Reduction of the Actuator Number in Magnetic Suspension

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ABSTRACT

This paper proposes a new approach for the design of magnetic suspensions. It is based on the feasibility of non linear control of electromagnets involved in such suspension. It suppresses the necessity of symmetry in electromagnet location, and it is shown that only (n+1) actuators are required to actively control n degrees of freedom. The synthesis of control scheme is detailed for the levitation of a beam with two degrees of freedom. Simulation results illustrate the good behavior of position and control variables when minimum number of actuators are activated at one time.

KEYWORDS

Active magnetic bearing, non linear control, minimisation.

I - INTRODUCTION

Since electromagnets are able to produce only attractive forces, they are usually located by pairs along one direction. Thus it is possible to obtain the required force with the right sign along each direction. Furthermore it makes possible the design of control schemes based on linearized models assuming biasing currents in opposite electromagnets.

The aim of this paper is to show that the total number of actuators can be reduced, thus reducing the dimensions of the stator, without lost of controllability of the positioning of the rotor. It can be shown that only (n+1) electromagnets are necessary to actively control n degrees of freedom of a body in space, provided that they are properly located in space. This means that the use of opposite electromagnets is only required when one degree of freedom is actively controlled.

However, this is only possible by using a non linear control scheme. In fact this kind of control structure was developed for classical design of active magnetic bearings. But it is easy to extend this principle to a novel situation where few actuators are involved in the positioning of the shaft.

The paper is organized as follows. Section II shows the location of electromagnets in the cases of 1, 2, and 3 degrees of freedom. Section III is devoted to the modeling of the two degrees of freedom case : it consists in the positioning of a beam in a plane and a passive thrust is considered to make the situation closer to the case of a rotating shaft in active magnetic bearings. Section IV describes the switching conditions between electromagnets. Section V presents the general outlines of the hierarchical non linear control scheme, including path planning and motion stabilization. Section VI gives some simulation results which illustrate the feasibility of the design, as well as the good behavior of control and position variables. A conclusion ends the paper with some considerations about the case of four actively controlled degrees of freedom which is often encountered in industrial applications.

II - ILLUSTRATION THROUGH SIMPLE SITUATIONS

The objective of the location of electromagnets is to introduce some redundancy in resulting efforts production in such a way that they can be realized by means of positive forces. Of course, in the case of one degree of freedom, since only one electromagnet is not able to produce a force with an arbitrary sign, the minimal number of actuator is two, see figure 1.

figure 1 : Motion along one direction

Let us now consider the case of a beam in a plane : two degrees of freedom need to be controlled. This is possible by means of three electromagnets, located as shown in figure 2 for example. Roughly speaking it is possible to control the resulting effort by means of the difference between $(F_1 + F_3)$ and F_2, while the torque at the center of mass is controlled by a proper balance between F_1 an F_3. Of course other locations of F_2 are possible, but they cannot be chosen arbitrarily as shown in the fourth section.

figure 2 : Motions in the plane

The third example deals with three degrees of freedom. It consists in the motion of a sphere in space, which means that three translations along orthogonal directions are actively controlled. In that case, four electromagnets are enough to ensure this task, as shown in figure 3.

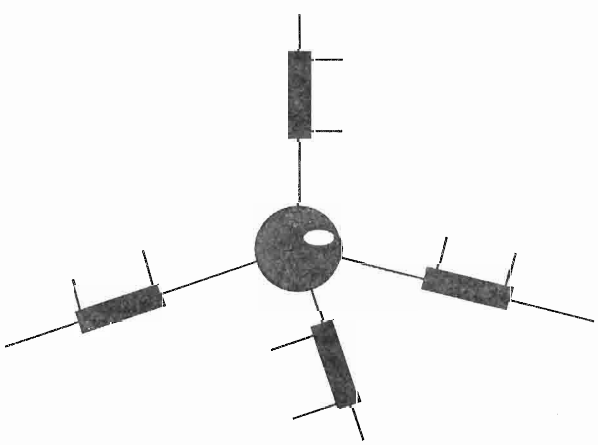

figure 3 : Motions of a ball in space

We stop here this description of simple situations which illustrate that only (n+1) electromagnets are necessary to control n degrees of freedom. The reader is invited to derive actuators location for cases involving four up to six degrees of freedom, and let us now focus on the problem of designing a control scheme for the two degrees of freedom.

III - MODELING OF A BEAM

Let us consider the problem of positioning a beam in a plane which represents the motion of a shaft in a plane parallel with its main axis of inertia. For that the action of a passive thrust is taken into consideration, as shown in the figure 4.

