

ROBUST CONTROL OF HIGH SPEED SPINDLE-MAGNETIC BEARING SYSTEM USING SLIDING MODE CONTROL AND VSS DISTURBANCE OBSERVER

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Abstract: This paper is concerned with the design and control of a spindle-magnetic bearing systems using with the discrete time sliding mode control method. A variable structure system (VSS) disturbance observer, with which the state vector and disturbance can be estimated simultaneously, is investigated to compensate the mismatched disturbance loaded on the rotor. A new discrete time sliding mode servo controller which can attenuate the chattering phenomenon is proposed by using the VSS observer. The usefulness of the proposed controller is demonstrated through the computer simulations and experiments.

1. Introduction

In the design of magnetic bearing systems, it is necessary to use an asymptotically stable and robust controller for magnetic bearings to support flexible rotor systems which pass higher-order flexible modes. However, the controller should be constructed with reduced order models to control the full order vibration system because of the limitation of computation time. Recently the linear robust control theory has recently been developed and used to control the flexible magnetic bearing systems [1-2]. But the plant model used in these references was simplified and idealized.

As a precise and robust nonlinear control theory, the variable structure system (VSS) with sliding mode control (SMC) has been considered for the magnetic levitation and magnetic bearing systems[3-4]. However, there exists some drawbacks which was imposed on SMC in many practical applications[5-6]. One of these drawbacks is the resultant control may yield over conservative feedback gains due to overestimated bounds on system perturbations, which can cause an inevitable chattering. Another drawback is the uncertainties and disturbances should be required to guaranteed the matching conditions.

Based on the studies of linear observer design, some research has recently been conducted on SMC with a

linear disturbance observer to obtain the lower switching gains and less chatter[7-8]. However, the switching gains of SMC in these schemes are difficult to be determined and the mismatched disturbance problem is not discussed. In the previous works[9-10], the discrete time sliding mode controller was designed to flexible rotor-magnetic bearing systems which is shown that the chattering phenomenon can be alleviated in sliding motion. In this paper, we present a new design of discrete time sliding mode controller by using a discrete robust VSS disturbance observer for the flexible rotor-magnetic bearing system(FR-MBS) used in the grinding spindle. The simulations and experiments for this flexible rotor-magnetic bearing system are given which verify that the discrete time control system is robust to uncertainty and can compensate the imbalance.

2. Modeling of plant

The flexible-rotor magnetic bearing model is made from a high speed grinding spindle, which can be regarded as consisting of 12 elements of a model based on the finite element method (FEM) shown in Fig.1.

2.1 Flexible rotor dynamics

According to the FEM model shown in Fig.1, the discrete model of twenty-six orders can be written as

$$M\ddot{q} + C\dot{q} + Kq = 0 \quad (1)$$

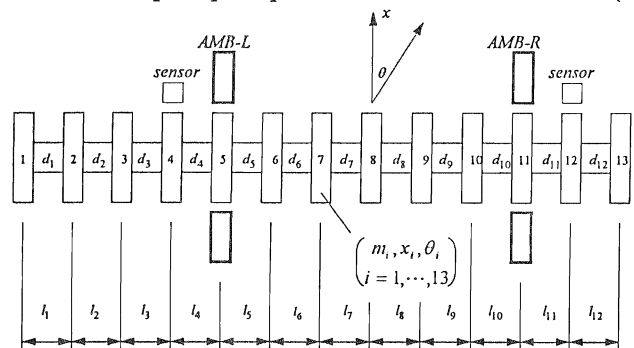


Figure1 Model of flexible rotor-magnetic bearing system

the case in which only the rigid modes and the first bending mode ($r=3$) are calculated and $m=2, l=2$.

