Rotor Disturbance Attenuation Using An H∞ Controller for Active Magnetic Bearings

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Abstract

The purpose of this paper is to demonstrate the performance of an H∞ optimized controller for a magnetic bearing, in theory and experimentally. The application of magnetic bearings to practical rotating machinery is becoming more and more widespread in recent years. Often, machinery is operated under conditions different from the design point, such as compressors or pumps at low load conditions, which results in high fluid loads at low frequency. These forces may create vibration problems. This paper develops a control methodology to stabilize the magnetic bearing/rotor control loop, in the presence of uncertainty, and minimize the worst-case excursion of rotor vibrations due to low frequency disturbances. The results are compared to an equivalent PD controller.

1 Introduction

There have been great advances in the techniques for the design of robust uncertainty tolerant feedback control systems in the past two decades. Using this theory enables designers to achieve very precise frequency domain loop shaping via suitable weighting functions.

Magnetic bearing controller design is the subject of many research activities at this time. A general Introduction to magnetic bearing actuator and PID(proportional, integral, derivative) controller design is given by Allaire, et al.[1]. Humphris et al.[2] reported on magnetic bearing control of a rotor using PD controls and investigated the stability limits of the system. Matsushita, et al.[3] modeled a multimass vertical flexible rotor and developed a PID controller with some notch filters.

Pinkney and Keesee [4] presented the design data for a four stage natural gas pipeline compressor operating between the first and second rotor modes. In the design process, they reported a relatively low original bearing stiffness, with an average value of approximately 50,000N/mm in the operating speed range, which was increased to an average value of approximately 90,000N/mm. The authors reported increasing the controller gain at all frequencies and the phase lead by approximately 45 degrees above 150Hz. Dhar et al.[5] discussed the use of constrained parameter optimization for low order decentralized magnetic bearing controllers.

Optimal regulator methods are used to design magnetic bearing controllers. Akishita et al.[6] discussed controller design for a flexible rotor using an optimal regulator containing a low pass filter. The filter design was based only upon the two rotor rigid body modes and a frequency shaped regulator was developed. Matsushita et al.[7] considered magnetic levitation with an PID controller including rigid body modes and first rotor bending mode. They discussed the implementation of a 3rd order low pass filter (LPF), which replaces a conventional 1st order LPF, to improve on rotor system instability above the second rotor bending modes due to spillover problems. A linear quadratic regulator (LQR) was used to obtain the controller. The location of the center frequency of the additional

2nd order LPF is obtained from the eigenvalue of the first rotor bending mode.

Recently, magnetic bearing controllers have employed H∞ controllers to improve performance for flexible rotors. Herzog and Bleuler [8] evaluated the use of H∞ controls to attain high stiffness magnetic bearings when the rotor system is subjected to unknown disturbance forces over a frequency range, such as a milling spindle. The discussion developed two H∞ controller approaches applied to a simple two mass plant to illustrate the design trade off conditions. Stabilizing controllers can be found which yield arbitrarily low compliance for either mass but not for both simultaneously. This work was extended to more general systems in [9] where several theorems were developed concerning mechanical ladder and non-ladder systems. The paper also discussed other properties 1) only a limited control effort could be expended due to limitations on achievable gains and bandwidths, 2) the "size" of certain closed loop functions must not exceed given bounds, 3) robust stability and/or robust performance for a given class of "neighbored" plants is required.

Fujita et al.[10] discussed the use of an H∞ controller for a flexible beam with the MATLAB Robust-Control Toolbox [11]. This controller design employed two indefinite Riccati equations to design controller. Nonami, et al.[12] reported on the use of an H∞ controller to remove spillover problems in magnetic bearing control. Cui and Nonami [13] considered a three mass rotor on magnetic bearings. Two reduced order models were developed, one with only rotor rigid body modes and another by including both rotor rigid body modes and the first rotor bending mode, to design an H∞ output feedback control. The controllers were found to avoid spillover problems.

Recently, Kanemitsu, et al.[14] compared several different control approaches for a simple magnetic suspension. Included were PID, H∞, LQG(Linear Quadratic Gaussian), sliding mode, TDC(Time Delay Control). The authors noted that the H∞ controller had the properties of excellent disturbance rejection, stability, and robustness. However, they stated that weighting functions must be included in the model to allow for disturbance and error which results in a time consuming process to design the controller.

