Fifth International Symposium on Magnetic Bearings
Kanazawa, Japan, August 1996



# Current Feedback Stabilization of Tuned Magnetic Suspension System

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***Abstract*** — *Suspension techniques employing a tuned LC circuit have the advantage that stable suspension can be achieved without using independent displacement sensors. Therefore, they are suitable for levitating micro mechanical objects by either magnetic or electrostatic force. However, tuned LC circuit suspension systems suffer from inherent dynamic instability, which has seriously hampered the research and applications of this technique. In this paper, a current feedback stabilizition method is proposed, and its effectiveness is confirmed experimentally.*

## Nomenclature

The symbols used in this paper are listed below. The superscript "~" denotes the total value, and the subscript "e" denotes the value at the equilibrium state. Symbols without superscript "~" and subscript "e" are small variable terms from the equilibrium state.

$E$: Amplitude of source voltage (V)
$E_c$: Amplitude of control voltage (V)
$f$: Source frequency (Hz)
$\omega$: Source angular frequency (rad/s)
$C$: Capacitance of LC circuit (F)
$L$: Inductance of electromagnet (H)
$R$: Coil resistance (ohm)
$x$: Air gap (m)
$F$: Magnetic force (N)
$i$: Coil current (A)
$A$: Coefficient of sin($\omega$t) (A)
$B$: Coefficient of cos($\omega$t) (A)
$H$: Feedback signal ($A^2$)
$m$: Mass of suspended object
$F_d$: Disturbance force (N)
$t$: Times (s)
$T$: Phase lead control parameter
$K_p$: Proportional control parameter
$n$: Phase lead control parameter
$K_s$: spring constant of leaf spring (N/m)

## 1. Introduction

The novel aspect of the suspension technique using tuned LC circuits is that stable suspension can be achieved without employing independent displacement sensors, via either magnetic or electrostatic forces. Therefore, compared to other suspension techniques, it is more suitable to the following systems:

1) Micro mechanical systems such as micro magnetic bearings and micro electrostatic actuators and motors [1], [2]. In such systems, the use of independent displacement sensors increases not only system total size, but also system cost. Traditional displacement sensors are generally big and cannot be directly introduced into such systems, new types of sensors need to be developed.

2) Long distance levitation and transportation systems. For example, in a levitation and transportation system using controlled DC electromagnets with several meters lengths, several hundreds of sensors were used to keep the suspension state [3]. This led to high system cost and difficulty of management.

3) Electrostatic levitation systems for some materials such as glass. In such a system, it is very difficult to detect the displacement of a glass plate by the traditional capacitance, eddy-current or optical sensors.

Although the suspension technique using tuned LC circuits has the above advantages and great industrial and scientific application potential, it has found applications only in some specialized fields. The reason is its inherent dynamical instability. A suspension system using tuned LC circuits is dynami-

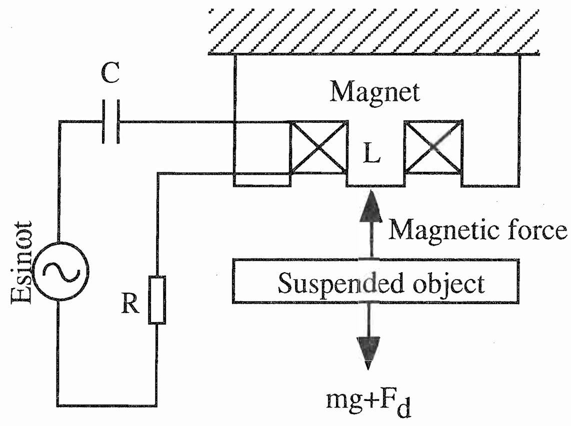


Fig. 1 Magnetic suspension using tuned LC circuit

cally unstable, and the suspended object tends to vibrate divergently if additional damping is not applied on it [4].

In this paper, a new dynamic stabilization method, current feedback damping method is proposed. It uses the coil current, which contains the displacement information of the suspended object, as a feedback signal to control the amplitude of the source voltage and thus to stabilize the suspension system.