3. Discrete Time VSS Disturbance Observer Design

To compensate the mismatched disturbance directly, a disturbance estimation should be utilized to produce a signal of the disturbance. The discrete time VSS observer will be described in this section to estimate the system state vector and the mismatched disturbance simultaneously. The state equation of this disturbance can be expressed in

$$\dot{w} = A_w w, \quad d = Lw \quad (10)$$

where

$$w = \begin{bmatrix} w_1 \\ \dot{w}_1 \end{bmatrix}, \quad A_w = \begin{bmatrix} 0 & 1 \\ -\omega_0^2 & 0 \end{bmatrix}, \quad L = \begin{bmatrix} 1 & 0 \end{bmatrix}$$

and $w_1 = a \sin \omega_0 t$ is the imbalance with frequency ω_0 , d is the output. Combining Eq.(10) with Eq.(9). The augmented state equation can be written as

$$\dot{x}_d = A_d x_d + B_d u + D_d \quad (11a)$$

$$y_d = C_d x_d \quad (11b)$$

where

$$x_d = \begin{bmatrix} x_r^T & w^T \end{bmatrix}^T, \quad A_d = \begin{bmatrix} A_r & D_{r1} L \\ 0_{2 \times 8} & A_w \end{bmatrix}, \quad B_d = \begin{bmatrix} B_r \\ 0_{2 \times 2} \end{bmatrix}$$

$$C_d = \begin{bmatrix} C_r & 0_{2 \times 2} \end{bmatrix}, \quad D_d = \begin{bmatrix} D_{r2}^T & 0_{1 \times 2} \end{bmatrix}^T$$

By assuming that the u is the output of a zero-order holder, the equivalent discrete-time system of Eq.(11) is as follows

$$x_d(k+1) = \Phi_d x_d(k) + \Gamma_{d1} u(k) + \Gamma_{d2}(k) \quad (12a)$$

$$y_d(k) = C_d x_d(k) \quad (12b)$$

Here we assume the following matching condition is guaranteed by the uncertainties Γ_{d2}

$$\Gamma_{d2}(k) = \Gamma_{d1} h(k) \quad (13)$$

where $h(k)$ is assumed to be bounded

$$\|h(k)\| \leq \eta, \quad \eta > 0$$

The objective is to design a VSS observer that would estimate the system state vector $x_r(k)$ and imbalance force $w_1(k)$. If selecting the error of state as $\bar{x}_d(k) = \hat{x}_d(k) - x_d(k)$, the robust VSS observer can be described by following equation

$$\hat{x}_d(k+1) = \Phi_0 \hat{x}_d(k) + G_0 y_d(k) + M(\bar{y}_d(k)) + \Gamma_{d1} u(k) \quad (14)$$

where

$$M(\bar{y}_d(k)) = -\Gamma_{d1} \frac{F_1 \bar{y}_d(k)}{\|F_1 \bar{y}_d(k)\| + \gamma} \rho, \quad \hat{x}_d = \begin{bmatrix} \hat{x}_r^T & \hat{w}^T \end{bmatrix}^T,$$

$$\bar{y}_d(k) = y_d(k) - \hat{y}_d(k) = y_d(k) - C_d \hat{x}_d(k)$$

here parameter ρ and $\gamma > 0$ are positive numbers.

4. Design of Discrete Time Sliding Mode Controller

This section will discuss the design of digital sliding mode controller using the state vector and disturbance estimated in above section. To have a zero steady state error

for this system, the conventional reduced system of Eq.(9) is augmented by the integral variables as follows:

$$\dot{z} = r - \hat{x} \quad (15)$$

where

$$r = \begin{bmatrix} r_5 & r_{11} \end{bmatrix}^T, \quad \hat{x} = \begin{bmatrix} \hat{x}_5 & \hat{x}_{11} \end{bmatrix}^T$$

and r is the reference input vector at actuator positions. The extended state equation can be given by

$$\dot{x}_i = A_i x_i + B_i u + D_i + Gr \quad (16a)$$

$$y = C_i x_i = \begin{bmatrix} x_4 & x_{12} \end{bmatrix}^T \quad (16b)$$

where

$$x_i = \begin{bmatrix} \hat{x}_r^T & z^T \end{bmatrix}^T, \quad z = \begin{bmatrix} z_5 & z_{11} \end{bmatrix}^T, \quad r = \begin{bmatrix} r_5 & r_{11} \end{bmatrix}^T$$

