Stabilization of Elastic Rotors with Fluid Components by Magnetic Bearings

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It is wellknown that fluid components in rotating systems may cause instabilities. This paper deals with the problem of mastering these instabilities by applying magnetic bearings and developing suitable control concepts. In the first part, we focus on the mathematical description of the coupled fluid-rotor system in a way to making it possible to carry out control design. The results already obtained show that instabilities caused by the rotor-fluid interactions can be avoided by applying appropriate control forces generated by a magnetic bearing. The design of more sophisticated control concepts is still in progress. For verifying the mathematical model of the system as well as the effectiveness of the developed control concepts, a test rig is introduced.

1 Introduction

Industrial centrifuges and separators often work with only partially filled tanks. Hence, there are couplings between the motions of the rotor and the motions of the rotating fluid. These couplings lead to a flow of energy into the rotor's bending motions, thereby absorbing rotational energy provided by the motor, which in turn causes under certain operating conditions unstable oscillations of the rotor with violently increasing amplitudes.

There have been several investigations dealing with this phenomenon both theoretically and experimentally ([1], [3], [5]). They differ concerning the modelling of the boundary conditions of the rotor, the tank's shape and the consideration or neglecting of damping and viscosity. The major result common to all investigations is the detection of unstable oscillations within a certain range of speed which cannot be removed by applying external and passive damping.

There are only a few papers dealing with active control to stabilize a rotor partially or completely filled with liquid ([2], [4]). HENDRICKS [2] theoretically investigates linear control techniques to avoid such instabilities. MATSUSHITA [4] considers a rotor completely filled with liquid.

Almost all of the previous examinations are restricted to the calculation of the regions of instability whereas in this investigation, the equations of motion of the rotor-fluid-system are derived with respect to an easy, subsequent application of control techniques. Furthermore, the unstable oscillations of the centrifuge shall be removed using active control.

2 Modelling and Equations of Motion

2.1 Rotor

Figure 1 shows a model of the centrifuge. It consists of a shaft (elasticity c_{ij}) running in isotropic self-aligning ball bearings (stiffness c_1 , c_2). The casing which is assumed to be rigid, has a cylindrical tank and is mounted on top of the shaft in overhung position. Gravitational forces are neglected.

