Anisotropic Stiffness Effect for Improving Stability of Two-Axes Controlled Rotor by Magnetic Bearings

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Abstract:
This paper describes a stability analysis of anisotropic bearing stiffness effect for improving damping characteristics of a two-axes controlled rotor by magnetic bearings. The method of averaging is applied for simplifying the mathematical model of tilting motion of the rotor. The anisotropic stiffness effect is found to be efficient for improving the damping characteristics but it depends on the ratio of moments of inertia of the rotor.

1 INTRODUCTION

The use of a magnetic bearing system provides many remarkable benefits such as friction-free rotation, reduction of energy consumption, long life time, and high reliability. Recently research and development for introducing a magnetic bearing system into a industrial machinery system becomes increasingly popular. Five types of magnetic suspension utilizing attractive force between stator and rotor can be categorized according to the actively controlled degrees of freedom. Each type has its own advantages and disadvantages. Generally speaking, complexity in the control electronics increases with the number of the actively controlled degrees of freedom (hereafter referred to as "axis"). Of the five types , the two-axes controlled magnetic bearing system as well as the one-axis controlled bearing system has attracted intensive attention due to its simplicity and compactness of structural configuration[1,2].

As shown schematically in Fig.1 a two-axes controlled magnetic bearing system contains the active control along the radial two axes in the midplane of the rotor. As pointed out earlier[2], this design of the electro-magnet is fitted to the flat rotor configuration. But many different configurations shall be investigated to expand its application area where its benefits of affording the wide variety of the bearing stiffness are enjoyed along the active controlled axes. For example, a slender configuration of the rotor shall be investigated to apply it to a turbo-machinery. One of the problems we should cope with for applying the two-axes controlled type to a slender configuration is to maintain the limiting speed of rotation above the required operational speed of rotation. As already pointed out, the motion stability of the rotor along the passively stabilized axes decreases with the rotational speed, which means the tilting motion of the two-axes controlled rotor may become unstable with increasing the rotational speed. We treated the same kind of problem at one-axis controlled rotor[3]. The rotor contains an actively controlled axis only along the rotational axis in the one-axis controlled magnetic bearing, where the rotor motion in the four axes, the translational motions along the two axes and the tilting motion around the two axes, is passively stabilized. One of measures for maintaining the passive motion stability up to high speed of rotation is to introduce anisotropic bearing stiffness effect[3]. The analysis to predict the anisotropic stiffness effect to stabilize the whirl motion of the rotor was conducted applying the method of averaging. The anisotropic stiffness effect will work also in the two-axes controlled rotor for increasing the stability of its tilting motion, that is expected. The analysis conducted in this paper to predict the anisotropic stiffness effect which causes the increase in the stability of the tilting motion, utilizing the method of averaging.

First, modelling of the tilting motion of the rotor is conducted assuming a rigid rotor with a elastic stator. The anisotropic stiffness effect in the elastic stator is transferred into the bearing stiffness anisotropy. The method of averaging [4] is applied for simplifying the mathematical model which describes the damping characteristics of the tilting motion of the rotor. The anisotropic stiffness effect is found to depend on the ratio of the moment of inertia of the rotor, different with a case of the one-axis controlled rotor, because the gyroscopic effect essential in the tilting motion works as a stabilizer of the whirl motion. Lastly, the design to realize the anisotropic stiffness effect is discussed.

2 MODELLING

Figure 1a shows schematically a two-axes controlled rotor. The rotor is suspended by the magnetic bearing and can move without contact within the gap clearance of the bearing. The rotor structural stiffness is assumed high enough for it to be treated as a rigid body. Then rotor motion may be separated into a motion of translational mode and that of tilting mode. The translational motion is dropped in the following sections, because that is actively controlled in a two-axes controlled magnetic bearing. The damping characteristics in the passively stabilized axes can be deduced with only the tilting mode motion. The stator is tilted elastically around x-axis and y-axis. The stator tilts at $(\theta_\xi,\ \theta_\eta)$ due to the elastic deformation from its nominal position. Figure 1b schematically defines tilt angles of the rotor. The rotor tilts at $(\theta_x,\ \theta_y)$ due to its whirling motion. The equations of tilting motion of the rotor are described as

$$\ddot{\theta}_x + 2(\delta_o+\delta_i)\dot{\theta}_x + \kappa\Omega\dot{\theta}_y + 2\delta_i\Omega\theta_y + \omega_c^2\theta_x + \alpha\theta_\xi = 0 \quad (1a)$$