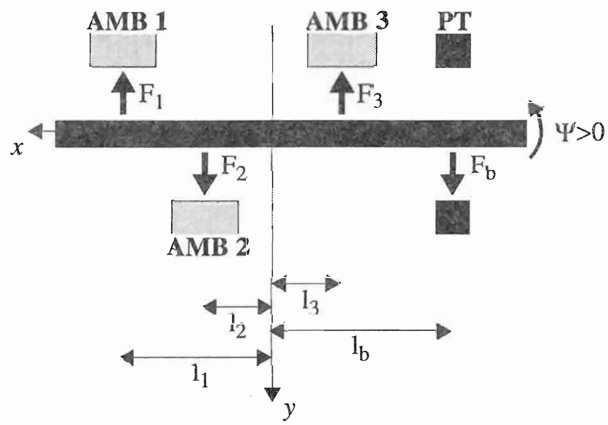

figure 4 : Beam with passive thrust

Denoting by y the difference between the lateral position of the beam's center of mass and its nominal position, ψ the angle between the longitudinal axis of the shaft and its nominal position, e_j the nominal air gap in the y direction, l_i the longitudinal distance between the electromagnet i and the center of mass (i=1..3), l_b the longitudinal distance between the passive thrust and the center of mass, F_i the forces created by the electromagnet i, and F_b the force created by passive thrust, we have :

$$\begin{cases} m \cdot \ddot{y} = -F_1 + F_2 - F_3 + F_b + mg \\ J \cdot \ddot{\psi} = -F_1 \cdot l_1 + F_2 \cdot l_2 + F_3 \cdot l_3 - F_b \cdot l_b \end{cases} \tag{1}$$

with

$$F_1 = \frac{\lambda_1 \cdot i_1^2}{[e_1 + (y + l_1 \cdot \sin\psi)]^2} \tag{2}$$

$$F_2 = \frac{\lambda_2 \cdot i_2^2}{[e_2 - (y + l_2 \cdot \sin\psi)]^2} \tag{3}$$

$$F_3 = \frac{\lambda_3 \cdot i_3^2}{[e_3 + (y - l_3 \cdot \sin\psi)]^2} \tag{4}$$

$$F_b = -k_b \cdot (y - l_b \cdot \sin\psi) \tag{5}$$

Let us introduce R and Γ which are resulting efforts at the center of mass :

$$\begin{cases} R = -F_1 + F_2 - F_3 = m \cdot (\ddot{y} - g) \\ \Gamma = -F_1 \cdot l_1 + F_2 \cdot l_2 + F_3 \cdot l_3 = J \cdot \ddot{\psi} + F_b \cdot l_b \end{cases} \tag{6}$$

IV - SWITCHING CONDITIONS

Let us show that it is possible to find F_i, i=1..3, such that it is possible to get arbitrary R and Γ with $F_i \geq 0 \quad i = 1...3$.

Furthermore, in order to reduce energy consumption in electromagnets, we want to limit the use of electromagnet to those which are necessary to satisfy constraint in equation (6), which means that at least one F_i is null at every instant.

Thus three cases have to be considered. If :

$$\begin{cases} l_1 - l_2 \geq 0 \\ l_2 + l_3 \geq 0 \\ l_1 + l_3 \geq 0 \end{cases} \tag{7}$$

then the following conditions must be satisfy to ensure positiveness of each electromagnetic force :

Case $F_1 = 0$:

$$\begin{cases} F_1 = 0 \\ F_2 = \dfrac{1}{l_2 + l_3} \cdot (l_3 \cdot R + \Gamma) > 0 \\ F_3 = \dfrac{1}{l_2 + l_3} \cdot (-l_2 \cdot R + \Gamma) > 0 \end{cases} \tag{8}$$

Case $F_2 = 0$:

$$\begin{cases} F_1 = \dfrac{-1}{l_1 + l_3} \cdot (l_3 \cdot R + \Gamma) > 0 \\ F_2 = 0 \\ F_3 = \dfrac{-1}{l_1 + l_3} \cdot (l_1 \cdot R - \Gamma) > 0 \end{cases} \tag{9}$$

Case $F_3 = 0$:

$$\begin{cases} F_1 = \dfrac{-1}{l_1 - l_2} \cdot (-l_2 \cdot R + \Gamma) > 0 \\ F_2 = \dfrac{1}{l_1 - l_2} \cdot (l_1 \cdot R - \Gamma) > 0 \\ F_3 = 0 \end{cases} \tag{10}$$

These constraints may be described by the figure 5.

It is easily seen that a solution exists for every pair (R, Γ).

Obviously, the case of biasing forces could also be treated in the same way.