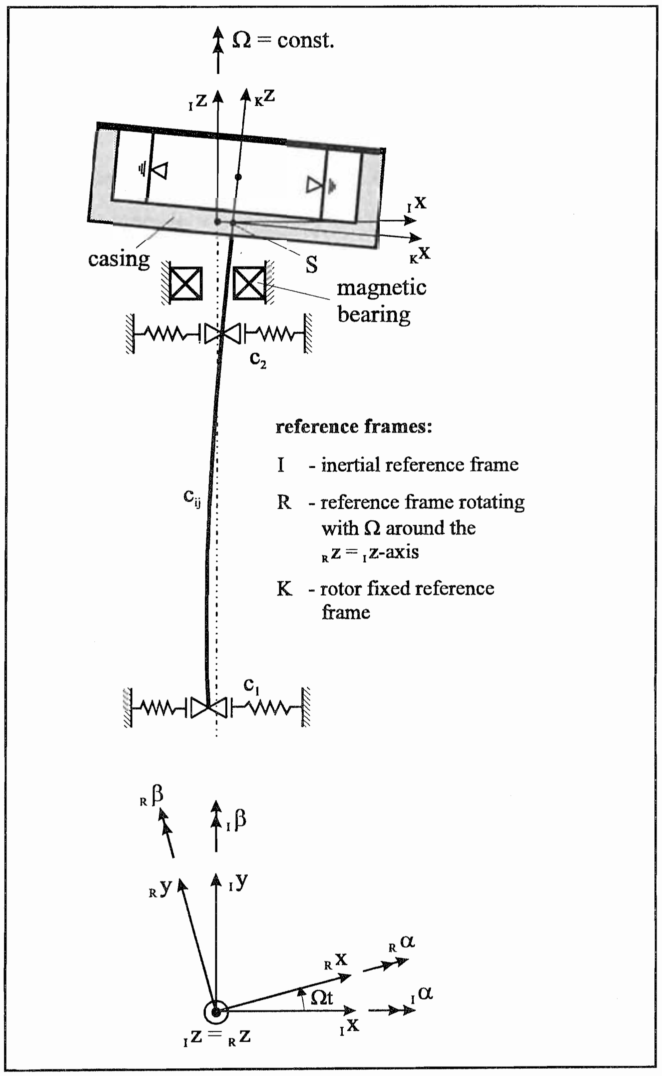

Fig. 1: Model of the centrifuge

The casing's displacements in linear approximation can be completely described by the displacement of the centre of gravity and the angular deflections. Thus, the vector $\boldsymbol{q}$, containing the generalized co-ordinates of the casing, can be written as

$$\boldsymbol{q} = [x \quad y \quad \alpha \quad \beta]^T . \tag{1}$$

The equations of motion may be developed by applying D'ALEMBERT's principle. With respect to section 2.2 (modelling of the liquid), they are described using a reference system of co-ordinates (R) which rotates with the rotor's angular velocity Ω (see fig. 1):

$$\boldsymbol{M}\,{}_R\ddot{\boldsymbol{q}} + \boldsymbol{P}\,{}_R\dot{\boldsymbol{q}} + \boldsymbol{Q}\,{}_R\boldsymbol{q} = {}_R\boldsymbol{F}_u + {}_R\boldsymbol{F}_F \ . \tag{2}$$

${}_R\boldsymbol{F}_u$ represents the vector of forces caused by a small, static unbalance ε and ${}_R\boldsymbol{F}_F = [F_{Fx} \quad F_{Fy} \quad M_{Fx} \quad M_{Fy}]^T$ contains the resulting forces and moments caused by the motions of the liquid.

2.2 Liquid

The liquid is supposed to be incompressible and homogeneous. Viscosity is neglected, i.e., the forces resulting from shearing tension on the case's surface are small compared to the forces resulting from liquid pressure. In dynamic equilibrium, the liquid has a cylindrical free surface and is rotating with the tank's angular velocity Ω .

Using EULER'S hydrodynamic equations, the equations of motion in casing-fixed cylindrical co-ordinates (Z) can be written as

$$\begin{aligned} &{}_Z\ddot{\boldsymbol{r}} + 2\,{}_Z(\omega \times \dot{\boldsymbol{r}}) + {}_Z\,\dot{\omega} \times {}_Z\boldsymbol{r} + {}_Z\omega \times {}_Z(\omega \times \boldsymbol{r}) \\ &+\boldsymbol{A}_{ZI}\ {}_I\ddot{\boldsymbol{r}}_s = {}_Z\left\{ -\mathrm{grad}\left(\frac{p - p_0}{\rho_F} \right) \right\}, \end{aligned} \tag{3}$$

where ${}_Z\boldsymbol{r} = [r \quad \varphi \quad z]^T$, ${}_I\ddot{\boldsymbol{r}}_s$ contains the acceleration of the centre of gravity calculated in co-ordinates of the inertial reference frame, and $\boldsymbol{A}_{ZI}$ represents the transformation matrix from inertial co-ordinates to cylindrical co-ordinates. Furthermore, the equation of continuity

$$\mathrm{div}\,({}_Z v) = {}_Z\left\{ \frac{\partial v_r}{\partial r} + \frac{1}{r} v_r + \frac{1}{r}\frac{\partial v_\varphi}{\partial \varphi} + \frac{\partial v_z}{\partial z} \right\} = 0 \tag{4}$$

has to be satisfied.