## 2. Principle of levitation

Earnshaw's theorem and Braunbeck's subsequent work claim that stable suspension or levitation cannot be achieved by electromagnets whose coil current is constant, whether they are DC or AC. As indicated by its name, tuned LC circuits magnetic suspension modifies the coil current using tuned LC circuits. More in detail, it uses the variation of the inductance of the electromagnet, which is governed by the gap between the electromagnet and the suspended object, to modify the coil current and hence the attractive force. Fig. 1 shows a single degree-of-freedom (DOF) version of magnetic suspension systems using a tuned LC circuit. Here it is supposed that the suspended object can move only in the vertical direction and the effective resistance of coil is constant. The electromagnet is the inductive part of the LC circuit. The LC circuit is designed in such a way that when the suspended object moves away from the electromagnet, the inductance decreases. The LC circuit tends to become resonant, increasing coil current and hence the attractive force, and thus restores the suspended object to its original position. On the other hand, when the suspended object moves towards the electromagnet, inductance increases. The LC circuit goes away from the resonant state, coil current and the attractive force decrease. As a result, the suspended object is pulled down by gravity. Therefore, if the attractive force at a certain position, is balanced against that of gravity, it is possible to obtain a stable equilibrium position for the suspended object.

## 3. System Description

### 3.1 Circuit Equation

The LC circuit in Fig. 1 can be represented as

$$\frac{d(\tilde{L}\tilde{i})}{dt} + R\tilde{i} + \frac{\oint_0^t \tilde{i}dt}{C} = Esin(\omega t) \tag{1}$$

### 3.2 Current Equation

In real suspension systems it has been found: (i) the coil current is a suppressed carrier amplitude modulation signal in the form

$$\tilde{i}(t) = \tilde{A}\sin(\omega t) + \tilde{B}\cos(\omega t) \tag{2}$$

(ii) A and B are functions of the gap

$\tilde{A} = \tilde{A}(\tilde{x}, t)$, $\tilde{B} = \tilde{B}(\tilde{x}, t)$

### 3.3 Force Equation

The attractive force acting on the suspended object is determined by

$$\tilde{F} = -\frac{1}{2}\frac{d\tilde{L}}{d\tilde{x}}\tilde{i}^2 = -\frac{1}{2}\frac{d\tilde{L}}{d\tilde{x}} \times \left[\frac{\tilde{A}^2 + \tilde{B}^2}{2} + \frac{\tilde{B}^2 - \tilde{A}^2}{2}\cos(2\omega t) + \tilde{A}\tilde{B}\sin(2\omega t)\right] \tag{3}$$

Compared to the source frequency, the movement of the suspended object is relatively slow, only the low frequency components in the above equation is important to the movement of the suspended object. Therefore, the components with 2ω can be neglected. As a consequent, the force equation (3) can be approximates by

$$\tilde{F} \approx -\frac{1}{4}\frac{d\tilde{L}}{d\tilde{x}} \times \left(\tilde{A}^2 + \tilde{B}^2\right) \tag{4}$$

### 3.4 Relationship Between Inductance and Gap

In order to calculate the attractive force using (4), $d\tilde{L}/d\tilde{x}$ should be available. The functional relation of inductance $\tilde{L}$ and gap $\tilde{x}$ can be determined experimentally. The following approximate formula is often used to express this relation.

$$\tilde{L}(\tilde{x}) = L_{\infty} + \frac{k_1}{\tilde{x} + k_2} \tag{5}$$

where $\kappa_1$, $\kappa_2$ are positive constants and $L_\infty$ denotes the inductance when the gap is infinite. From (5), $d\tilde{L}/d\tilde{x}$ can be obtained.

### 3.5 Equation of Motion

The equation of motion of the suspended object is

$$\ddot{\tilde{x}}^2 = g - \frac{\tilde{F}}{m} + \frac{F_d}{m} \qquad (6)$$

These five equations, (1), (2), (4), (5) and (6), together represent the suspension system shown in Fig. 1.

## 4. Static Behaviors

### 4.1 Current Equation

Next, let us consider the static behaviors of the magnetic suspension system using a tuned LC circuit shown in Fig. 1. When the suspended object stays in the equilibrium position, inductance $L_e$ is a constant, the LC circuit becomes a linear network, and (1) becomes an ordinary differential equation.

$$L_e\frac{d^2 i_e}{dt^2} + R\frac{di_e}{dt} + \frac{i_e}{C} = E_e\omega\cos(\omega t) \qquad (7)$$

By solving it, we can obtain the coil current at the equilibrium state.

$$i_e = A_e\sin(\omega t) + B_e\cos(\omega t) \qquad (8)$$

where

$$A_e = \frac{E_e R}{\rho^2 + R^2} \qquad B_e = \frac{E_e\rho}{\rho^2 + R^2} \qquad \rho = \frac{1}{C\omega} - L_e\omega \qquad (9)$$