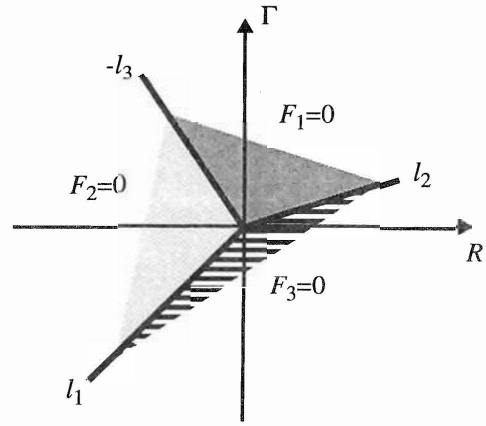

figure 5 : Switching between electromagnets

V - OVERVIEW OF THE CONTROL SCHEME

The true control variables of the system are coil voltages. To complete the mechanical model given in section II, we must add the following equations (i=1..3) :

$$U_j = r_j \cdot i_j + L_j \cdot \frac{di_j}{dt} + i_j \cdot \frac{dL_j}{dt} \tag{11}$$

$$L_j = \frac{\lambda}{w(y, \psi)} \tag{12}$$

$$F_j = \frac{\lambda \cdot i_j^2}{[w(y, \psi)]^2} \tag{13}$$

In fact, it is much interesting to take advantage of the distinct dynamics of electrical part on the one hand and mechanical part on the other hand. This makes the design of a hierarchical control possible. It means that two levels are considered. The high-level computes the required currents in electromagnets in order to produce the required forces, while three low-level loops take in charge the control of coil voltages so that coil currents are well driven according to the references provided by the higher level.

Let us now focus on the higher level. The objective is the computation of currents such that :

- trajectories of y(t) and ψ(t) are well stabilized,
- the minimum number of actuators are activated at one time.

It has been shown in the previous section that there is always a solution for the second condition, when forces are considered. Then by means of equations (2), (3) and (4), it is possible to derive reference currents in electromagnets. In order to keep a good dynamic behavior, it is useful to introduce path planning in the control scheme according to the following figure :

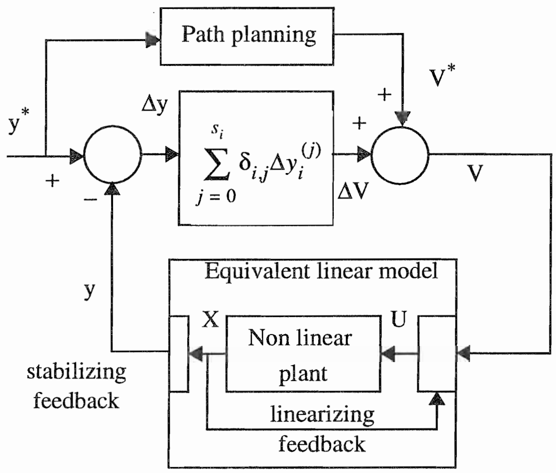

figure 6 : Path planning and stabilization

V-1 - Path planning

Analysis of flatness properties of the system shows that there are three flat outputs which can be the two positions of the beam and one of the three current variables.

This means that reference trajectories can be arbitrarily chosen for these flat outputs, provided that they are sufficiently differentiable, in fact twice for *y(t)* as well as $\psi(t)$ and no constraint on the third one. Then it is possible to derive trajectories for other system variables and control input without integrating differential equation.

For example, let us consider the rising of the beam from a steady state position with $y(0), \dot{y}(0)$ and $\psi(0), \dot{\psi}(0)$ to a final position $y(H), \dot{y}(H)$ and $\psi(H), \dot{\psi}(H)$. A possible trajectory may be found in the class of polynomial of *t*. In this case, initial and final conditions lead to a third order polynomial for *y(t)* and $\psi(t)$. Then the mechanical model allows computation of resulting efforts at the center of mass and the switching conditions given in equations (9), (10) and (11), lead to the determination of each electromagnetic force and then to the current reference trajectories by means of relations (14).

However, supplementary constraints can be introduced in the design of flat output reference trajectories, in order for example to take into account the boundedness of the true inputs of the process, namely the coil voltages. This problem occurs every time when one current becomes null, that is at each switching instant and more specifically at the beginning and the end of trajectory planning.

