Experimental Results of μ-Synthesis Applied to Point Compliance Minimization

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Abstract: Many practical problems in magnetic bearing control concern essentially the minimization of the compliance of the rotor at a particular point, often not a bearing or sensor location. Experimental results are presented which demonstrate that controller design problems of this type can be tackled via μ-synthesis. The problem of minimizing the peak compliance at the midspan of a rotor test rig is examined. A significant improvement in performance is obtained by a multivariable μ controller over a benchmark optimal decentralized PD controller. Interestingly, the μ controller obtained is unstable. Therefore, the rotor is first levitated with a PD controller and then the multivariable algorithm is implemented. Careful attention is given to developing an accurate system model, especially the transfer function of the magnetic actuators with solid stators.

1 Introduction

An important area of research for magnetic bearing control systems is the reduction of the compliance of the rotor at a specified point. Point compliance is of interest in any system that must deal with a poorly characterized externally applied force. Some applications where this problem is of interest are high-speed machining spindles and textile feeder rolls.

Decentralized PD and PID control systems are presently the standard type of control for magnetic bearing applications. However, the performance of these controllers is limited due to both their poor loop shaping capability and their inherent lack of coordination between bearings. Also, PD controllers lack systematic design tools for obtaining a specified performance in spite of model uncertainties or plant variations. In contrast to the meager tools available for decentralized PD controller design, μ–synthesis provides a methodology that includes both performance specifications and guaranteed robustness.

In this paper, we examine the effectiveness of μ–synthesis in designing a controller for the reduction of rotor point compliance. The goal of the experiments described herein was to minimize the point compliance of the rotor at the midspan location using the bearings and sensors at the rotor's ends. This task is interesting (and difficult) since the location of the disturbance is so far from the bearings. For a rigid rotor, the midspan compliance could be reduced using very stiff bearings. However, since the shaft of the test rig is quite thin and flexible, this approach would not be successful. The performance of the multivariable controller is also compared to that obtained with an optimized PD controller.

2 Theory

The μ-synthesis procedure permits the design of multivariable controllers for complex linear systems with uncertainties in their model representations. It is a rather natural augmentation of H^∞ control theory with the analysis techniques of the structured singular value [1]. Since the application of μ-synthesis to the point compliance problem is discussed in detail elsewhere in this proceeding [2], it is not elaborated upon here.

The problem of minimizing the point compliance of a rotor using actuators non-collocated with the disturbance was examined by Herzog and Bleuler [3]. They pointed out several important theoretical results for this problem:

- Whether there is a limit on achievable H^∞ attenuation strongly depends on the relative position of the disturbance (point of compliance measurement) and the actuators.

- If no limit exists, greater H^{∞} performance can be obtained by using greater control effort; perfect attenuation is achieved with infinite controller gain.

- The presence of right half-plane (RHP) transmission zeros in the transfer function between the actuator and the disturbance location determines whether a limit on H^{∞} attenuation exists.

For the experiments considered herein, RHP zeros are present in the transfer function of interest. This establishes a limit on the achievable minimum compliance. Our goal in synthesis is to close in on this limit while maintaining robustness and moderate controller gain.

3 Apparatus and Modeling

In order to investigate the ability of μ–synthesis design to minimize point compliance under plant uncertainty, a very flexible demonstration rig was used. (Figure 1) The rotor system is made up of a 12 mm diameter shaft with a span of 508 mm supported at either end by radial magnetic bearings. The radial bearings have solid (non-laminated) stators and a laminated journal. Eddy currents in the bearing stators have a profound effect on system performance; accurately modeling this was a major effort as will be discussed. Adjacent to each radial bearing is an eddy current position sensor used for feedback. While the rig has a midspan sensor, this was not used for feedback control. A 0.82 kg disk is mounted at the midspan, and the rotor is driven through a coupling by a servo motor at one end.