$$\ddot{\theta}_y + 2(\delta_o+\delta_i)\dot{\theta}_y - \kappa\Omega\dot{\theta}_x - 2\delta_i\Omega\theta_x + \omega_c^2\theta_y + \alpha\theta_\eta = 0 \quad (1b)$$

where $\dot{\theta}_x$, $\dot{\theta}_y$, $\ddot{\theta}_x$, and $\ddot{\theta}_y$ are the first and second derivatives of x and y with time, respectively; the other symbols are defined as follows: Ω is the angular speed of rotor, ω_c is the critical speed of rotor caused by finite bearing stiffness, δ_o is the damping factor due to eddy current loss in the stator, δ_i is the damping factor due to eddy current loss in the rotor, κ is the ratio of moment of inertia (I_z/I_x or I_z/I_y), and α is the reaction factor due to stator deflection. The terms δ_i and δ_o represent electromagnetic induction effect related with the relative speed between the rotor and the stator. The terms $2\delta_i\Omega\theta_y$ in Eq.(1a) and $-2\delta_i\Omega\theta_x$ in Eq.(1b) represent the coupling effect where tilt angle in the one axis induces another tilt angle in the other axis due to the rotation. The terms $\kappa\Omega\dot{\theta}_y$ in Eq.(1a) and $-\kappa\Omega\dot{\theta}_x$ are called the gyroscopic effect. Their effect will be re-examined later in more detail. The terms $\alpha\theta_\xi$ in Eq.(1a) and $\alpha\theta_\eta$ in Eq.(1b) represent the coupling effect between the rotor and the stator due to tilting. The equations of tilting motion of the stator are described as

$$\ddot{\theta}_\xi + 2\delta_r\dot{\theta}_\xi + \omega_{sx}^2\theta_\xi + \alpha'\theta_x = 0 \quad (2a)$$

$$\ddot{\theta}_\eta + 2\delta_r\dot{\theta}_\eta + \omega_{sy}^2\theta_\eta + \alpha'\theta_y = 0 \quad (2b)$$

where $\dot{\theta}_\xi$, $\dot{\theta}_\eta$, $\ddot{\theta}_\xi$, and $\ddot{\theta}_\eta$ are the first and second derivatives of θ_ξ and θ_η with time, respectively; the other symbols are defined as follows: ω_{sx} and ω_{sy} are the mode angular frequencyof the stator along the x-axis and y-axis due to its structural elasticity, δ_r is the damping factor of the stator due to its elastic deflection, and α' is the reaction factor due to the rotor tilt. The symbols ω_{sx} and ω_{sy} are distinctive to each other to prepare for the introduction of the anisotropy of the structural stiffness in the stator. The anisotropy of the structural stiffness will turn out later to be equivalent to that of the bearing stiffness. The terms $\alpha'\theta_x$ and $\alpha'\theta_y$ represent the tilting restoring force effect due to the rotor displacement. Eqs.(1a-2b) describe the motion of the rotor and the stator considering the anisotropy of the stator structural stiffness.

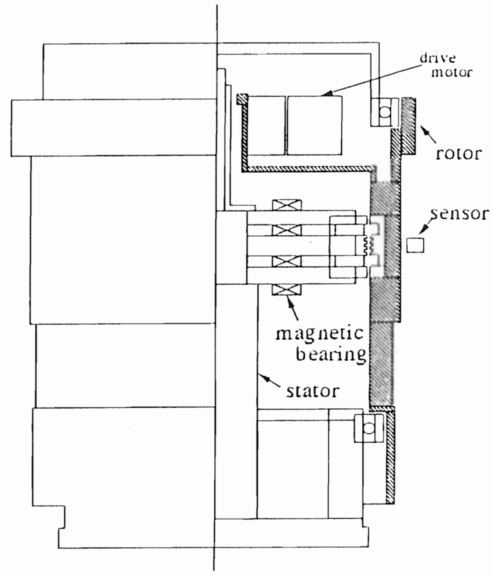

Fig. 1a Configuration of a two-axes controlled wheel.

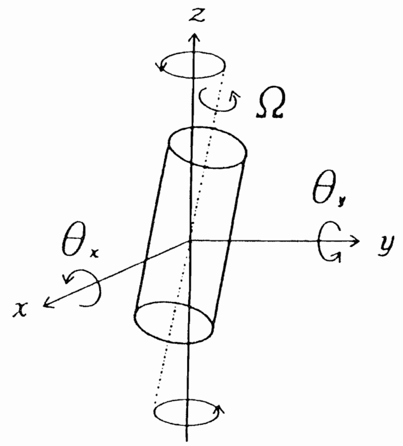

Fig. 1b Definition of tilting angles of the rotor.