The analysis in depth of the first derivative of the current $i_j(t)$ shows a singularity which can be circumvented by a proper choice of the second and third derivatives of the reference trajectories for *y(t)* and $\psi(t)$. It is why the path planning problem is solved in two steps :

- choose a fifth order polynomial of *t* for *y(t)* and $\psi(t)$ that satisfy initial and final constraints $y(0), \dot{y}(0), \ddot{y}(0)$, $\psi(0), \dot{\psi}(0), \ddot{\psi}(0)$, $y(H), \dot{y}(H), \ddot{y}(H)$ and $\psi(H), \dot{\psi}(H), \ddot{\psi}(H)$: second derivatives are introduced in order to smooth transient behavior at the beginning and the end of motion,

- by means of the previous trajectories, determine switching instants, and introduce supplementary conditions concerning third derivative, either left or right, according to the case, which is forced to zero. Then, within each time interval defined by switching instants, compute new polynomials with higher degree, i. e. six or seven according to the time interval which is considered, with initial and final conditions that are deduced from first step plus supplementary constraint if necessary.

V-2 - Stabilization

Since dynamical aspects only appear in the control of the mechanical part, the stabilization problem concerns a linear multi-input multi-output model. Thus a state feedback is easily designed such that the transient behavior has the appropriate dynamics.

It consists in computing

$$\begin{aligned} V_y &= \ddot{y}^* + \delta_{y,0}(y^* - y) + \delta_{y,1}\left(\dot{y}^* - \dot{y}\right) \\ V_\psi &= \ddot{\psi}^* + \delta_{\psi,0}(\psi^* - \psi) + \delta_{\psi,1}\left(\dot{\psi}^* - \dot{\psi}\right) \end{aligned} \qquad (14)$$

where the symbol * refers to the reference trajectories.

Then resulting efforts *R* and Γ are determined by replacing $\ddot{y}$ and $\ddot{\psi}$ by V_y and V_ψ respectively in the mechanical model (6).

VI - SIMULATION RESULTS

Simulations have been carried out in order to illustrate the behavior of current and voltage variables when the beam moves from an horizontal steady state position at *y(0)=-4e-4 m* to the equilibrium which corresponds to a "centered" position with respect to electromagnets. Let us notice that air gap of electromagnets n°1 and 3 varies from *1e-4 m* to *5e-*

4 m, so it induces an important change in the behavior of the electromagnets. Furthermore, in order to test robustness of the hierarchical control scheme, trajectory planning is based on a single fifth order polynomial for each variable $y(t)$ and $\psi(t)$. It means that intermediate switchings don't receive particular attention. The horizon that is used for path planning is determined in accordance with process capabilities, namely maximum amplitude of coil currents and coil voltages. The main characteristics of the process are given below :

Tableau 1 : Characteristics of the process

Parameters	Beam	Units
l_1	0.2	m
l_2	0.1	m
l_3	0.3	m
J	1e-3	kg.m^2
m	1.5	kg
k_b	-10000	N/m
λ_j	5e-6	N.m^2/A^2
e_j	5e-4	m

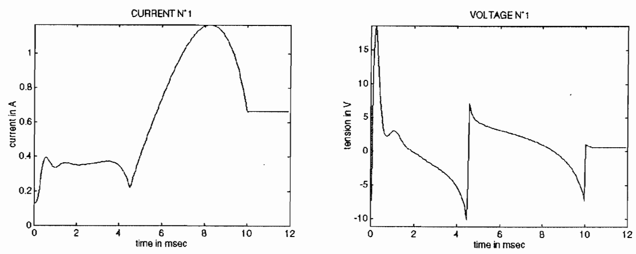

figure 8 : Current and voltage in coil n°1

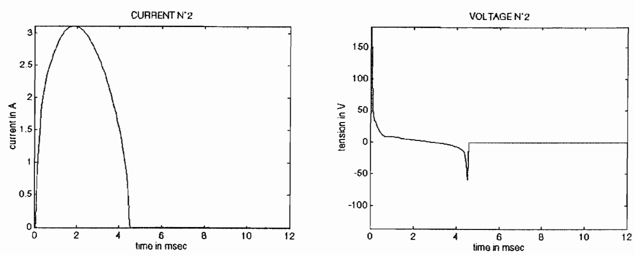

figure 9 : Current and voltage in coil n°2

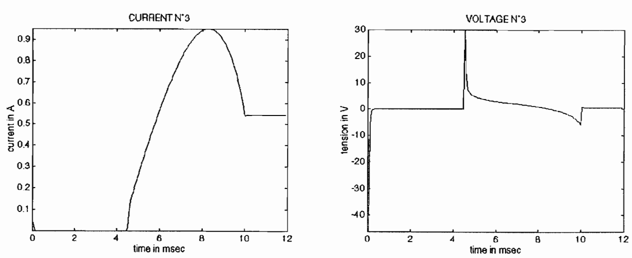

figure 10 : Current and voltage in coil n°3

VI-1 - No voltage saturation

The first test given below correspond to the simulation of the process and its control structure, without taking into consideration electrical variable saturations.

