study of the behaviour on a curved track is performed, or aerodynamic forces, while control forces are thought to be applied in point H. Note that in the case the control forces are due to electromagnets with their axis directed in y direction, vector $\{F_{1c}\}$ should vanish and the handling behaviour of the system is supposed uncontrolled. Actually their effect does not vanishes completely, usually due to misalignment, and their effect may be considered parasitic and modeled as an external disturbance.

Moreover, the second equation in (13) is uncoupled from the others, as the supporting devices have been assumed to be located symmetrically with respect to the center of mass. Also the first equation is weakly coupled. As the present study deals with the stability of the vehicle and not with running performances, the first equation will be solved in $m\ddot{x}$, which will be substituted in the third equation. The two following equations for heave and pitch motions are so obtained

$$m\ddot{z}' + 4C_z\dot{z}' + 4K_z z' = F_{z_e} \tag{14}$$

$$J_y\ddot{\theta} + 4C_z\overline{d}^2\dot{\theta} + \left(4K_z\overline{d}^2 - mgh\right)\theta = M_{y_e} \tag{15}$$

The system behaves in these modes as a simple mass-spring-damper system. The very low value of the damping however causes a strongly oscillatory behaviour, suggesting the use of either active or passive electromagnetic dampers in an actual application.

Handling behaviour

In the study of the handling of the vehicle the forward velocity and acceleration can be considered as stated functions of time. The set of equations for the handling behaviour is

$$[M_2]\{\ddot{q}_2\} + [C_2]\{\dot{q}_2\} + [K_2]\{q_2\} = \{F_{e2}\} + \{F_{c2}\} \tag{16}$$

where

$$\{q_2\} = \{\ y' \quad \varphi \quad \psi\ \}^T$$

$$[M_2] = \begin{bmatrix} m & -mh & 0 \\ -mh & J_x - mh^2 & -J_{xz} \\ 0 & -J_{xz} & J_z \end{bmatrix}$$

$$[C_2] = 4C_z\overline{d}^2 \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$[K_2] = \begin{bmatrix} 4K_y & 0 & 0 \\ 0 & 4K_z\overline{d}^2 - mgh & mh\dot{V} \\ 0 & mh\dot{V} & 4K_z\overline{d}^2 \end{bmatrix}$$

$$\{F_{e2}\} = \{\ F_{y_e} \quad M_{x_e} + hF_{y_e} \quad M_{z_e}\ \}^T$$

$$\{F_{c2}\} = \{\ F_{y_c} \quad M_{x_c} \quad M_{z_c}\ \}^T$$

Note that when running at constant speed ($\dot{V} = 0$) the only coupling of the yaw equation is through the inertia matrix J_{xz}: if yz is a plane of symmetry also for the distribution of masses the yaw behaviour is completely uncoupled from lateral and roll behaviour except for the control action, as this is the case.

The forces exerted by each pair of electromagnets are applied at a distance a_2 from the center gravity and the motion in the xy plane may be conveniently expressed in terms of the new coordinates directly measured by the (colocated) displacement sensors:

$$\begin{aligned} y_a &= y' + a_2\sin\psi \cong y' + a_2\psi \\ y_a &= y' + a_2\sin\psi \cong y' + a_2\psi \end{aligned}$$

in matricial form

$$\{q''\} = \begin{Bmatrix} y_a \\ y_b \end{Bmatrix} \cong \begin{bmatrix} 1 & a_2 \\ 1 & -a_2 \end{bmatrix} \begin{Bmatrix} y' \\ \psi \end{Bmatrix} = [T]\{q'\} \tag{17}$$

Note that when the sensors are aligned to the y axis the roll motion is not only completely uncontrollable but also completely unobservable. Of course, the model to be used for the control design will be reduced to the completely observable and controllable part, i.e. neglecting the roll motion. On the contrary, a given misalignment of both sensors and actuators may be used to control roll motion acting on the bias command.