### 4.2 Force Equation

The attractive force at the equilibrium state can be obtained by substituting (9) into (4).

$$F_e = -\frac{1}{4}\left.\frac{d\tilde{L}}{d\tilde{x}}\right|_{x_e} \times \left(A_e^2 + B_e^2\right) = -\frac{1}{4}\left.\frac{d\tilde{L}}{d\tilde{x}}\right|_{x_e} \frac{E^2}{\rho^2 + R^2} \qquad (10)$$

### 4.3 Force Equilibrium Equation

At the equilibrium state, the attractive force should be equal to the weight of the suspended object.

$$F_e = mg \qquad (11)$$

## 5. Current Feedback Stabilization

### 5.1 Basic Concept of Current feedback

As mentioned in section 3.2, the coil current contains the displacement information of the suspended object. This fact provides for the possibility of stabilizing the suspension system using current feedback instead of direct position feedback. The basic idea of the current feedback control is to control the amplitude of the source voltage and thus to stabilize the suspension system using the current as feedback signals, as shown in Fig. 2.

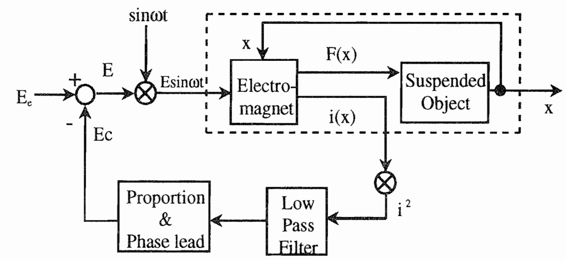


Fig. 2 Current feedback stablization method

In order to make the suspension system stable, it is supposed that the amplitude of the source voltage should be controlled as

$$E = E_e - E_c \qquad (12)$$

where $E_e$ is the source voltage at the equilibrium state, and $E_c$ is the control voltage. We take the change of amplitude of the coil current as the feedback signal,

$$H = (A_e + A)^2 + (B_e + B)^2 - (A_e + B_e)^2 \approx 2A_eA + 2B_eB \qquad (13)$$

The control law we have employed is a proportion and phase lead compensation,

$$E_c(s) = K_pH(s) + \frac{1+nTs}{1+Ts}H(s) \qquad (14)$$

where n is an integral number greater than one, T and $K_p$ are positive constants, and "s" denotes the Laplace transformation.

### 5.2 New Circuit Equation

It is clear from the motion equation, (6), that in order to obtain the closed system transfer function, i.e., the relationship between the movement of the suspended object, $\tilde{x}$, and the disturbance force $F_d$, we must first get the relation of the magnetic force $\tilde{F}$ and the gap $\tilde{x}$. From (4), we can see that the function relation of $\tilde{F}$ and the gap $\tilde{x}$ is depending on the relations between $\tilde{A}$, $\tilde{B}$ and $\tilde{x}$. Therefore, we should start our analysis from solving (1), the equation of the tuned LC circuit.

Inserting (12) into (1), we have the new equation representing the LC circuit with current feedback control.

$$\frac{d(\tilde{L}\tilde{i})}{dt} + R\tilde{i} + \frac{\oint_0^t \tilde{i}\,dt}{C} = \left(E_e - E_c\right)\sin(\omega t)$$

or

$$\frac{d^2(\tilde{L}\tilde{i})}{dt^2} + R\frac{d\tilde{i}}{dt} + \frac{\tilde{i}}{C} = \left(E_e - E_c\right)\omega\cos\left(\omega t\right) - \frac{dE_c}{dt}\sin(\omega t) \qquad (15)$$

### 5.3 New Current Equation

Inserting (2) into (15), and comparing the coefficients of $\sin(\omega t)$ and $\cos(\omega t)$ separately, the next two equations can be obtained.

$$\begin{cases} \dfrac{d^2(\tilde{L}\tilde{A})}{dt^2} + R\dfrac{d\tilde{A}}{dt} - 2\omega\dfrac{d(\tilde{L}\tilde{B})}{dt} + \left[\dfrac{1}{C} - \tilde{L}\omega^2\right]\tilde{A} - R\omega\tilde{B} \\ = -\dfrac{dE_c}{dt} \\ \dfrac{d^2(\tilde{L}\tilde{B})}{dt^2} + R\dfrac{d\tilde{B}}{dt} + 2\omega\dfrac{d(\tilde{L}\tilde{A})}{dt} + \left[\dfrac{1}{C} - \tilde{L}\omega^2\right]\tilde{B} + R\omega\tilde{A} \\ = \left[E_e - E_c\right]\omega \end{cases} \quad (16)$$