Figure 7 shows the reference trajectory for $y(t)$, since $\psi(t)=0$, and tracking error which remains weak everywhere. Then, following figures 8-9-10 give evolution of currents and voltages in each coil. One may notice sudden changes of voltage variables at switching instants.

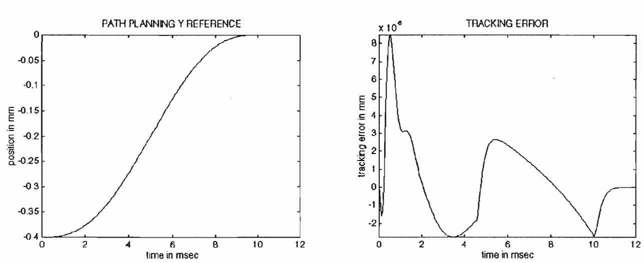

figure 7 : Reference trajectory and tracking error

VI-2 - Voltage saturation

The second test differs slightly from the preceding since saturation of electrical variables is now taken into account in the simulation. We don't notice important changes in the behavior of the beam.

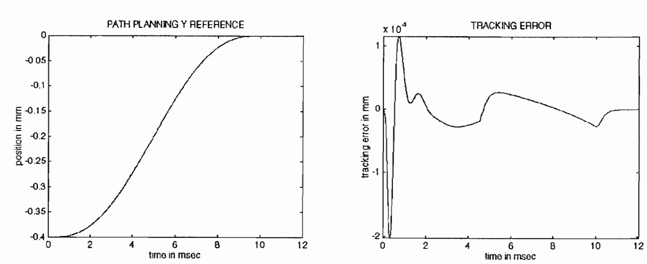

figure 11 : Reference trajectory and tracking error

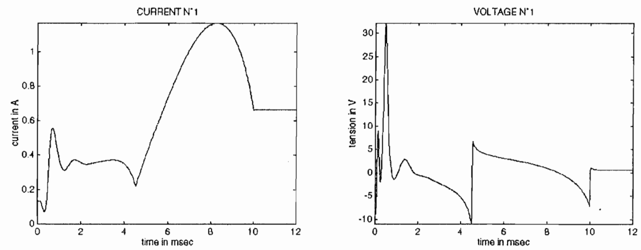

figure 12 : Current and voltage in coil n°1

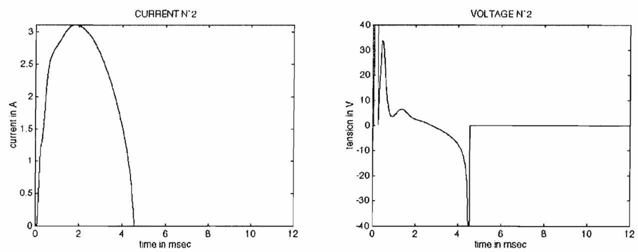

figure 13 : Current and voltage in coil n°2

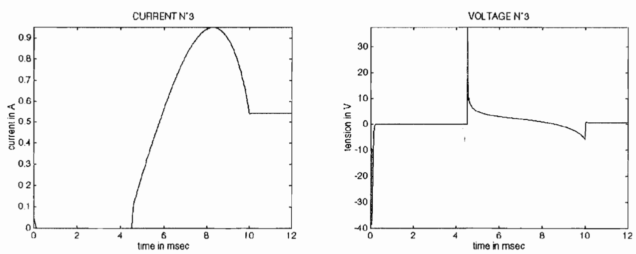

figure 14 : Current and voltage in coil n°3

VII - CONCLUSIONS

We have shown in this paper the feasibility of a new design for magnetic suspension that is not based on the use of electromagnets by pairs along each direction to be controlled. The reduction of the actuator number may be a crucial problem in some industrial applications. We have shown that the asymmetry which appears in the electromagnet location is not a problem for the control scheme design.

In this paper, we have discussed the special case of a beam with two active degrees of freedom and the complementarity condition for switchings between electromagnets. This example was retained since it makes easier the presentation of the whole procedure, concerning both electromagnets location as well as control scheme design. For the more usual situation of four degrees of freedom which is encountered in industrial applications, five electromagnets are enough to control the positioning of the shaft. The design of the non linear control scheme follows the same outlines as in the simpler case dealt with in this paper.

For robustness aspects it could be useful to introduce small biasing currents in the neighborhood of switching instants. This could be easily taken into account in the control scheme presented in this paper.

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