2 Experimental Test Rig

A magnetic bearing test rig [15,16,17], illustrated in Fig.1 was set up for the controls experiments. The rotor consisted of a 12.7mm (0.050 in) diameter shaft with three attached masses. The midspan disk measured 73.2mm(2.88in) in diameter and 25.4mm(1.0in) in length. The two outboard disks were the bearing journals, and measured 58.4mm (2.3in) in diameter and 25.4mm(1.0in) in length. An small electric motor was used to drive the test rig at various speeds. The coupling stiffness was evaluated as 17.5N/mm(100lbf/in) and damping of 0.011N-s/mm(0.06lbf-s/in).

The magnetic bearings were eight pole radial bearings. The leg width was 12.7mm(0.50in) for each pole. The stators were constructed of solid soft magnetic iron and the journals were constructed of 0.18mm(0.007in) laminations of non oriented 3% silicon iron. The bearings had a 1.0mm(0.039in) air gap. The initial control feedback system used an analog PD controller with four linear power amplifiers.

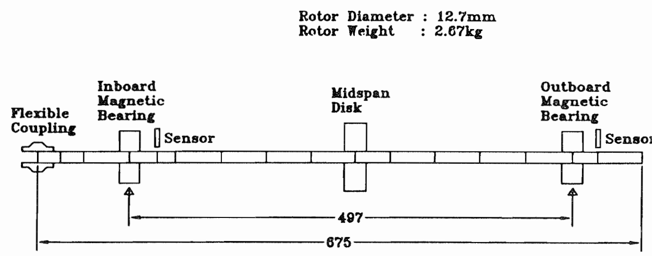

Fig.1 Experimental test rig

3 Theoretical Rotor Model

The rotor equations of motion have the general matrix form

$$[M]\frac{d^2}{dt^2}[U]+\{[G]+[C]\}\frac{d}{dt}[U]+[K][U]=[F] \tag{3-1}$$

where

[M] : mass matrix, $4n \times 4n$
[U] : displacement vector, $4n \times 1$
[G] : gyroscopic matrix, $4n \times 4n$
[C] : damping matrix, $4n \times 4n$
[K] : stiffness matrix, $4n \times 4n$
[F] : force vector, $4n \times 1$
n : number of rotor mass stations

Since the order of a controller is depend on degrees of freedom of the rotor model, it is needed to reduce the dimension of equation(3-1). Using the modal matrix , which are found by solution of the eigenvalue problem, we obtain the expression,

$$\frac{d^2}{dt^2}[q]+[\phi]^t\{[G]+[C]\}[\phi]\frac{d}{dt}[q]+[\lambda][q]=[\phi]^t[F] \tag{3-2}$$

where

[q] : modal coordinate, $2m \times 1$

[ϕ] : modal matrix(mass normalized), $2m \times 2m$

[λ] : eigenvalue matrix, $2m \times 2m$

m : number of modes

In this rotor model, we consider two rigid modes and one bending mode. Therefore, the matrix size in equation(3-2) is 6×6 and much smaller than that of equation(3-1). The stability of the higher modes which we ignore here are guaranteed by uncertainty weighting function.

Equation (3-2) can be represented as a set of first order differential equations. In state-space form, the rotor model can be written as

$$\frac{d}{dt}\begin{bmatrix} q \\ \dot{q} \end{bmatrix}=\begin{bmatrix} O & I \\ -[\lambda] & -[\phi]^t\{[G]+[C]\}[\phi] \end{bmatrix}\cdot\begin{bmatrix} q \\ \dot{q} \end{bmatrix}+\begin{bmatrix} O \\ [\phi]^t \end{bmatrix}[F]$$

$$[U]=[\phi][q] \tag{3-3}$$

The transfer function from the outboard bearing to each sensor is shown in Fig.2.