$$A_i = \begin{bmatrix} A_r & 0_{8 \times 2} \\ -E_2 & 0_{2 \times 2} \end{bmatrix}, \quad B_i = \begin{bmatrix} B_r \\ 0_{2 \times 2} \end{bmatrix}, \quad D_i = \begin{bmatrix} D_r \\ 0_{2 \times 1} \end{bmatrix}, \quad G = \begin{bmatrix} 0_{8 \times 2} \\ I_{2 \times 2} \end{bmatrix}$$

$$E_2 = \begin{bmatrix} (F^T \Psi)_{2 \times 6} & 0_{2 \times 2} \end{bmatrix}, \quad C_i = \begin{bmatrix} C_r & 0_{2 \times 2} \end{bmatrix}$$

Similarly with Eq.(9), the disturbances D_i in Eq.(16) can be divided into

$$D_i = D_{i1} + D_{i2} = \begin{bmatrix} 0_{3 \times 1} \\ \Pi_{r \times 1} \\ 0_{2 \times 1} \\ 0_{2 \times 1} \end{bmatrix} \hat{d} + \begin{bmatrix} 0_{3 \times 1} \\ 0_{3 \times 1} \\ E \\ 0_{2 \times 1} \end{bmatrix} \quad (17)$$

which shows that D_{i2} satisfies the matching condition and D_{i1} does not guarantee the matching condition. \hat{d} is estimated by VSS observer designed in Eq.(14). According to Eq.(16) and Eq.(17) the equivalent discrete-time system is given by

$$\begin{aligned} x_i(k+1) &= \Phi x_i(k) + \Gamma_1 u(k) + \Gamma_2 r(k) + \Gamma_3 \hat{d}(k) + \Xi_2(k) \\ y(k) &= C_i x_i(k) \end{aligned} \quad (18)$$

where

$$\Phi = e^{A_i \Delta}, \quad \Gamma_1 = \int_0^{\Delta} e^{A_i \tau} B_i d\tau, \quad \Gamma_2 = \int_0^{\Delta} e^{A_i \tau} G d\tau$$

$$\Gamma_3 = \int_0^{\Delta} e^{A_i \tau} D_{i1} ((k+1)\Delta - \tau) d\tau$$

$$u(t) = u(k\Delta) = u(k), \quad k\Delta \leq t \leq (k+1)\Delta$$

and the following matching condition is guaranteed by disturbance D_{i2} .

$$\Xi_2(k) = D_{i2}(k) = \Gamma_1 h(k) \quad (19)$$

For compensating the mismatched disturbance, the whole control input can be divided into following two parts

$$u(k) = u_1(k) + u_2(k) \quad (20)$$

where $u_1(k)$ is the sliding mode controller part which will be designed later. $u_2(k)$ is used to cancel the mismatched disturbance by following equation

$$\Gamma_1 u_2 + \Gamma_3 \hat{d}(k) = 0 \quad (21)$$

the solution of the compensation control become

$$u_2 = -\Gamma_1^{-1} \Gamma_3 \hat{d}(k) \quad (22)$$

where Γ_1^{-1} is the pseudoinverse of Γ_1 . In the result, Eq.(18a) may be rewritten as

$$x_i(k+1) = \Phi x_i(k) + \Gamma_1(u_1(k) + h(k)) + \Gamma_2 r(k) \quad (23)$$

4.1 Design of switching manifolds

For discrete sliding mode system the design procedure is divided into two steps. The first step is to select the sliding manifold which is defined by

$$\sigma(k) = Sx_i(k) \quad (24)$$

After the system state is driven into the sliding manifolds at the time of $k_1\Delta$, the following condition is guaranteed

$$\sigma(k) = \sigma(k+1) \quad \text{for } k > k_1 \quad (25)$$

Using Eq.(23), Eq.(24) and Eq.(25), the equivalent control in the sliding mode can be given by

$$u_{eq}(k) = -(\Gamma_1^{-1})^{-1}[S(\Phi - I)x_i(k) + S\Gamma_2 r(k)] - h(k) \quad (26)$$