Electromechanical model

Limiting the mechanical degrees of freedom to those completely controllable and observable, the expressions of the energies stored in the system reduce to

$$\begin{aligned} \mathcal{T} &= \frac{1}{2}\{\dot{q}''\}^T[T^{-1}]^T \begin{bmatrix} m & 0 \\ 0 & J_z \end{bmatrix} [T^{-1}]\{\dot{q}''\} \\ &= \frac{1}{2}\begin{Bmatrix} y_a \\ y_b \end{Bmatrix}^T \begin{bmatrix} m + \frac{J_z}{a_2^2} & m - \frac{J_z}{a_2^2} \\ m - \frac{J_z}{a_2^2} & m + \frac{J_z}{a_2^2} \end{bmatrix} \begin{Bmatrix} y_a \\ y_b \end{Bmatrix} \end{aligned} \tag{18}$$

$$\begin{aligned} \mathcal{U}_m &= \frac{1}{2}\{q''\}^T[T^{-1}]^T \sum_{i=1}^{4}\int_{a_1}^{a_2} \begin{bmatrix} 1 \\ \pm\xi \end{bmatrix} k_y \begin{bmatrix} 1 & \pm\xi \end{bmatrix} d\xi [T^{-1}]\{q''\} \\ &= \frac{1}{2}\begin{Bmatrix} y_a \\ y_b \end{Bmatrix}^T 4K_y \begin{bmatrix} 1 + \frac{\overline{d}^2}{a_2^2} & 1 - \frac{\overline{d}^2}{a_2^2} \\ 1 - \frac{\overline{d}^2}{a_2^2} & 1 + \frac{\overline{d}^2}{a_2^2} \end{bmatrix} \begin{Bmatrix} y_a \\ y_b \end{Bmatrix} \end{aligned} \tag{19}$$

The magnetic energy stored in the electromagnet is expressed using charges as generalized coordinates, i.e currents as generalized velocities

$$\{\dot{q}'''\} = \{\ \dot{q}_1 = i_1 \quad \dot{q}_2 = i_2 \quad \dot{q}_3 = i_3 \quad \dot{q}_4 = i_4\ \}^T$$

and then

$$\mathcal{W}_m = \frac{1}{2}\{\dot{q}'''\}^T\Gamma\,\mathrm{diag}\left(\ \tfrac{1}{d+y_a} \quad \tfrac{1}{d-y_a} \quad \tfrac{1}{d+y_b} \quad \tfrac{1}{d-y_b}\ \right)\{\dot{q}'''\} \tag{20}$$

where d is the nominal air gap, $\Gamma = \frac{\mu_0 N^2 A}{2}$ summarizes the magnetic properties of the coil (N number of turns, A flux section).

The dissipation due to the resistance R of the coils is represented as

$$\mathcal{D}_m = \frac{1}{2}\{\dot{q}'''\}^T \operatorname{diag}\left(R \quad R \quad R \quad R \right)\{\dot{q}'''\} \qquad (21)$$

Following the standard lagrangian approach the governing differential equations are obtained and linearization about the nominal mid-gap position d and bias current i_0 yields the following dynamic equation

$$[M]\{\ddot{q}\} + [D+G]\{\dot{q}\} + [K]\{q\} = [S_f]\{f\} \qquad (22)$$

where

$$\{q\} = \{\ y_a \quad y_b \quad q_1 \quad q_2 \quad q_3 \quad q_4\ \}^T$$

$$[M] = \begin{bmatrix} m + \frac{J_z}{a_2^2} & m - \frac{J_z}{a_2^2} & \mathbf{0}_{2\times4} \\ m - \frac{J_z}{a_2^2} & m + \frac{J_z}{a_2^2} & \\ \multicolumn{2}{c}{\mathbf{0}_{4\times2}} & L\,\mathbf{I}_{4\times4} \end{bmatrix}$$

$$[D] = \operatorname{diag}\left(\ 0 \quad 0 \quad R \quad R \quad R \quad R\ \right)$$

$$[G] = \begin{bmatrix} \multicolumn{2}{c}{\mathbf{0}_{2\times2}} & k_i & -k_i & 0 & 0 \\ & & 0 & 0 & k_i & -k_i \\ -k_i & 0 & & & & \\ k_i & 0 & & \mathbf{0}_{4\times4} & & \\ 0 & -k_i & & & & \\ 0 & k_i & & & & \end{bmatrix}$$

$$[K] = \begin{bmatrix} k_1 & k_2 & \mathbf{0}_{2\times4} \\ k_2 & k_1 & \\ \multicolumn{2}{c}{\mathbf{0}_{4\times2}} & \mathbf{0}_{4\times4} \end{bmatrix}$$

$$k_1 = 4K_y\left(1 + \frac{\bar{d}^2}{a_2^2}\right) - k_b$$

$$k_2 = 4K_y\left(1 - \frac{\bar{d}^2}{a_2^2}\right)$$

$$[S_f] = \begin{bmatrix} T & \\ & \mathbf{I}_{4\times4} \end{bmatrix}$$

$$\{f\} = \{\ F_y \quad M_z \quad v_1 \quad v_2 \quad v_3 \quad v_4\ \}^T$$

with $L = \frac{\Gamma}{d}$ the coil inductance with nominal air gap, $k_i = L\frac{i_0}{d}$ the current to force constant, $k_b = k_i\frac{i_0}{d}$ the negative stiffness introduced by the electromagnets driven by the bias current, that is behaving has permanent ones and summing up their contributions. It may be interesting to note the skew-symmetry of the $[G]$ "gyroscopic" matrix for the back electromotive force (potential) in the coils and the symmetry of the $[D]$ "damping" matrix for the coils resistance.