———— : response at the outboard bearing

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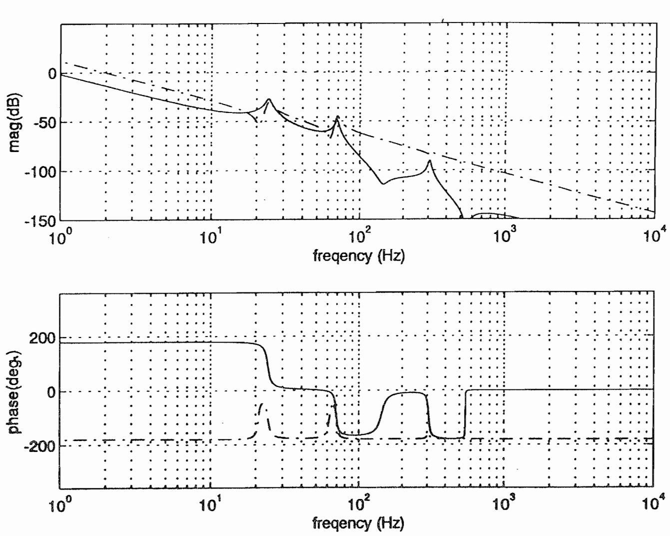


Fig.2 Rotor transfer functions
(Force is added at outboard bearing)

4 Controller Design

4.1 PD Controller

A 3rd order PD magnetic bearing controller was designed for the rotor/bearing system with the transfer function

$$K_{PD}(S)=\frac{-2.47\times10^6S^2+1.20\times10^{10}S+3.10\times10^{12}}{S^3+8.28\times10^4S^2+8.28\times10^8S+10^{12}} \tag{4-1}$$

4.2 Rotor/Bearing System

The theoretical model for the rotor/bearing system is shown in Fig.3,4 in block diagram form. Fig.3 indicates the components of the inboard bearing model. Fig.4 shows the system block diagram where the block representing the inboard bearing contains all of the detail from Fig.3. The outboard controller has single input and single output. The input is the signal from the eddy current sensor located at outboard side and the output is the command for the amplifier. An H∞ controller is applied to the outboard bearing.

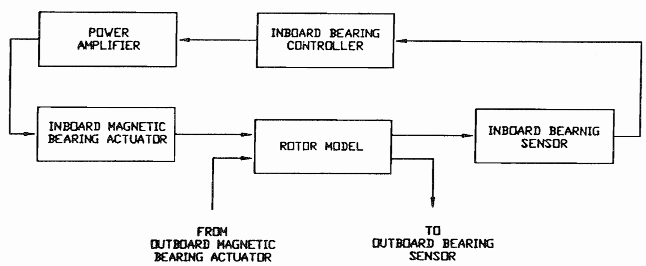


Fig.3 Inboard bearing model

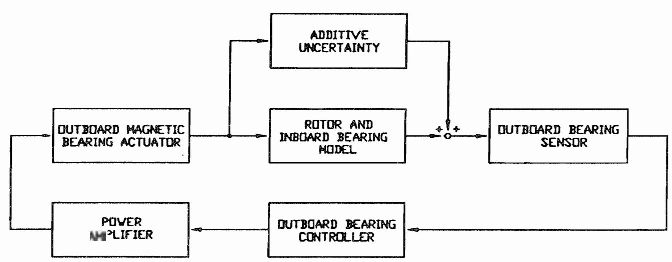


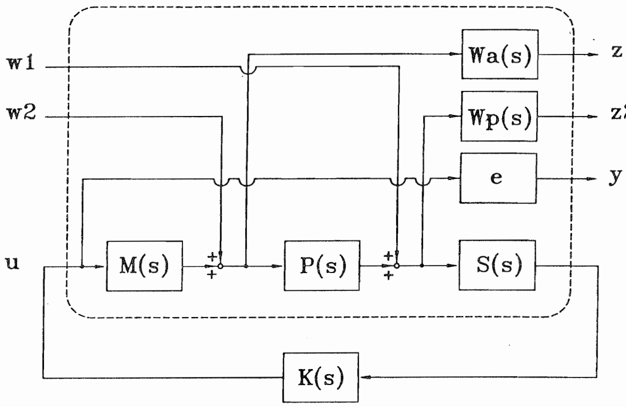
Fig.4 Rotor bearing system model

4.3 Augmented Plant

In order to design the H∞ controller, an augmented plant is developed, as shown Fig.5. Here w1 represents noise input to the rotor model and w2 is noise in the external force signal. Block "e" is a dummy block to solve H∞ problem.

4.4 Weighting function

Fig.5 also gives the additive uncertainty



M(s) : Power Amplifier and Actuator

P(s) : Rotor and Inboard Magnetic Bearing

S(s) : Sensor

K(s) : Controller

Fig.5 Augmented plant

weighting function Wa(s) and the performance weighting function Wp(s) employed in the model.