Substituting Eq.(26) into Eq.(23), we have the following equivalent control system

$$x_i(k+1) = [\Phi - \Gamma_1(\Gamma_1^{-1})^{-1}S(\Phi - I)]x_i(k) + [I - (\Gamma_1^{-1})^{-1}S]\Gamma_2 r(k) \quad (27a)$$

$$\sigma(k) = Sx_i(k) = Sx_i(k+1) \quad (27b)$$

Next, switching matrix S must be determined so that the eigenvalues of Eq.(27a) lie within the unit circle. For discrete time system the switching matrix S can be given by

$$S^T = (R_2 + \Gamma_1^T P \Gamma_1)^{-1} \Gamma_1^T P \Phi_\varepsilon \quad (28)$$

here P is the solution of the algebraic matrix Riccati equation

$$P - \Phi_\varepsilon^T P \Phi_\varepsilon + \Phi_\varepsilon^T P \Gamma_1 (R_2 + \Gamma_1^T P \Gamma_1)^{-1} \Gamma_1^T P \Phi_\varepsilon - Q_2 = 0$$

and

$$Q_2 \geq 0, R_2 > 0, \Phi_\varepsilon = \Phi + \varepsilon I, \text{ with } \varepsilon \geq 0$$

where ε is a positive number, I is the identity matrix.

4.2 Design of controller

A new type of sliding mode controller considered here is directly defined by a desired derivative of discrete Lyapunov function. We assume

$$V(k) = 0.5\sigma(k)^2 \quad (29)$$

as a candidate Lyapunov function. One can find that if the difference of the Lyapunov function of Eq.(29) is

$$V(k+1) < V(k) \quad (30)$$

the ensure stability condition for discrete time sliding mode can be satisfied. Defining the incremental change of surface $\sigma(k)$ as

$$d\sigma(k+1) = \sigma(k+1) - \sigma(k) \quad (31)$$

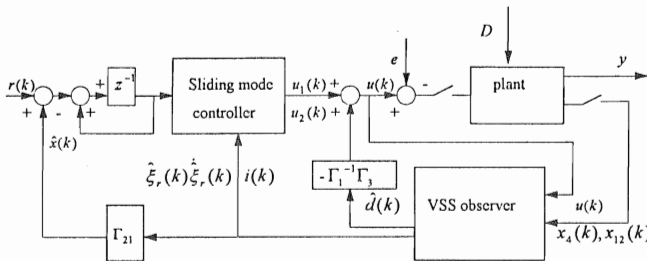


Figure 2 Block diagram of sliding mode control system

A solution for Eq.(30) can be find as

$$d\sigma(k+1) = -\Delta G_1 \sigma(k) = -J\sigma(k) \quad (32)$$

where

$$G_1 = \text{diag}[g_1, \dots, g_m], g_i > 0, (i = 1, \dots, m)$$

$$J = \text{diag}[\Delta g_1, \dots, \Delta g_m]$$

$$= \text{diag}[j_1, \dots, j_m], \quad 0 < j_i < 1, (i = 1, \dots, m)$$

and Δ is the sampling period. Using Eq.(19),(23) and Eq.(24), the left hand part of Eq.(32) can be expressed as

$$d\sigma(k+1) = Sx_i(k+1) - Sx_i(k) = S\Phi x_i(k) + S\Gamma_1 u(k) + S\Gamma_{d2}(k) + S\Gamma_2 r(k) - Sx_i(k) \quad (33)$$

Comparing Eq.(33) with Eq.(32) gives

$$-J\sigma(k) = S\Phi x_i(k) + S\Gamma_1 u(k) + S\Gamma_{d2}(k) + S\Gamma_2 r(k) - Sx_i(k) \quad (34)$$