A suitable choice of state variables allows to decouple the system model into a pure electrical part governing the bias dynamics (z_{el}) and an electromechanical part driven by the voltage differences on each magnet and whose current states are the deviation from the bias (equilibrium) current (z_{elm}):

$$\left\{ \begin{matrix} \{z_{elm}\} \\ \{z_{el}\} \end{matrix} \right\} = \left\{ \begin{matrix} \{(\dot{q}_1 - \dot{q}_2) \quad (\dot{q}_3 - \dot{q}_4) \quad \dot{y}_a \quad \dot{y}_b \quad y_a \quad y_b\}^T \\ \{\ (\dot{q}_1 + \dot{q}_2) \quad (\dot{q}_3 + \dot{q}_4)\ \}^T \end{matrix} \right\}^T \qquad (23)$$

Control design and experimental results

In order to assure the so-called "virtual zero power consumption" (as quoted by Vischer and Bleuler [3])the state space is augmented with the states describing the cumulative error of the deviation currents from zero (z_{error}), i.e. each current from its bias value:

$$\{z_{error}\} = \{\ \textstyle\int(R_s i_1 - R_s i_2) \quad \int(R_s i_3 - R_s i_4)\ \}^T \qquad (24)$$

where R_s is the shunt resistor used to measure the coil currents. In practice, when an external load is applied to the system the control will try to keep the current deviation to zero counteracting the external force with that produced by the permanent magnets and by the bias command moving the system toward the incoming disturbance. The augmented system is an application example of a far more general theory named by Francis and Wonham [4] after the "internal model" of the disturbance: in this case a constant (step) function modelling the load, $\frac{1}{s}$ in the frequency domain, $\int$ in the time domain.

Equation (24) may be conveniently re-written in term of the general coordinates

$$\begin{aligned} \{z_{error}\} &= R_s\{\ \textstyle\int(\dot{q}_1 - \dot{q}_2) \quad \int(\dot{q}_3 - \dot{q}_4)\ \}^T \\ &= R_s\{\ (q_1 - q_2) \quad (q_3 - q_4)\ \}^T \end{aligned} \qquad (25)$$

and thus chosing the following as state space variables for the electromechanical part of the system

$$\{z\} = \{(\dot{q}_1 - \dot{q}_2)\ (\dot{q}_3 - \dot{q}_4)\ \dot{y}_a\ \dot{y}_b\ (q_1 - q_2)\ (q_3 - q_4)\ y_a\ y_b\}^T \qquad (26)$$

and the following as control inputs

$$\{u\} = \{\ (v_1 - v_2) \quad (v_3 - v_4)\ \}^T \qquad (27)$$

Since the current is defined as a charge variation per unit time, the cumulative error, i.e. the integral, of the deviation currents is coherently trasduced in charge quantity, to be kept to zero to achieve the stated objective of zero power consumption.

The state space matrices characterizing the linear behaviour of the electromechanical part of the system are the following

$$\{\dot{z}\} = [A]\{z\} + [B]\{u\} \qquad (28)$$

$$\{y\} = [C]\{z\} \qquad (29)$$

where

$$[A] = \begin{bmatrix} \frac{R}{L} & 0 & \frac{2k_i}{L} & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{R}{L} & 0 & \frac{2k_i}{L} & 0 & 0 & 0 & 0 \\ -\frac{k_i}{m_1} & \frac{k_i}{m_2} & 0 & 0 & 0 & 0 & \frac{k_1}{m_1} - \frac{k_2}{m_2} & \frac{k_2}{m_1} - \frac{k_1}{m_2} \\ \frac{k_i}{m2} & -\frac{k_i}{m_1} & 0 & 0 & 0 & 0 & \frac{k_2}{m_1} - \frac{k_1}{m_2} & \frac{k_1}{m_1} - \frac{k_2}{m_2} \\ \mathbf{I}_{4\times4} & & & & \mathbf{0}_{4\times4} & & & \end{bmatrix}$$