As has been mentioned before, the modes which are higher than 2nd bending mode are ignored in the rotor model. In order to stabilize these modes, we defined a weighting function Wa(s) as

$$W_a(S) = \frac{1.43\times10^{-4}\times\dfrac{S}{2\pi\times400}}{\dfrac{S}{2\pi\times5000}+1} \qquad (4\text{-}2)$$

The aim of control here is to minimize the worst-case excursion of rotor vibrations resulting from the aerodynamic disturbance. For this purpose, a performance weighting function Wp(s) is defined as

$$W_p(S) = \frac{12}{\left(\dfrac{S}{2\pi\times20}\right)^2+\left(\dfrac{S}{2\pi\times20}\right)+1} \cdot \frac{1}{\left(\dfrac{S}{2\pi\times90}\right)^2+0.4\left(\dfrac{S}{2\pi\times90}\right)+1} \qquad (4\text{-}3)$$

The bode diagram of weighting functions are shown in Fig.6. As the figure indicates, Wa(s) takes high gain at high frequency range, while Wp(s) takes high gain at low frequency range.

4.5 H∞ Optimal Controller

An H∞ controller was designed by the MATLAB Robust Control Toolbox [11] and the result for a 16th

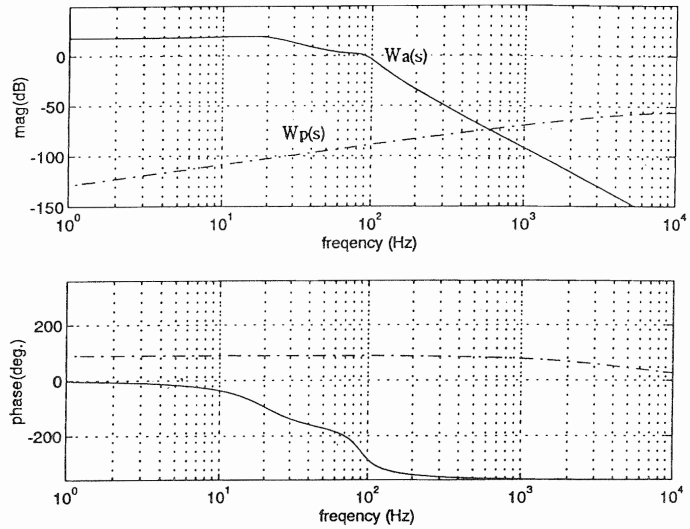


Fig.6 Weighting functions

order controller is shown in Fig.7 with the PD controller transfer function. The low frequency gain is higher than the PD controller resulting in higher stiffness. Table 1 gives the calculated poles and zeros of the controller.

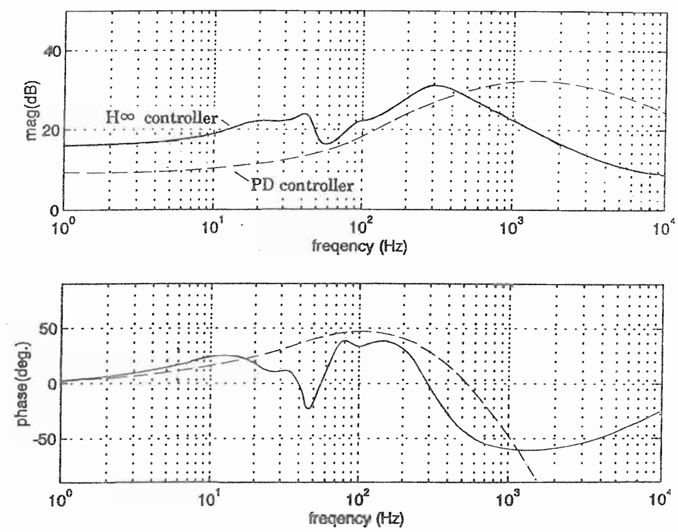


Fig.7 Transfer function of H∞ controller

4.6 Controller Order Reduction

The order of the controller is 16th and about five times higher than that of PD controller. From the view point of realization by a digital controller, it is desirable to reduce the order of the controller. It is easily seen that the H∞ controller has pole and zero cancellation in the poles and zeros labeled 2 through 5 in Table 1. All of these high frequency pole zero cancellations are at high frequency relative to the