With Eq.(13) and Eq.(34), we can obtain the control law

$$u_1(k) = -(\Gamma_1^{-1})^{-1}[S(\Phi - I)x_i(k) + S\Gamma_2 r(k)] - h(k) - (\Gamma_1^{-1})^{-1}JSx_i(k) \quad (35)$$

Comparing Eq.(35) with Eq.(26) one can find

$$u_1(k) = u_{eq}(k) - (\Gamma_1^{-1})^{-1}JSx_i(k) \quad (36)$$

Figure 2 shows the total block diagram of the proposed sliding mode control system based on a VSS observer.

5. Simulations

The sampling time of discrete time sliding mode controller was $\Delta = 100 \mu s$. The coefficient γ is selected to be 0.001 and ε is 10. For simplicity, the value of control gain matrix J is selected as $j_i = 0.8, (i = 1, \dots, m)$. Simulations were performed for the control system designed with the rigid modes and the first bending mode.

In servo control simulation, the position command of flexible rotor to lift off is defined to be $0.12mm$. The simulation results of the step reference responses proposed by discrete time SMC at positions x_5 (solid line) and x_{11} (dashed line) are shown in Fig.3. It shows that the rotor can track the position command with good stability and no overshoot in the case of the flexible modes. It has been made clear that the robust stability to spillover caused by the truncation of higher modes is sufficiently guaranteed by sliding mode control. The step reference responses, using linear PID compensator under the same condition, is shown in Fig.4. Comparing this result with that of Fig.3, we can find the responses of PID have a big overshoot. The above results show the proposed discrete time sliding mode controller has superior tracking performance without any chattering problems.

Under the mismatched load disturbance caused by an imbalance acting on position x_1 , Fig. 5 shows the time history displacement responses of position x_5 . The disturbance frequency ω_0 is 400 Hz and the disturbance amplitude $a = I(N)$. In this figure, the solid line indicates the response of system with the disturbance compensation control design and dashed line shows the system response

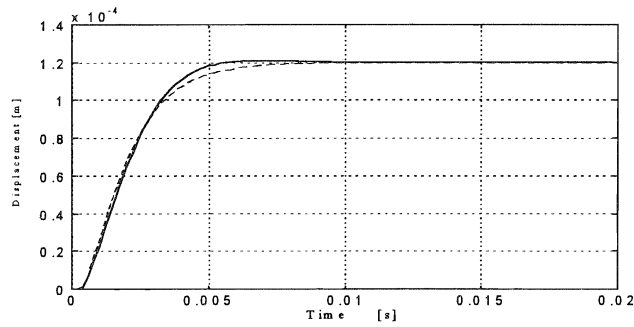


Figure 3 Step reference responses at positions x_5 and x_{11}

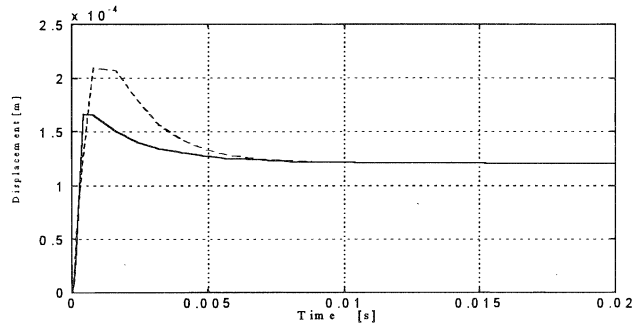


Figure 4 Step reference responses using PID controller

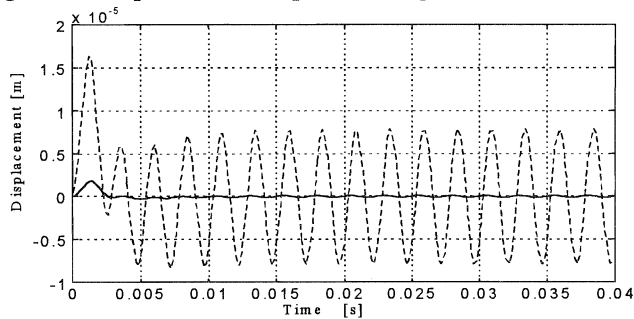


Figure 5 Time history responses with imbalance disturbance in case of VSS observer ($\omega_0 = 400\text{Hz}$)