Near the equilibrium state, system parameters can be approximated in the forms

$$\begin{cases} \tilde{x} = x_e + x, \quad \tilde{L} = L_e + L, \\ \tilde{A} = A_e + A, \quad \tilde{B} = B_e + B, \quad L = K_x x \\ \dfrac{d\tilde{L}}{dt} = \dfrac{d\tilde{L}}{d\tilde{x}}\dfrac{d\tilde{x}}{dt} \quad K_x = \left.\dfrac{d\tilde{L}}{d\tilde{x}}\right|_{x_e} \quad K_{2x} = \left.\dfrac{d^2\tilde{L}}{d\tilde{x}^2}\right|_{x_e} \\ \left.\dfrac{d\tilde{L}}{d\tilde{x}}\right|_{\tilde{x}} = \left.\dfrac{d\tilde{L}}{d\tilde{x}}\right|_{x_e} + \left.\dfrac{d^2\tilde{L}}{d\tilde{x}^2}\right|_{x_e} x = K_x + K_{2x}x \end{cases} \quad (17)$$

Inserting these formulae and system static characteristics, (9) into (16), and neglecting the nonlinear components of minute variation, we have

$$\begin{cases} L_e\dfrac{d^2A}{dt^2} + R\dfrac{dA}{dt} + \rho\omega A - a\dfrac{dB}{dt} - bB \\ = c\dfrac{d^2x}{dt^2} + d\dfrac{dx}{dt} + ex - \dfrac{dE_c}{dt} \\ a\dfrac{dA}{dt} + bA + L_e\dfrac{d^2B}{dt^2} + R\dfrac{dB}{dt} + \rho\omega B \\ = f\dfrac{d^2x}{dt^2} + g\dfrac{dx}{dt} + hx - \omega E_c \end{cases} \quad (18)$$

Their Laplace transformations are

$$\begin{cases} \left(L_e s^2 + Rs + \rho\omega\right)A(s) - (as + b)B(s) \\ = \left(cs^2 + ds + e\right)X(s) - sE_c(s) \\ (as + b)A(s) + \left(L_e s^2 + Rs + \rho\omega\right)B(s) \\ = \left(fs^2 + gs + h\right)X(s) - \omega E_c(s) \end{cases} \quad (19)$$

where

$$\begin{cases} a = 2L_e\omega \quad b = R\omega \\ c = -A_e K_x \quad d = 2\omega B_e K_x \\ e = \omega^2 A_e K_x \quad f = -B_e K_x \\ g = -2\omega A_e K_x \; h = \omega^2 B_e K_x \end{cases}$$

Inserting control equations (13) and (14) into (19), the following two equations are derived.

$$\begin{cases} \sum_{k=0}^{3} \theta_k s^k A(s) + \sum_{k=0}^{3} \chi_k s^k B(s) = \sum_{k=0}^{3} \delta_k s^k X(s) \\ \sum_{k=0}^{3} \tau_k s^k A(s) + \sum_{k=0}^{3} \psi_k s^k B(s) = \sum_{k=0}^{3} \sigma_k s^k X(s) \end{cases} \quad (20)$$

where

$$\begin{cases} \theta_3 = TL_e \\ \theta_2 = L_e + T\left[R + 2\left(n + K_p\right)A_e\right] \\ \theta_1 = R + T\rho\omega + 2\left(1 + K_p\right)A_e \\ \theta_0 = \rho\omega \end{cases} \quad \begin{cases} \delta_3 = Tc \\ \delta_2 = Td + c \\ \delta_1 = Te + d \\ \delta_0 = e \end{cases}$$

$$\begin{cases} \chi_3 = 0 \\ \chi_2 = T\left[2\left(n + K_p\right)B_e - a\right] \\ \chi_1 = 2\left(1 + K_p\right)B_e - Tb - a \\ \chi_0 = -b \end{cases} \quad \begin{cases} \sigma_3 = Tf \\ \sigma_2 = Tg + f \\ \sigma_1 = Th + g \\ \sigma_0 = h \end{cases}$$

$$\begin{cases} \tau_3 = 0 \\ \tau_2 = Ta \\ \tau_1 = T\left[2\omega\left(n + K_p\right)A_e + b\right] + a \\ \tau_0 = 2\omega\left(1 + K_p\right)A_e + b \end{cases}$$

$$\begin{cases} \psi_3 = TL_e \\ \psi_2 = TR + L_e \\ \psi_1 = T\left[2\omega\left(n + K_p\right)B_e + \rho\omega\right] + R \\ \psi_0 = 2\omega\left(1 + K_p\right)B_e + \rho\omega \end{cases}$$