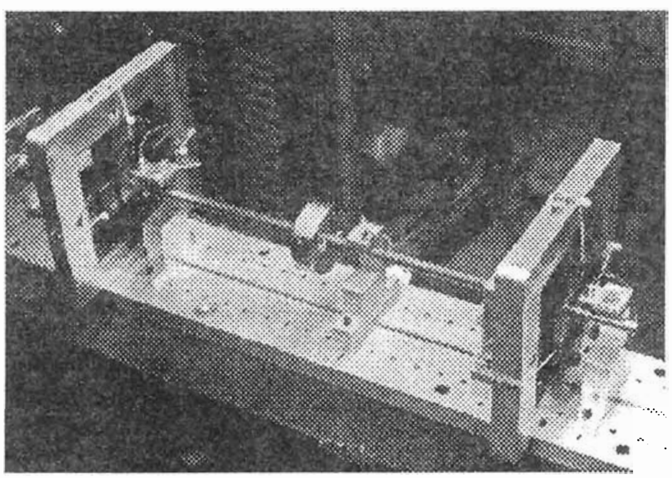

Figure 1: Experimental test rig

A model of the rotor was developed that included the first six free-free modes. The natural frequencies of the first three bending modes are 383, 1835, and 2746 rad/s. Models of the other system components were also developed and compared to experimental data. While the MPW switching amplifier is inherently a nonlinear system, experimental results demonstrate that it behaves quite linearly for typical amplitude signals up to its bandwidth of approximately 1 KHz. Therefore, the amplifiers were modeled as a constant gain. (Any ignored amplifier dynamics are, in fact, accounted for in modeling the bearing transfer functions - see below.) Tests were also performed in order to determine the magnetic bearings' open loop stiffness (K_x) and current gain (K_i). Their values were determined to be -40.8 N/mm and 44.5 N/A respectively.

In order to further refine the system model, the rotor was suspended with a stabilizing PD controller and a swept sine perturbation signal was injected into the amplifiers. While the resulting frequency response compared fairly well to that of the system model, the discrepancies indicated greater error in the model than we felt allowable as a basis for design of a high performance, multivariable controller. After the elimination of all other possible error sources, it was determined that these discrepancies were due to experimental errors in K_i and K_x as well as stator eddy current effects. In order to improve the overall system model, closed loop data that was taken without the motor coupling was used to estimate the values of K_i and K_x as well as a transfer function representation (force/current) of the eddy current effects.

Using the initial values of K_i and K_x, the complex gain of the actuators at each frequency was backed out of the experimental data. This sequence of discrete frequencies and associated complex gains characterizes the frequency response of the actuator. Matching this experimentally determined actuator frequency response with a stable rational transfer function was very difficult due to the frequency response behavior of this element: significant magnitude attenuation with little associated phase lag. Quite simply, this magnitude and phase relationship cannot be realized with a stable rational transfer function. (Unstable filters, however, could match the magnitude and phase curves). In order to overcome this problem, the delay due to the digital controller was lumped with the actuator model. That is, the actuator frequency response data was modified to also account for the phase lag of the delay element. Since the delay element imposes a phase lag with no magnitude attenuation, the combined delay-actuator frequency response has a significant phase lag accompanying the magnitude roll-off. The combined delay-actuator element could then be accurately approximated by a stable rational (6th order) transfer function. Using this model, much better agreement was found between the theoretical and experimental frequency responses. The values for K_i and K_x were then readjusted so as to better match the experimental data. The motor coupling was then attached and more frequency response data was taken. The stiffness and damping coefficients of the coupling were chosen to obtain a best fit of the data.

To further verify our system model, the rig was operated with a number of different PD controllers which effectively varied the system natural frequencies. A good match was found between the model and experimental data in all of these cases. The remaining discrepancies between the component models and experimental data were taken into account in the μ-synthesis design procedure by utilizing structured uncertainty blocks.

4 Benchmark PD Controller

A benchmark decentralized PD controller was designed so that we could accurately assess the performance improvement obtained with multivariable control. Due to the lack of a design methodology with the capability to include the performance specification, an exhaustive search (employing optimization methods) was performed to find the best performing PD controller for the two bearing locations. The performance of a candidate design was evaluated by determining the maximum point compliance as measured by the H^{∞} norm. The resulting PD design was as follows:

$$G_{PD}(s) = 13.57\frac{s+110}{s+12566}$$

The maximum point compliance for the nominal rotor model with this controller was 26.8 μm/N. The frequency response of benchmark PD controller is shown in Figure 2