The liquid is bounded by the case's surface. Hence, the normal velocities of the liquid disappear at the casing's surface. This leads to the following boundary conditions:

- curved boundary : ${}_Z v_r = 0$ (5.1)
- plane boundary (top) : ${}_Z v_z = 0$ (5.2)
- plane boundary (bottom) : ${}_Z v_z = 0$ (5.3)

The radial deflection of the free surface of the liquid is supposed to be

$${}_Z r = b + \eta(\varphi, z, t) \tag{6}$$

where b is the radius of the free surface in equilibrium (see Figure 2) and η describes the small deviations from equilibrium. This leads to the kinematical boundary condition

$${}_Z v = \frac{\partial \eta}{\partial t} \ . \tag{7}$$

Furthermore, the pressure on the free boundary equals the ambient pressure:

$$p({}_Z\boldsymbol{r}_0, t) - p_0 = 0 \ . \tag{8}$$

(7) and (8) can be comprised to only one boundary condition for the free surface, thus eliminating the unknown function η (φ, z, t).

To get the complete set of equations of motion, it is required to interpret the equation of EULER (determination of the kinematical relations). The forces F_{Fx} and F_{Fy} in equation (2) result from the integration of the fluid pressure perpendicular to the surface of the tank:

$${}_Z[F_{Fx} \quad F_{Fy} \quad F_{Fz}]^T = {}_Z\left\{ \int_A \boldsymbol{n}\,p\,dA \right\}, \tag{9}$$

where $\boldsymbol{n}$ is the normal vector onto the considered element of the surface of the tank (using cylindrical co-ordinates). Analogously, the moments resulting from the motions of the liquid are determined by

$${}_Z[M_{Fx} \quad M_{Fy} \quad M_{Fz}]^T = {}_Z\left\{ \int_A (\boldsymbol{r} \times \boldsymbol{n})\,p\,dA \right\}, \tag{10}$$

where ${}_Z\boldsymbol{r}$ represents the vector from the origin of the cylindrical system of co-ordinates to the considered element of the tank's surface.

Finally, the complete set of equations of motion consists of

- 4 equations concerning the rotor (2),
- hydrodynamic equations of EULER (3) in interpreted form,
- 4 boundary conditions.

It contains ordinary differential equations as well as partial differential equations.

A solution of the equations of motion requires the elimination of time by assuming that all variables are proportional $e^{\lambda t}$. After several calculations using the equation of continuity and of EULER, one finally gets the liquid pressure in terms of a transformed function Q:

$$Q = Q_0 + \sum_{i=1}^{\infty} Q_i \ . \qquad (11)$$

The term Q_0 describes the horizontal motions of the liquid (in radial and circumference direction), while the Q_i arise from the flow in axial direction. Several researches have shown that the influence of Q_i to the regions of instability is infinitely small ([1] and [3]), so it will be neglected for further considerations.

The function Q_0 can be written as

$$Q_0 = \left(K_1 r + K_2 \frac{1}{r} \right) (K_3 \cos\varphi + K_4 \sin\varphi) \ . \qquad (12)$$

The coefficients K_1, K_2, K_3 and K_4 have to be fitted to the boundary conditions. The K_i have no constant values, but they depend on λ and thus on the actual state of motion:

$$K_i = K_i(\lambda) = \frac{Z_i(\lambda)}{N(\lambda)} \ . \qquad (13)$$

This clearly shows that the function Q_0 is frequency-dependent. Substitution of (13) into (12) allows to calculate the pressure $p(\lambda)$ from $Q_0(\lambda)$ and thus the frequency-depending fluid forces and moments, using (9) and (10), respectively. After changing from frequency domain to time domain, one finally obtains the ordinary differential equation (ODE) for calculating the forces and moments, using co-ordinates of the rotating R-system:

$$\ddot{\boldsymbol{F}}_F + \boldsymbol{A}_F \dot{\boldsymbol{F}}_F + \boldsymbol{B}_F \boldsymbol{F}_F = \boldsymbol{C}_4 \boldsymbol{q}^{(4)} + \boldsymbol{C}_3 \dddot{\boldsymbol{q}} + .. + \boldsymbol{C}_0 \boldsymbol{q} \ . \qquad (14)$$

The (4x4)-matrices $\boldsymbol{A}_F$, $\boldsymbol{B}_F$, $\boldsymbol{C}_4$, ...,$\boldsymbol{C}_0$ depend on the filling ratio f, the angular velocity Ω, the mass m_F of the completely filled tank, the tank's diameter and the distance s between the tank's centre and the centre of mass S (see Figure 2).