Solving (20), we can get the functional relations between A(s), B(s) and X(s).

$$A(s) = \frac{\sum_{k=0}^{6} p_k s^k}{\sum_{k=0}^{6} \gamma_k s^k} X(s) \tag{21}$$

$$B(s) = \frac{\sum_{k=0}^{6} \eta_k s^k}{\sum_{k=0}^{6} \gamma_k s^k} X(s) \tag{22}$$

where

$$\begin{cases} p_k = \sum_{j=0}^{k} \delta_j \psi_{k-j} - \sum_{j=0}^{k} \chi_j \sigma_{k-j} & (k = 0,1,2,3) \\ p_k = \sum_{j=k-3}^{3} \delta_j \psi_{k-j} - \sum_{j=k-3}^{3} \chi_j \sigma_{k-j} & (k = 4,5,6) \end{cases}$$

$$\begin{cases} \eta_k = \sum_{j=0}^{k} \theta_j \sigma_{k-j} - \sum_{j=0}^{k} \delta_j \tau_{k-j} & (k = 0,1,2,3) \\ \eta_k = \sum_{j=k-3}^{3} \theta_j \sigma_{k-j} - \sum_{j=k-3}^{3} \delta_j \tau_{k-j} & (k = 4,5,6) \end{cases}$$

$$\begin{cases} \gamma_k = \sum_{j=0}^{k} \theta_j \psi_{k-j} - \sum_{j=0}^{k} \chi_j \tau_{k-j} & (k = 0,1,2,3) \\ \gamma_k = \sum_{j=k-3}^{3} \theta_j \psi_{k-j} - \sum_{j=k-3}^{3} \chi_j \tau_{k-j} & (k = 4,5,6) \end{cases}$$

### 5.4 New Force Equation

Near the equilibrium state, the force equation (4) cab be simplified to

$$\begin{aligned} \tilde{F} &= -\frac{1}{4}\frac{d\tilde{L}}{d\tilde{x}} \times \left(\tilde{A}^2 + \tilde{B}^2\right) \\ &\approx F_e - q_1 A - q_2 B - q_3 x \end{aligned}$$

Change of force due to gap variation is

$$F = -q_1 A - q_2 B - q_3 x \tag{23}$$

where

$$q_1 = K_x A_e / 2, \quad q_2 = K_x V_e / 2, \quad q_3 = K_{2x}(A_e^2 + B_e^2)/ 4$$

Inserting (21) and (22) into (23), we have the new force equation as

$$F(s) = \frac{\sum_{k=0}^{6} \beta_k s^k}{\sum_{k=0}^{6} \gamma_k s^k} X(s) \tag{24}$$

where

$$\beta_k = q_1 p_k + q_2 \eta_k + q_3 \gamma_k$$

### 5.5 Closed System Equation

Inserting (24) into (6), the closed system transfer function is available.

$$X(s) = \frac{\sum_{k=0}^{6} \gamma_k s^k}{\sum_{k=0}^{8} \alpha_k s^k} F_d(s) \tag{25}$$

where

$$\alpha_k = m\gamma_{k-2} + \beta_k \tag{26}$$

Equation (25) is what we are searching for, the transfer function model for the tuned LC circuit magnetic suspension system controlled by current feedback.

### 5.6 System Stability

The characteristic equation of the closed system is

$$\sum_{k=0}^{8} \alpha_k s^k = 0 \tag{27}$$

From the definitions of $\alpha_k$, formulae (26), we can see that it is a very hard work to give a formula which specify the values of $K_p$, T, n, to make sure all the solutions of (27) have negative real parts and thus ensure the system stability. An practical way is to choose them via a numerical calculation.