3 STABILITY ANALYSIS

3.1 Isotropic stiffness

Taking the stiffness of stator as infinite, Eqs.(2) are discarded, and $\theta_\xi = \theta_\eta = 0$ are assumed in Eqs.(1). The remaining equations represent the motion of the rotor suspended by isotropic bearing:

$$\ddot{\theta}_x + 2\zeta\omega_c\dot{\theta}_x + \kappa\Omega\dot{\theta}_y + 2\zeta_i\omega_c\Omega\theta_y + \omega_c^2\theta_x = 0 \quad (3a)$$

$$\ddot{\theta}_y + 2\zeta\omega_c\dot{\theta}_y - \kappa\Omega\dot{\theta}_x - 2\zeta_i\omega_c\Omega\theta_x + \omega_c^2\theta_y = 0 \quad (3b)$$

where ζ, ζ_i are defined as $(\delta_o+\delta_i)/\omega_c$, δ_i/ω_c. The coupling effects are expressed explicitly in the last terms in Eqs.(3a) and Eqs.(3b). To obtain an approximate solution for the dynamic characteristics of the rotor, the method of averaging is applied as follows.

The homogeneous equation of Eqs.(3) are

$$\ddot{\theta}_x + \kappa\Omega\dot{\theta}_y + \omega_c^2\theta_x = 0 \quad (4a)$$

$$\ddot{\theta}_y - \kappa\Omega\dot{\theta}_x + \omega_c^2\theta_y = 0 \quad (4b)$$

Eqs.(4) are transferred as follows;

$$M\ddot{q} + G\dot{q} + Kq = 0, \quad q = (\theta_x, \theta_y)^T \tag{5}$$

where M, G, and K are defined as

$$M = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \; G = \begin{bmatrix} 0 & \kappa\Omega \\ -\kappa\Omega & 0 \end{bmatrix}, \; K = \begin{bmatrix} \omega_c^2 & 0 \\ 0 & \omega_c^2 \end{bmatrix}$$

Further, Eqs.(5) are transferred as follows;

$$M_o\dot{x} + G_o x = 0, \quad x = (q^T, \dot{q}^T)^T \tag{6}$$

where M_o, G_o are defined as

$$M_o = \begin{bmatrix} K & 0 \\ 0 & M \end{bmatrix}, \; G_o = \begin{bmatrix} 0 & -K \\ K & G \end{bmatrix}$$

The eigenvalues of the homogeneous solution are described as

$$\lambda_1 = j\left(\frac{\kappa\Omega}{2} - \sqrt{(\frac{\kappa\Omega}{2})^2 + \omega_c^2}\right) = j\omega_1$$

$$\lambda_2 = j\left(\frac{\kappa\Omega}{2} + \sqrt{(\frac{\kappa\Omega}{2})^2 + \omega_c^2}\right) = j\omega_2$$

$$\lambda_3 = -j\omega_1, \quad \lambda_4 = -j\omega_2 \tag{7}$$

To obtain an approximate solution of Eqs.(3), we transfer the variables θ_x, θ_y to a harmonic functions with ω_2 such as

$$\theta_x = b(t)e^{j\omega_2 t} + b^*(t)e^{-j\omega_2 t} \tag{8a}$$

$$\theta_y = -j\{b(t)e^{j\omega_2 t} - b^*(t)e^{-j\omega_2 t}\} \tag{8b}$$

where b^* denotes the conjugate complex variables of b. Inserting Eqs.(8) into Eqs.(3a). Neglecting the terms with $\ddot{a}$, $\ddot{a}^*$, and the other high-order terms, we obtain the following equation by multiplying the remaining terms by $e^{-j\omega_2 t}$:

$$j2\omega_2\dot{a} + \kappa\Omega\omega_2 a + (\omega_c^2 - \omega_2^2)a + j2\zeta\omega_c\omega_2 a - j2\zeta_i\omega_c\Omega a$$
$$+ f(\dot{a}^*, a^*)e^{-2j\omega_2 t} = 0 \tag{9}$$

where f is the linear function of $\dot{a}^*$, a^*. The sixth terms in the left side of Eq.(9) are of the same magnitude as others. The sinusoidal function has a frequency $2\omega_2$. When focussing the slow variation of $\dot{a}$, we can eliminate the last term on the left side of Eq.(9) by averaging them over the period (π/ω_2). This approximation is valid for the frequency below (ω_2/π). Then we obtain the following Equations, which approximate Eq.(9):

$$\dot{a} + \omega_c\left(\zeta - \zeta_i\frac{\Omega}{\omega_2}\right)a - j\left(\frac{\kappa\Omega}{2} + \frac{\omega_c^2 - \omega_2^2}{2\omega_2}\right)a = 0 \tag{10}$$