Table 1 Poles and zeros of the H∞ controller

No.	poles	zeros
1	- 1.1677e6	- 3.1416e4
2	- 2.7843e4	- 2.7843e4
3	- 9.8715e3	- 9.8715e3
4	- 4.1432e3	- 4.1433e3
5	- 7.5961e2	- 7.5765e2
6	- 2.4097e3	- 1.6667e3
7	- 8.7010e2 + i*1.6565e3	- 1.4896e2
8	- 8.7010e2 - i*1.6565e3	- 1.6279e2 + i*5.5362e2
9	- 1.1310e2 + i*5.5406e2	- 1.6279e2 - i*5.5362e2
10	- 1.1310e2 - i*5.5406e2	- 1.4248e2 + i*4.7234e2
11	- 1.6309e2 + i*4.7020e2	- 1.4248e2 - i*4.7234e2
12	- 1.6309e2 - i*4.7020e2	- 6.2235e1 + i*3.1997e2
13	- 4.0366e1 + i*2.7346e2	- 6.2235e1 - i*3.1997e2
14	- 4.0366e1 - i*2.7346e2	- 1.5525e2 + i*8.2931e1
15	- 6.2832e1 + i*1.0883e2	- 1.5525e2 - i*8.2931e1
16	- 6.2832e1 - i*1.0883e2	

rotor operating speed and can be dropped from the actual controller.

The bode diagram of the reduced order controller and original controller are shown in Fig.8. There is no big difference between them under 2kHz.

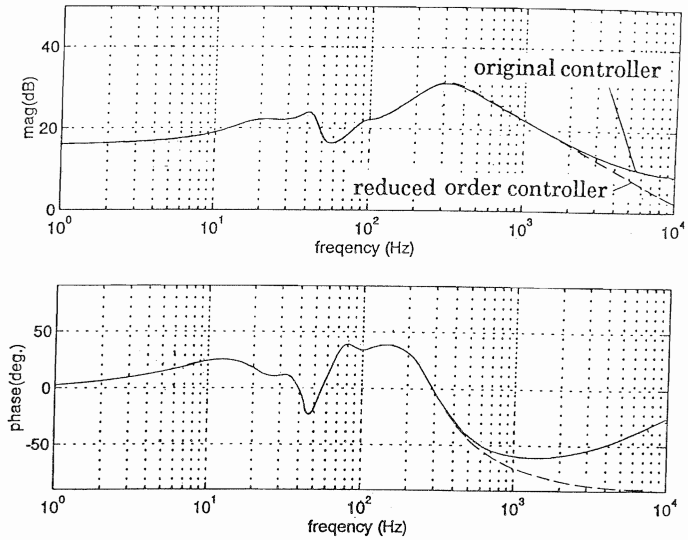


Fig.8 Reduced order controller

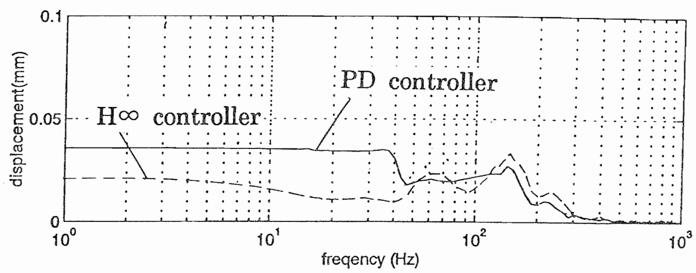


Fig.9 Experimental result

5 Test Results

Excitation tests were made to verify the controller performance. The H∞ controller is converted to the DSP system with the sampling time of 50 μs.

The vibration response of the rotor was measured while the rotor was excited at the outboard magnetic bearing position. In order to excite the rotor, a sinusoidal signal was added to the control signal. The force amplitude was 4.3N constant. The response is compared between the H∞controller and the PD controller in Fig.9. The response for H∞ controller is about 1/3 compared to the that of PD controller at low frequency.

We also made rotating tests to the speed of 4200rpm, and made sure that each controller kept the rotor/bearing system stable within the speed range.

6 Conclusion

An H∞ optimized controller was applied to a magnetic bearing system. The purpose of the controller is to minimize rotor vibrations due to low frequency disturbances. The experimental result demonstrates that the designed H∞ controller is superior to the conventional PD controller in rotor disturbance attenuation.

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