## 6. Suspension Experiment

Using the current feedback stabilization method mentioned above, we have carried out a suspension experiment in an one-degree-of-freedom system shown in Fig. 3. The electromagnet is fastened to a parallel leaf spring, which moves in the horizontal direction and its restoring force balances against the magnetic attractive force when the spring moves to the left of its natural position. If the suspension is successful, the electromagnet should rest stably in a position to the left, with a certain gap respect to the target. The experimental parameters are listed in Table I. Fig. 4 shows changes of the source volt-

Table I System parameters

| | | | |
|---|---|---|---|
| C | 9.54 (μF) | E | 42.4 (V) |
| R | 16.4 (ohm) | f | 400 (Hz) |
| $L_e$ | 19.2 (mH) | $x_e$ | 1.4 (mm) |
| $K_x$ | -5.75 (H/m) | $A_e$ | 2.23 (A) |
| $K_{2x}$ | 5000 (H/m$^2$) | $B_e$ | -0.889 (A) |
| m | 1.07 (kg) | n | 5 |
| $K_p$ | 0.94 | T | 0.01 |
| $K_s$ | 1.37 (kgf/mm) | | |

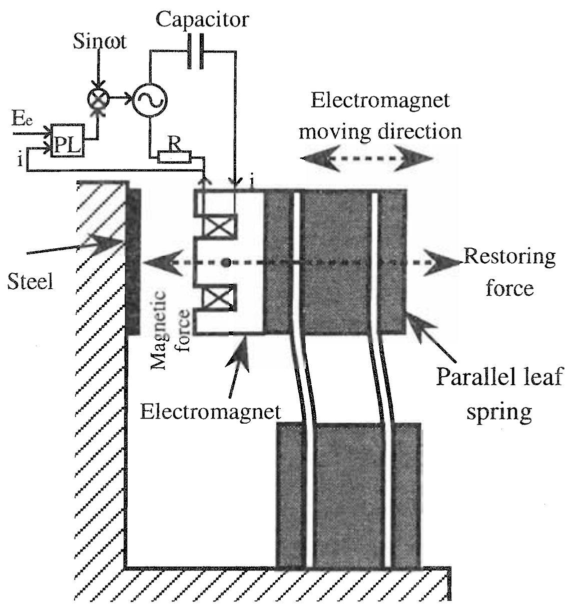


Fig. 3 Schematic of expeimental system

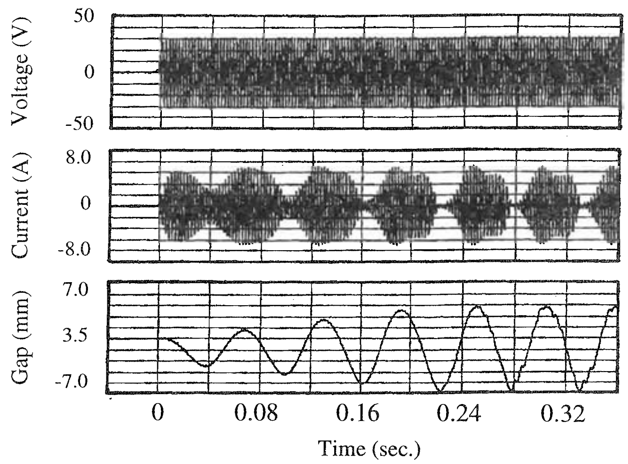


Fig. 4 System response withot control

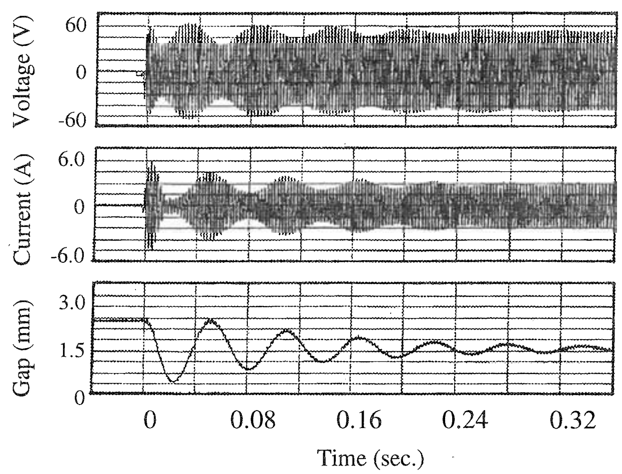


Fig. 5 System response with current-feedback control

age, coil current and the movement of the suspended object in the case of no control. It is noted that the movement of the magnet (suspended object) was divergent. Fig. 5 shows changes of the source voltage, coil current and the movement of the suspended object in the case that the current feedback control was introduced. The magnet finally rested stably at a 1.4 mm distance away from the steel target.

## 7. Conclusions

In this paper, a new dynamic stabilization method for tuned LC circuit magnetic suspension systems, the current feedback stabilization method, has been discussed. Its effectiveness is confirmed by experimental results from an one degree-of-freedom suspension system. We believe that this method will play an important role in the future research and applications of the electrostatic and magnetic suspension systems employing tuned LC circuits.

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