We obtain the real part denoted by Λ of the eigenvalue of the system such as

$$\Lambda = -\omega_c\left(\zeta - \zeta_i\frac{\Omega}{\omega_2}\right) \tag{11}$$

Immediately we can find the following relation when Ω becomes to infinity

$$\Lambda \to -\omega_c\left(\zeta - \frac{\zeta_i}{\kappa}\right) \tag{12}$$

The imaginary part of the eigenvalue is assumed to be $\pm\omega_2$, because when using them in Eq.(8), and then Eq.(11) represents the damping factor. From, it is found that the lower value of the damping factor decreases with the angular speed of rotor Ω and it becomes negative beyond the boundary angular speed at which the right side becomes zero. When $\kappa > \zeta_i/\zeta$, the real part of the eigenvalues is negative. Then this motion is stable. But, when $\kappa < \zeta_i/\zeta$, the real parts of the eigenvalues will be positive. Then the limiting speed of the rotor stabilization exists. Eq.(12) shows that the internal loss δ_i causes decrease in the limiting speed. To increase magnitude of the limiting speed, we must increase ω_c or incrèase δ_o, because as a practical matter we cannot decrease δ_i below some minimum value. An example of damping factor variation of a isotropic rotor with rotational speed is shown in Figure 2. As shown in Figure 2, when $\kappa \geq 0.5$, the damping factor is positive at all times. Then the motion is stable.

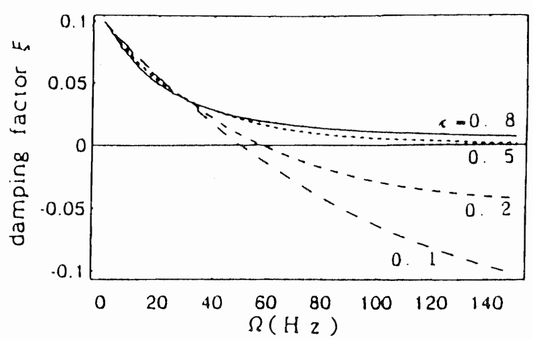

Fig.2 The damping factor variation of the isotropic rotor with rotational speed. (ω_c = 140 (rad/s), $\zeta = 0.1$, $\zeta_i = 0.05$)

3.2 Anisotropic stiffness

Eq.(11) has been deduced on the assumption of isotropic bearing stiffness. The reason why the damping factor decreases with Ω is explained with that the isotropy of bearing stiffness helps the growth of rotor whirling motion through a tight coupling effect between x and y axes of freedom of the motion, and a rotational energy of the rotor may be poured to the whirling motion. It is pointed out that the anisotropy of bearing stiffness prevents the internal loss factor from decreasing the mode damping factor(3). We formulate this effect from Eqs.(1a-2b). Now, we transfer θ_ξ to the following approximate equation focussing on Eq.(2a).

$$\theta_\xi = f(t)e^{j\omega_2 t} + f^*(t)e^{-j\omega_2 t} \tag{13}$$

where $f^*(t)$ is the conjugate complex of a function to be determined, $f(t)$. Inserting Eq.(13) into Eq(2a), we can obtain the following relation on $f(t)$ after neglecting the derivatives of $f(t)$:

$$f(t) = -\frac{\alpha'}{\omega_{rx}^2 - \omega_2^2 + 2j\omega_2\delta_r}b(t) \tag{14}$$

After using Eq.(14) in Eq.(13), and also Eq.(7), and neglecting the derivatives of $b(t)$, we can obtain the following expression:

$$\theta_\xi = -\frac{\alpha'}{\omega_{rx}^2 - \omega_2^2}\theta_x + \frac{2\alpha'\delta_r}{(\omega_{rx}^2 - \omega_2^2)^2}\dot{\theta}_x \quad (15)$$

After introducing Eq.(15) into Eq.(1a), we finally obtain

$$\ddot{\theta}_x + 2\left\{\delta_o + \frac{\alpha\alpha'}{(\omega_{rx}^2 - \omega_2^2)^2}\delta_r\right\}\dot{\theta}_x + 2\delta_i(\dot{\theta}_x + \Omega\theta_y + \kappa\Omega\dot{\theta}_y)$$

$$+\left(\omega_c^2 - \frac{\alpha\alpha'}{\omega_{rx}^2 - \omega_2^2}\right)\theta_x = 0 \quad (16)$$