2.3 Complete System

In order to study the stability of the equilibrium position of the liquid-containing centrifuge, the roots of the equations describing the whole system have to be determined. Positive real parts of the roots indicate instabilities, i.e., bounded perturbations lead to unbounded amplitudes.

Prior to calculating the roots of the system equations, the equations of the rotor and the liquid have to be combined and formulated as one set of state equations. Using (2), the fluid forces and moments can be written as

$$\boldsymbol{F}_F = \boldsymbol{M}\ddot{\boldsymbol{q}} + \boldsymbol{P}\dot{\boldsymbol{q}} + \boldsymbol{Q}\boldsymbol{q} - \boldsymbol{F}_u \ . \qquad (15)$$

Substituting (15) into (14) and considering that $\boldsymbol{F}_u$ is constant in rotating co-ordinates, the combined equations can be written as

$$\boldsymbol{D}_4 \boldsymbol{q}^{(4)} + \boldsymbol{D}_3 \dddot{\boldsymbol{q}} + \boldsymbol{D}_2 \ddot{\boldsymbol{q}} + \boldsymbol{D}_1 \dot{\boldsymbol{q}} + \boldsymbol{D}_0 \boldsymbol{q} = \boldsymbol{B}_F \boldsymbol{F}_u \ , \qquad (16)$$

where

$$\left.\begin{aligned} \boldsymbol{D}_4 &= \boldsymbol{M} - \boldsymbol{C}_4 \\ \boldsymbol{D}_3 &= \boldsymbol{P} + \boldsymbol{A}_F \boldsymbol{M} - \boldsymbol{C}_3 \\ \boldsymbol{D}_2 &= \boldsymbol{Q} + \boldsymbol{A}_F \boldsymbol{P} + \boldsymbol{B}_F \boldsymbol{M} - \boldsymbol{C}_2 \\ \boldsymbol{D}_1 &= \boldsymbol{A}_F \boldsymbol{Q} + \boldsymbol{B}_F \boldsymbol{P} - \boldsymbol{C}_1 \\ \boldsymbol{D}_0 &= \boldsymbol{B}_F \boldsymbol{Q} - \boldsymbol{C}_0 \end{aligned}\right\} . \qquad (17)$$

Using

$$\dot{q} = q_1 \ ; \quad \dot{q}_1 = q_2 \ ; \quad \dot{q}_2 = q_3 \qquad (18)$$

and introducing the vector

$$z = [q \quad q_1 \quad q_2 \quad q_3]^T \ , \qquad (19)$$

one obtains the equations of motion of the complete, partially liquid-filled centrifuge:

$$\dot{z} = \boldsymbol{A} z + \boldsymbol{h}_u \qquad (20)$$

where

$$\boldsymbol{A} = \begin{bmatrix} \boldsymbol{0} & \boldsymbol{E} & \boldsymbol{0} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{E} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{E} \\ -\boldsymbol{D}_4^{-1}\boldsymbol{D}_0 & -\boldsymbol{D}_4^{-1}\boldsymbol{D}_1 & -\boldsymbol{D}_4^{-1}\boldsymbol{D}_2 & -\boldsymbol{D}_4^{-1}\boldsymbol{D}_3 \end{bmatrix} \qquad (21)$$

and

$$\boldsymbol{h}_u = \begin{bmatrix} \boldsymbol{0} \\ \boldsymbol{0} \\ \boldsymbol{0} \\ \boldsymbol{D}_4^{-1}\boldsymbol{B}_F \boldsymbol{F}_u \end{bmatrix} . \qquad (22)$$

This state space representation allows an easy application of familiar methods for numerical evaluation. Thus, it is suitable for the determination of the regions of instability as well as for the application of control techniques. Furthermore, the vector $z(t)$ can be evaluated. Use of (14) allows to calculate the forces and moments depending on the motions of the liquid.