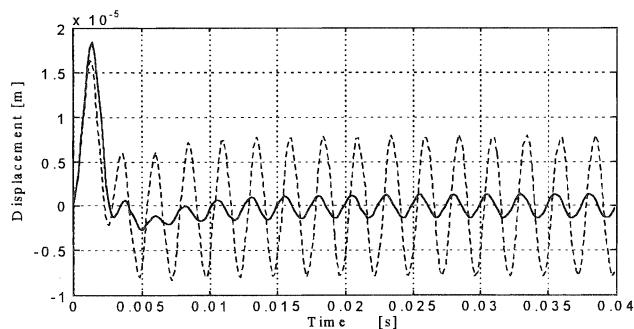


Figure 6 Time history responses with imbalance disturbance in case of linear observer ($\omega_0 = 400\text{Hz}$)

without the disturbance compensation control design. It is clear that the displacement of position x_5 can be made nearly zero amplitude by means of the disturbance compensation control. Fig. 6 shows the time history displacement responses of position x_5 using a linear observer. Comparing the results of Fig.6 with that of Fig.5, we can find that the VSS observer has superior control performance for mismatched disturbance in sliding mode control system.

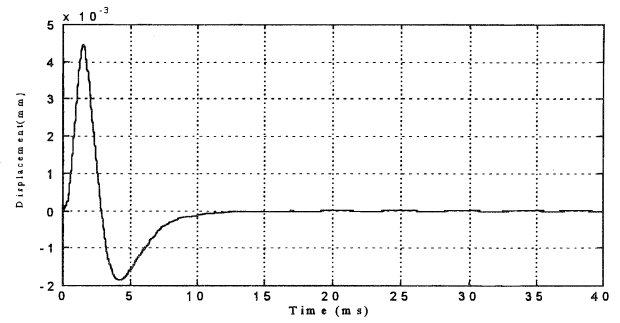


Figure 7 System response for impulse disturbance using sliding mode controller

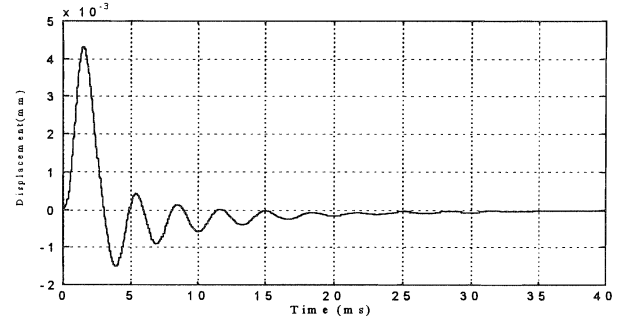


Figure 8 System response for impulse disturbance using conventional linear compensator

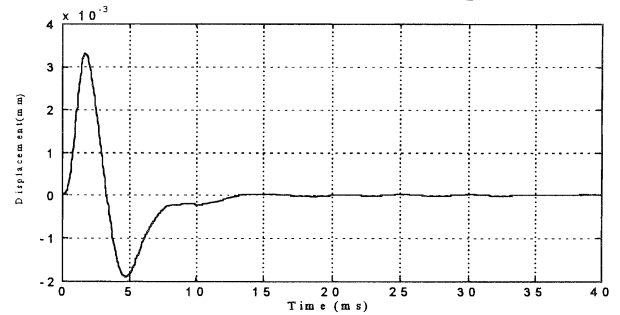


Figure 9 System response for impulse disturbance after parameter variation with SMC

6. Experiments

The experiments for the actual grinding MB spindle are carried out with discrete time sliding mode controller and conventional linear compensator. Two displacement signals measured by two sensors in the radial direction are sent into DSP(TMS320C40) through A/D converter and two control inputs given by DSP are supplied to two electromagnets through D/A converter and power amplifiers. The sampling time in experiments is set at 0.1ms(sampling frequency 10KHz) and other controller parameters are the same as those used in simulations.