To compare the factors in this equation with the corresponding factors in Eq.(1), we must modify ω_c and δ_o as follows:

$$\Delta\omega_{cx} = -\frac{\alpha\alpha'}{2\omega_2(\omega_{rx}^2 - \omega_2^2)}, \quad \Delta\delta_{ox} = \frac{\alpha\alpha'}{(\omega_{rx}^2 - \omega_2^2)^2}\delta_r \quad (17)$$

$$\Delta\omega_{cy} = -\frac{\alpha\alpha'}{2\omega_2(\omega_{ry}^2 - \omega_2^2)}, \quad \Delta\delta_{oy} = \frac{\alpha\alpha'}{(\omega_{ry}^2 - \omega_2^2)^2}\delta_r \quad (18)$$

The modification of $\Delta\delta_{ox}$ and $\Delta\delta_{oy}$ are practically negligible, because δ_r cannot be large. The anisotropy of structural stiffness in the stator can be transferred into that of the bearing stiffness. So, we replace ω_c with ω_{cx} and ω_{cy} as follows:

$$\omega_{cx} = \omega_c + \Delta\omega_{cx}, \quad \omega_{cy} = \omega_c + \Delta\omega_{cy} \quad (19)$$

The following equations express the modified equations of the rotor motion considering the anisotropy of bearing stiffness which can be caused by the structural anisotropy:

$$\ddot{\theta}_x + 2\zeta\omega_{cx}\dot{\theta}_x + \kappa\Omega\dot{\theta}_y + 2\zeta_i\omega_{cx}\Omega\theta_y + \omega_{cx}^2\theta_x = 0 \quad (20a)$$

$$\ddot{\theta}_y + 2\zeta\omega_{cx}\dot{\theta}_y - \kappa\Omega\dot{\theta}_x - 2\zeta_i\omega_{cx}\Omega\theta_x + \omega_{cy}^2\theta_y = 0 \quad (20b)$$

where ζ, ζ_i are defined as $(\delta_o + \delta_i)/\omega_{cx}$, δ_i/ω_{cx}, respectively. The transfer of the variables θ_x and θ_y with the harmonic function of the angular speed ω_{x2} is executed by the same way as in Eq.(9),

$$\theta_x = c(t)e^{j\omega_2 t} + c^*(t)e^{-j\omega_2 t}, \quad \theta_y = d(t)e^{j\omega_2 t} + d^*(t)e^{-j\omega_2 t} \quad (21)$$

$$\omega_{x2} = \frac{\kappa\Omega}{2} + \sqrt{(\frac{\kappa\Omega}{2})^2 + \omega_c^2} \quad (22)$$

Inserting Eqs.(21) into Eqs.(20), and neglecting $\ddot{c}$, $\ddot{d}$, $\ddot{c}^*$, $\ddot{d}^*$, and the other high-order terms, we obtain the following equations by multiplying the remaining terms by $e^{-j\omega_{x2}t}$. After applying the method of averaging, we obtain the following equations relating c and d,

$$\dot{c} + \left(\zeta\omega_{cx} - j\frac{\omega_{cx}^2 - \omega_{x2}^2}{2\omega_{x2}}\right)c + \left(\frac{\kappa\Omega}{2} - j\zeta_i\Omega\frac{\omega_{cx}}{\omega_{x2}}\right)d \quad (23a)$$

$$\dot{d} + \left(\zeta\omega_{cx} - j\frac{\omega_{cy}^2 - \omega_{x2}^2}{2\omega_{x2}}\right)d\left(\frac{\kappa\Omega}{2} - j\zeta_i\Omega\frac{\omega_{cx}}{\omega_{x2}}\right)c \quad (23b)$$

The characteristic equation is described as

$$\Lambda^2 + \{2\zeta\omega_{cx} + j(\kappa\Omega - \Delta')\}\Lambda$$

$$+\left\{(\zeta\omega_{cx})^2 - \left(\zeta_i\Omega\frac{\omega_{cx}}{\omega_{x2}}\right)^2 + \frac{\kappa\Omega}{2}\Delta'\right\}$$

$$+j\left\{\kappa\Omega\left(\zeta\omega_{cx} - \zeta_i\Omega\frac{\omega_{cx}}{\omega_{x2}}\right) - \Delta'\zeta\omega_{cx}\right\} = 0 \quad (24)$$

where the following notations are defined,

$$\frac{\omega_{cx}^2 - \omega_{x2}^2}{2\omega_{x2}} = -\frac{\kappa\Omega}{2}, \quad \frac{\omega_{cy}^2 - \omega_{x2}^2}{2\omega_{x2}} = \Delta\frac{\omega_{cx}}{\omega_{x2}} - \frac{\kappa\Omega}{2}$$

$$\Delta = \frac{\omega_{cy}^2 - \omega_{cx}^2}{2\omega_{cx}}, \quad \Delta' = \Delta\frac{\omega_{cx}}{\omega_{x2}} \quad (25)$$