3 Simulations

The simulations of the system are based upon the set of data shown in Figure 2.

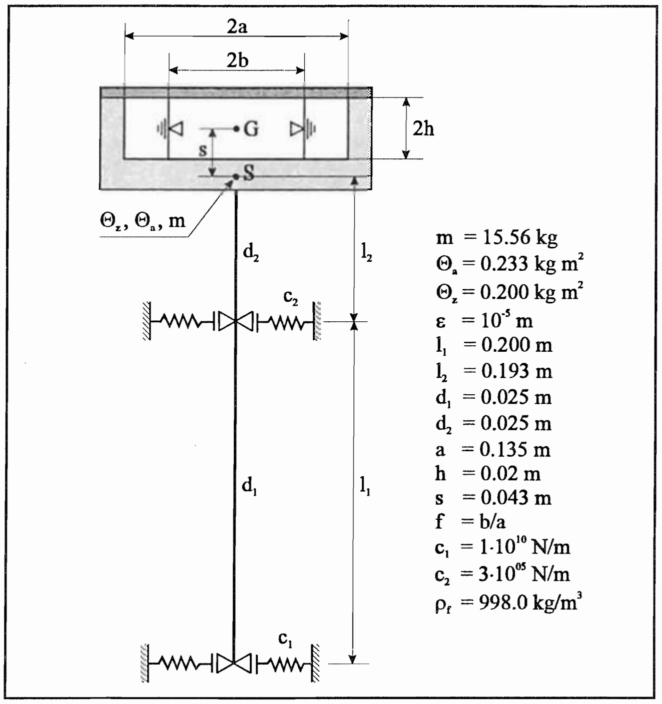

Fig. 2: Data for simulation

3.1 Eigenvalues

For the use of partially filled centrifuges, it is crucial to know the location of the instability regions. Consequently, the eigenvalues λ_i of the system are calculated from

$$\det(A-\lambda E)\overset{!}{=}0 \ , \qquad (23)$$

using (21). The filling ratio f varies from 0 to 1, the rotational speed Ω from 60 1/s to 140 1/s. The maximum of the real parts of eigenvalues is assigned to the corresponding pair of parameters (Ω, $1\text{-}f^2$). The result is shown in Figure 3.

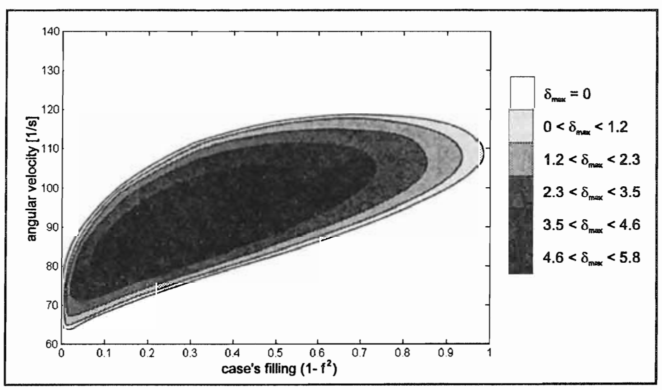

Fig. 3: Regions of instability

There exists a wide speed-range containing eigenvalues with positive real part, i.e., a wide range of parameters causing instabilities. For the given configuration, the worst parameter combination is found to be

$$\Omega \quad = \quad 92 \text{ 1/s}$$

$$1\text{-}f^2 \quad = \quad 0.26 \ ,$$

showing the maximum real part of all eigenvalues,

$$\delta \quad = \quad +5.78 \text{ 1/s} \ .$$

Choosing these parameters, the amplitude increases by factor 18 within 0.5 s (i.e. after 7 revolutions). This example not only shows the danger arising from the instabilities, but also reveals the necessity to take action against them using active control.