Considering the effect of disturbance, Figure 7 shows the impulse disturbance responses of this system at positions x_5 after lift off with sliding mode control. The same impulse disturbance responses, using linear PID compensator under the same condition, is shown in Fig.8. Comparing Fig.7 and Fig.8, one can find that the result of discrete time sliding mode controller indicates excellent performance and damped quickly. Furthermore, Fig.9 gives

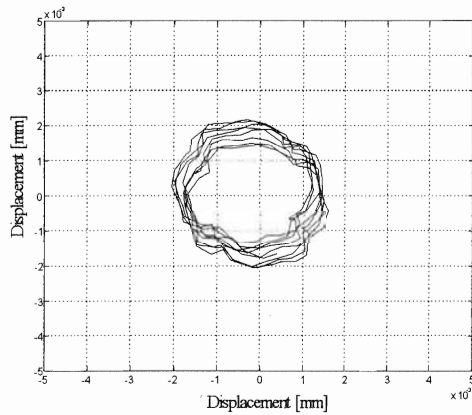


Figure 10 Orbit plot of shaft center at 100,000 rpm using sliding mode controller

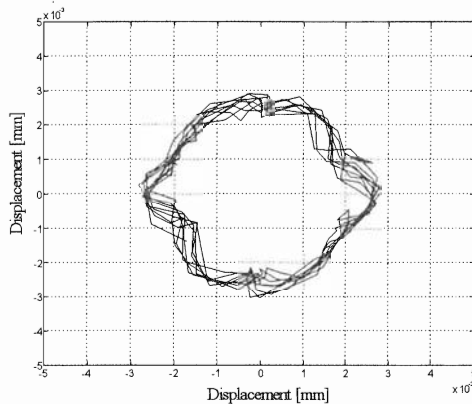


Figure 11 Orbit plot of shaft center at 100,000 rpm using conventional linear compensator

the impulse disturbance response of discrete time sliding mode controller after the mass of rotor is decreased by 20% of the nominal value. It shows that sliding mode controller has strong robustness by comparing this result with the result of fig. 7.

Next, we shall indicate the results of the rotating test with this spindle. Figure 10 gives the orbit plot of the shaft center when the shaft is rotated up to 100,000 rpm. The result of PID is shown in Fig. 11. We can find that the shaft vibration amplitudes under discrete time sliding mode controller is smaller than that of conventional linear compensator at this speed.

7. Conclusions

This paper presents a new control method for the flexible rotor-magnetic bearing systems. The plant dynamics, including the actuator dynamics and the flexible rotor dynamics, were described. A new discrete time VSS disturbance observer were proposed. A discrete time sliding mode controller, using a reduced-order model of the plant, was designed for active control of this magnetic bearing system. By carrying out the comparative simulation and experiment studies, the following results were obtained:

(1) The proposed discrete time sliding mode controller is computationally effective and easy to implement. The

resulting control signals are smooth, unlike the conventional sliding mode controllers, so that the possibility of exciting high order unmodelled dynamics is eliminated.

(2) By using the proposed control method, the unstable modes can be controlled with very strong stability and the prescribed tracking performance without overshoot is also obtained comparing with the linear PID controller.

(3) By means of studied control scheme, the effect of the parameter variations and disturbances in this system, which satisfy the matching condition, can be nullified to obtain robust performance.

(4) With the designed discrete time VSS disturbance observer, the non-matched imbalance disturbance can be canceled or reduced.

(5) With the discrete time sliding mode controller, the spindle was successfully performed up to 100,000 rpm without unstable vibration and have smaller vibration amplitude comparing with conventional linear compensator.

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