We obtain the eigenvalue Λ of the system as

$$\Lambda = -\zeta\omega_{cx} + j\left(\frac{\Delta'}{2} - \frac{\kappa\Omega}{2}\right)$$

$$\pm\sqrt{\left(\zeta_i\frac{\omega_{cx}}{\omega_{x2}} + j\frac{\kappa}{2}\right)^2\Omega^2 - \left(\frac{\Delta'}{2}\right)^2} \quad (26)$$

The real part of the eigenvalue, that is the damping factor ξ, is modified as follows: (1) At $\Omega = 0$:

$$\Lambda(\Omega = 0) = -\zeta\omega_{cx} + j\frac{\Delta'}{2} \pm j\frac{\Delta'}{2} \quad (27)$$

When focussing only on the real part of Eq.(27),

$$\xi = -\frac{\zeta\omega_{cx}}{\omega_{x2}}, \quad \omega_{x2}(\Omega = 0) = \omega_{cx} \quad (28)$$

Then, this motion is stable, because the real part of the eigenvalue is negative. (2) With $\Delta' = 0$, that means the isotropic stiffness. Transferring ω_{cx}, ω_{cy} to ω_c;

$$\xi = -\omega_c\left(\zeta - \zeta_i\frac{\Omega}{\omega_2}\right) \quad (29)$$

Then, Eq.(29) agrees with Eq.(11). (3) The following relation will be obtained, focussing only on the real part, with $\Omega \to \infty$,

$$\xi(\Omega \to \infty) = -\omega_{cx}\left(\zeta - \zeta_i\frac{\Omega}{\omega_{x2}}\right) \quad (30)$$

Then, the anisotropic stiffness and the isotropic stiffness have essentially the same effect. The damping factor variation of the anisotropic rotor with rotational speed is shown in Figure 3. As shown in Figure 3, when $\kappa \geq 0.5$, damping factor is positive at all times. Then, this motion is stable over the whole range.

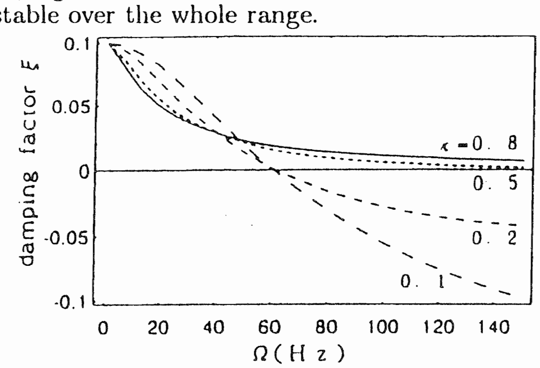

Fig.3 The damping factor variation of the anisotropic rotor with rotational speed. ($\omega_{cx} = 140$ (rad/s), $\omega_{cy} = 100$ (rad/s), $zeta = 0.1$, $\zeta_i = 0.05$)

4 EXAMPLES OF STABILITY PREDICTION

In case of $\kappa = 0.8$, the approximate solution compared with a exact solution is shown in Figure 4. As shown in Figure 4, it is found that the approximate solution is effective for predicting the characteristic of the exact solution in wide range.

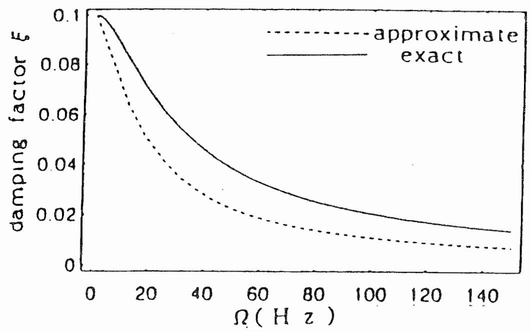

Fig.4 The approximate solution compared with the exact solution. ($\kappa = 0.8$, $\omega_{cx} = 140$ (rad/s), $\omega_{cy} = 100$ (rad/s), $\zeta = 0.1$, $\zeta_i = 0.05$)

In case of $\kappa = 0.8$ and 0.1, the anisotropic stiffness effect on the damping factor is shown in Figure 5a and Figure 5b. As shown in Figures 5, the anisotropic bearing stiffness improves the damping characteristics in higher rotational speed for the rotor which has the lower ratio of the moment of inertia.