3.2 Characteristic behaviour of the centrifuge

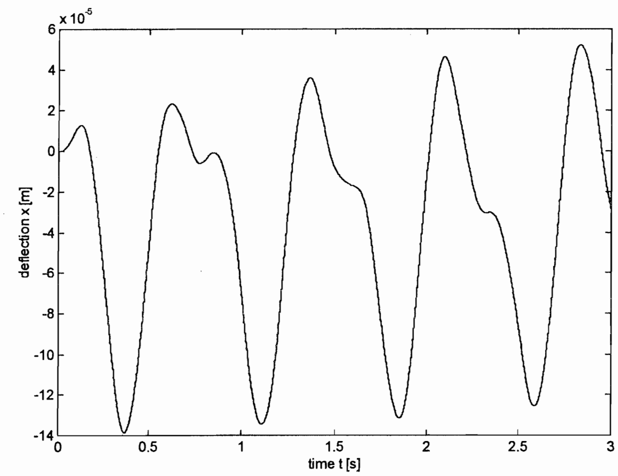

Fig. 4 : Development of ${}_R x$ (Ω = 70.0 1/s; $1\text{-}f^2$ = 0.26)

Figure 4 shows the displacement ${}_R x$ of the centre of mass (indicated by S - see Figure 1) for a set of parameters belonging to stable oscillations.

Obviously, this is the superposition of two oscillations. The corresponding frequencies, which are calculated for the system represented in the rotating reference frame R, can be identified by a FFT (see Figure 5):

$$f_1 \quad = \quad 1.4 \text{ Hz},$$

$$f_2 \quad = \quad 2.6 \text{ Hz}.$$

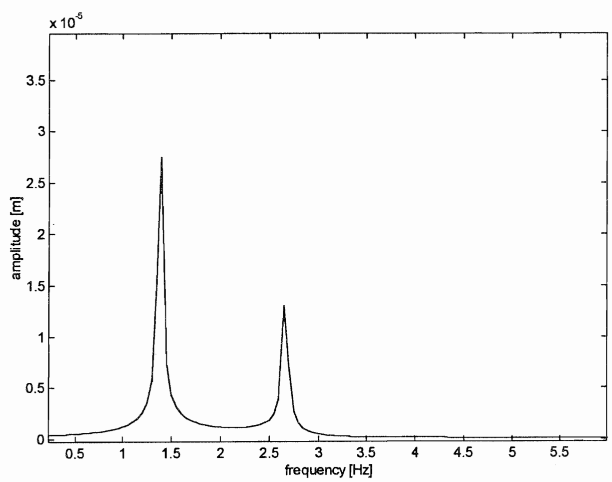

Fig. 5 : FFT of ${}_R x$ shown in Figure 4 (Ω = 70.0 1/s; $1\text{-}f^2$ = 0.26)

This result also holds for the other degrees of freedom. Hence, the coupling between rotor and liquid affects only two modes of vibration while the others remain unaffected.

4 Active Control

The design of linear and non-linear controllers has been treated very detailed in literature. There are numerous methods and concepts for the design and realization of controllers. The choice of a concept is determined by the aim and the dynamics of the regulated structure.

Concerning the partially filled centrifuge, the aim consists in removing unstable regions. Applying control forces which are generated by the magnetic bearing, (20) can be written as

$$\dot{z} = \boldsymbol{A}z + \boldsymbol{Bu} \tag{24}$$

where $\boldsymbol{B}$ represents the control matrix (depending on location and type of the actuator) and $\boldsymbol{u}$ is the control vector. The vector of unbalance $\boldsymbol{h}_u$ has been neglected for control design.