$$[B] = \frac{1}{2}\frac{K_a}{L}\begin{bmatrix} \mathbf{I}_{2\times2} \\ \mathbf{0}_{6\times2} \end{bmatrix}$$

$$[C] = \begin{bmatrix} & R_s & 0 & 0 & 0 \\ \mathbf{0}_{4\times4} & 0 & R_s & 0 & 0 \\ & 0 & 0 & K_s & 0 \\ & 0 & 0 & 0 & K_s \end{bmatrix}$$

with

$$\frac{1}{m_1} = \frac{m + \frac{J_z}{a_2^2}}{\left(m + \frac{J_z}{a_2^2}\right)^2 - \left(m - \frac{J_z}{a_2^2}\right)^2}$$

$$\frac{1}{m_1} = \frac{m - \frac{J_z}{a_2^2}}{\left(m + \frac{J_z}{a_2^2}\right)^2 - \left(m - \frac{J_z}{a_2^2}\right)^2}$$

A pole placement design method has been applied to obtain the gain to be applied to the state feedback for the electromechanical part (ref. [5] contains also a simplified description on the usage of the internal model theory). A state observer is used to obtain the velocities and to filter the position measures while the deviation currents are directly measured by means of shunt resistors (fig. 3). To note the fact that using an observer to filter the current measures would transform the integral control into a lag compensation. The purely electrical part is internally fedback analogically (high gain).

An example of the system response under a 100 N lateral load is reported in figure 4.

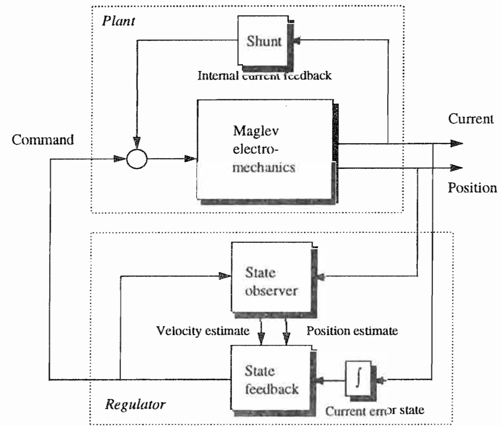

Figure 3: Control layout.

Acknowledgement

The present research was performed under the sponsorship of *Progetto Finalizzato Trasporti 2* of the Italian National Research Council.

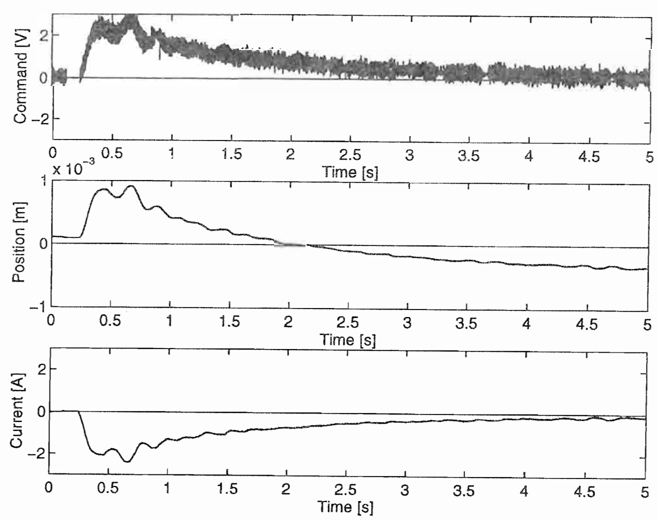

Figure 4: From above command, position and current of a single axis under constant lateral load: after a transient the position stabilizes "against" the load.

References

[1] F. Di Majo and G. Sciarrone, *Le altissime velocità nel futuro dei trasporti guidati.* Centro Studi sui Sistemi di Trasporto, Roma, 1990.

[2] G. Genta, *Meccanica dell'autoveicolo.* Torino: Levrotto & Bella, 1993.

[3] D. Vischer and H. Bleuler, "Self-sensing active magnetic levitation," *IEEE Trans. on Magnetics*, vol. 29, pp. 1276–1281, March 1993.

[4] B. A. Francis and W. M. Wonham, "The internal model principle of control theory," *Automatica*, vol. 12, no. 5, pp. 457–465, 1976.

[5] G. Franklin, J. Powell, and A. Emani-Naeini, *Feedback Control of Dynamic Systems.* Addison-Wesley, 1991, 1993.