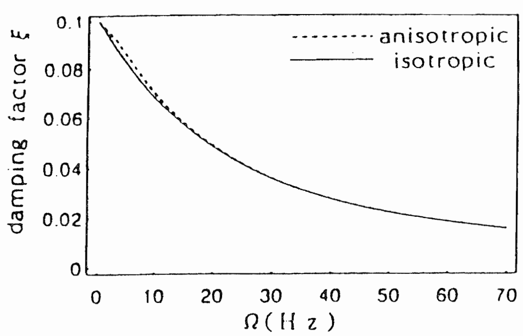

Fig.5a The anisotropic effect to the damping factor. ($\kappa = 0.8$, $\omega_{cx} = 140$ (rad/s), $\omega_{cy} = 100$ (rad/s), $\zeta = 0.1$, $\zeta_i = 0.05$)

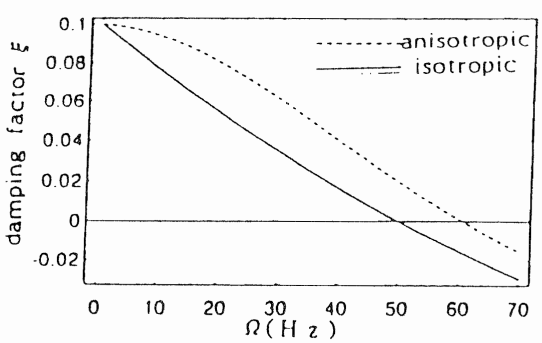

Fig.5b The anisotropic effect to the damping factor. ($\kappa = 0.1$, $\omega_{cx} = 140$ (rad/s), $\omega_{cy} = 100$ (rad/s), $\zeta = 0.1$, $\zeta_i = 0.05$)

The relation between the ratio of moment of inertia κ and the critical speed is shown in Figure 6. As shown in Figure 6, it is found that the anisotropic stiffness effect becomes large, as the ratio of moment of inertia κ becomes lower. On the other hand, when the ratio of moment of inertia κ is enough high, the anisotropic stiffness effect is small.

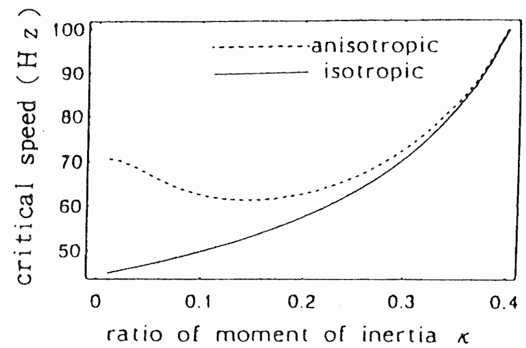

Fig.6 The critical speed variation with the ratio of the moment of inertia. ($\omega_{cx} = 140$ (rad/s), $\omega_{cy} = 100$ (rad/s), $\zeta = 0.1$, $\zeta_i = 0.05$)

In case of $\omega_{cy} = 40$, 90, 140, and 190(rad/s) for $\omega_{cx} = 140$(rad/s), the damping factor variation of the rotor with rotational speed is shown Figure 7.

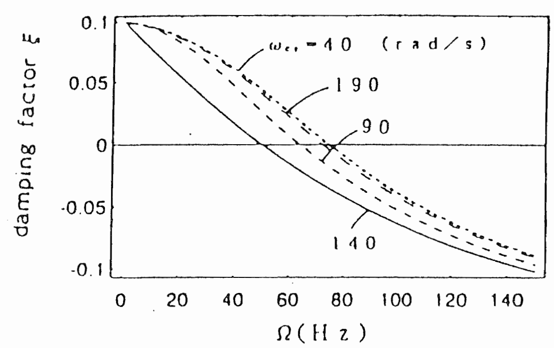

Fig.7 The damping factor variation with rotational speed. ($\kappa = 0.8$, $\omega_{cx} = 140$ (rad/s), $\omega_{cy} = 100$ (rad/s), $\zeta = 0.1$, $\zeta_i = 0.05$)

The relation between the ratio of anisotropic stiffness and the damping factor is shown in Figure 8. The relation between the ratio of anisotropic stiffness and the critical speed is shown Figure 9.