To check the controllability of the system which depends mainly on the choice of the magnetic bearing arrangement, a state feedback controller is used. In order to guarantee the stability of the system in the whole range of rotating speed, the rotor frequency Ω has to be taken into account for the control design. Applying digital concepts, this can be realized by dividing the rotational speed range into subsections $\Delta\Omega_i$ whose size depends on the actual operating point. Thus, the control vector $\boldsymbol{u}$ can be written as

$$\boldsymbol{u} = -\boldsymbol{K}(\Delta\Omega_i)z \tag{25}$$

with the feedback gain matrix $\boldsymbol{K}(\Delta\Omega_i)$. The determination of $\boldsymbol{K}$ has to be carried out for each subsection of rotational speed $\Delta\Omega_i$ by optimizing the quadratic integral criterion

$$J = \int_0^\infty (z^T \boldsymbol{Q} z + \boldsymbol{u}^T \boldsymbol{R} \boldsymbol{u})\, dt \tag{26}$$

with the weighting matrices $\boldsymbol{Q}$ and $\boldsymbol{R}$. The determination of these matrices depends on the different sizes of $\Delta\Omega_i$ which in turn have to be chosen with respect to the stability of the system.

In order to show the effect of the active control, the displacement ${}_Rx$ has been calculated for the most unstable point ($\Omega = 92$ 1/s, $1\text{-}f^2 = 0.26$) (see Figure 6). Starting from zero initial condition, the amplitude quickly increases giving evidence of the system's instability. At time t = 1 s, the controller is initiated and subsequently, asymptotical stability is imposed.

Of course, state feedback control is usually not technically practicable. In this investigation, the design is just carried out to check the technical feasibility and to evaluate the quality of controllers of simpler structure, such as an output controller. The design of more realistic control concepts is still in progress. The numerical results have to be verified experimentally.

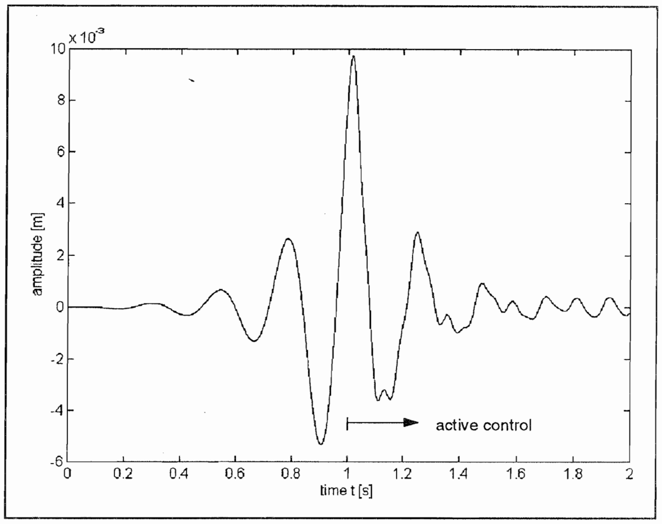

Fig. 6 : Amplitude ${}_Rx$ without / with active control

5 Test Rig

In order to verify the theory and different control concepts, a test rig has been designed (see Figure 7 and Figure 8).

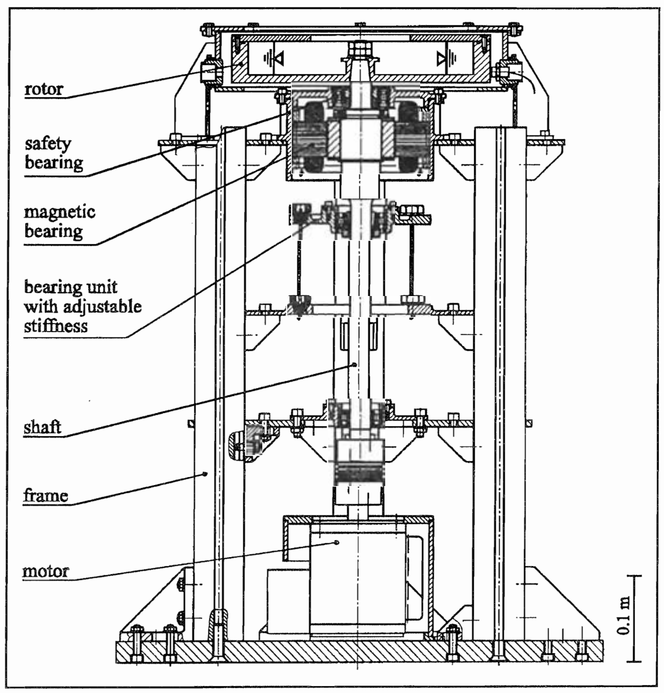

Fig. 7 : Model of the test rig

The rotor is mounted on a shaft and driven by a motor which is speed-controlled in order to maintain a constant rotor speed. The shaft is supported by two roller bearings one of which has an adjustable stiffness. To limit the shaft deflection in case of a controller failure, a safety bearing is located near the top of the shaft. For the application of control forces, an active magnetic bearing is placed below the rotor near the point of largest shaft deflection. The whole assembly is mounted in a frame in order to allow easy access.

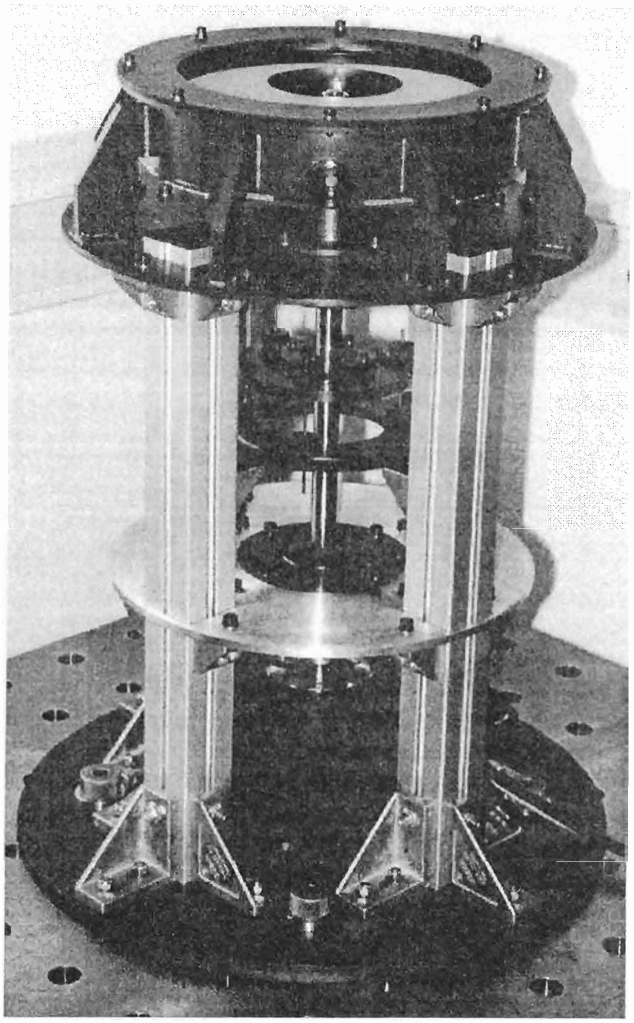

Fig. 8 : The designed test rig

6 Summary

It has been shown that centrifuges partially filled with liquid have wide regions of instabilities. These instabilities can be calculated from the equations of motion in time domain. This also allows to determine the displacements and deflections of the centrifuge as well as the forces and moments resulting from the flow of the liquid.

Introducing active control, significant improvements can be obtained providing asymptotical stability for formerly unstable parameter combinations. Further investigations will focus on the development of more sophisticated control strategies such as:

- adaptive control which is self-adjusting depending on the rotor speed Ω and the filling ratio f,
- adaptive control in combination with a disturbance observer for estimating the fluid forces and
- learning control for further optimization of the dynamics while the rotor system is running.

All of these investigations are in progress both numerically and experimentally.

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