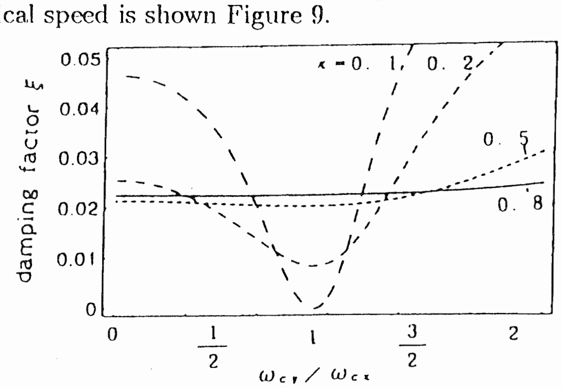

Fig.8 The damping factor variation with the ratio of the ratio.($\omega_{cx} = 140$ (rad/s), $\zeta = 0.1$, $\zeta_i = 0.05$)

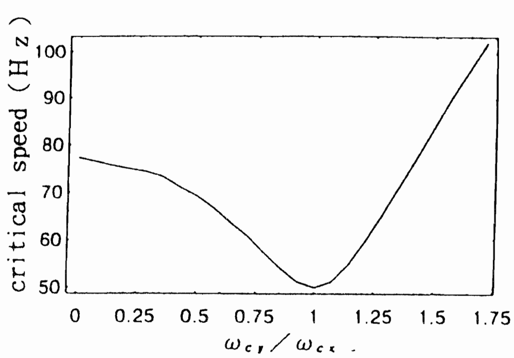

Fig.9 The critical speed variation with the ratio of the anisotropic stiffness.($\kappa = 0.1$, $\omega_{cx} = 140$ (rad/s), $\zeta = 0.1$, $\zeta_i = 0.05$)

As shown in Figure 8, it is found that the damping factor increases with the difference of bearing stiffness and the anisotropic effect of the rotor with the lower ratio of the moment of inertia is larger than that with the higher ratio of the moment of inertia. As shown in Figure 9, it is found that the critical speed increases with the difference of bearing stiffness .

5 DESIGN FOR OBTAINING ANISOTROPIC STIFFNESS

As discussed in the preceding section, the anisotropic bearing stiffness improves the damping characteristics at the higher rotational speed for the rotor which has the low ratio of the moment of inertia. The designing method for obtaining the anisotropic stiffness is investigated in this section.

We assume that the natural angular frequency ω_{cx}, ω_{cy} representing the bearing dynamic stiffness is proportional to the natural angular frequency $\omega'_{cx}, \omega'_{cy}$ representing the bearing static stiffness as follows,

$$\omega_{cx} \propto \omega'_{cx}, \omega_{cy} \propto \omega'_{cy} \tag{31}$$

Accordingly, when the difference of the static magnetic bearing stiffness $\omega'_{cx}, \omega'_{cy}$ become larger, the anisotropic stiffness will be larger. Figure 10 schematically shows the restoring force of the magnetic bearing due to the tilting motion of the rotor. Where f_U, f_L are magnetic attractive forces between the rotor and the stator, when the rotor tilts at small angle θ. We assume that the magnetic attractive forces f_U, f_L are concentrated at the teeth of the upper and lower parts, and they have a radial force f_{rU}, f_{rL}, and an axial force f_{zU}, f_{zL}, respectively. Then, the moment M due to the magnetic attractive force is described as (defining a positive angle counterclockwise)

$$M = (f_{rL} - f_{rU})l_z - (f_{zL} + f_{zU})R_m \tag{32}$$

The magnetic attractive forces are described as

$$f_U = f_s - k\theta, \quad f_L = f_s + k\theta \tag{33}$$

where f_s is the static magnetic attractive force and k is a tilting spring constant. Neglecting ok terms, we obtain the following equation:

$$M = 2(kl_z - f_s R_m)\theta \tag{35}$$

Restorability from the the tilting attitude of the rotor means

$$\frac{dM}{d\theta} = 2(kl_z - f_s R_m) < 0 \tag{36}$$

As the derivative $-\partial M/\partial\theta$ means the static bearing stiffness, then, when the different value of $\partial M/\partial\theta$ is realized around x-axis and y-axis, we can obtain the anisotropic bearing stiffness.

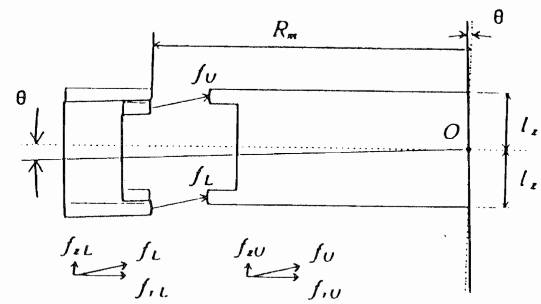

Fig.10 Definition of restoring force.

6 CONCLUDING REMARKS

The anisotropic stiffness effect to the stability characteristics in tilting motion of a rotor which is actively suspended in two radial axes by the magnetic attractive force is analyzed by utilizing the method of averaging. In the result, the anisotropic stiffness improves the damping characteristics at higher rotational speed for the rotor which has the lower ratio of the moment of inertia.

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