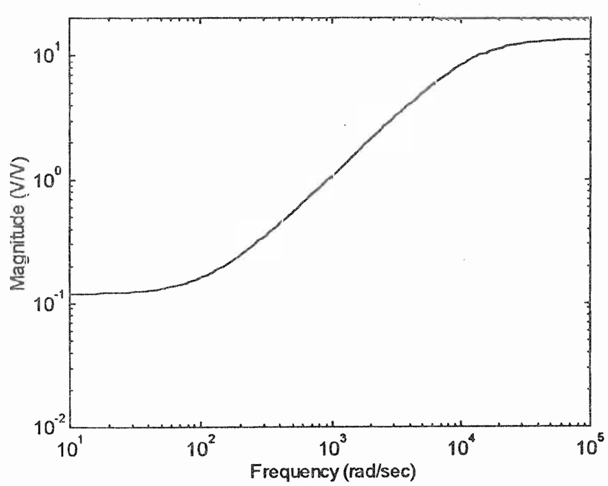

Figure 2: Benchmark PD controller frequency response

5 μ Controller Design

The insights gained from our experience with system modeling were employed in determining the proper representation for model uncertainties.

The uncertainties in K_i and K_x for the two bearings were modeled by four scalar real (non-repeated) Δ blocks representing an uncertainty of ±15% in each parameter. The magnitude and phase error associated with the actuator transfer function approximation was incorporated via a complex scalar block for each of the actuators. The weighting for these Δ blocks was chosen so that the uncertainty represented was negligible below 30 rad/s (5 Hz) and increased quickly afterward to a maximum variation of ± 86.6% in gain or ± 60° in phase at 3000 rad/s (500 Hz). Two real Δ blocks were included to represent uncertainties in the sensor gains of ± 2%.

The point compliance performance specification and a controller maximum gain constraint (including roll-off) were also integrated into the μ-synthesis design procedure via two additional complex Δ blocks [2]. The associated frequency weighting functions appended to the plant model are the inverse of the two desired specifications: point compliance and controller gain. The weight for the point compliance Δ block was chosen so that the resulting controller would have a maximum point compliance of less than 26.8 μm/N for all possible plants represented by the uncertainty description. The controller constraint was chosen so that the maximum gain was less than 3650 N/mm and rolled off at 20 dB/decade after 13000 rad/s (2 KHz).

The μ-synthesis procedure produced a controller design which robustly met the performance specifications above. A very interesting result is that the synthesized controller is open loop unstable. Since the number of states of the controller was large (56 states), a controller reduction procedure was used. The unstable dynamics of the controller were removed, the stable portion was reduced through balancing [4], and the unstable dynamics reintroduced. The minimum order for the reduced controller was obtained via checking the peak μ of the resulting closed loop system. When an incremental reduction in controller order resulted in a significant increase in peak μ, the reduction procedure was halted. With this method, the controller was reduced to 23rd order. The controller still had two unstable poles at 155.9 ± j518.2 rad/s. The frequency response of the μ controller is shown in Figure 3.

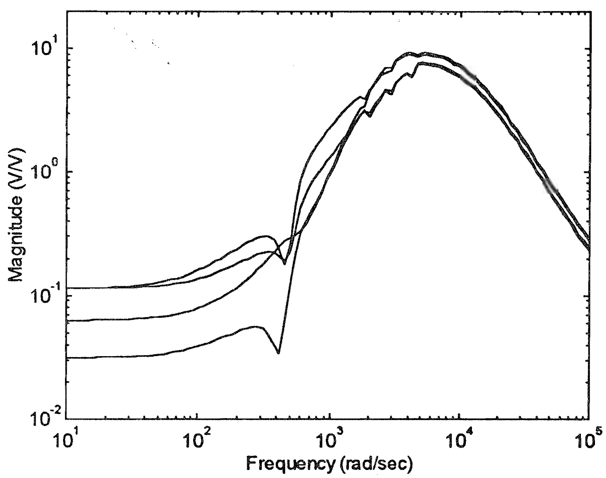

Figure 3: μ controller frequency response

6 Theoretical Comparison

The theoretical nominal point compliance of the benchmark PD and μ controllers are shown in Figure 4. The maximum point compliance of the nominal system with the μ controller was 18.8 μm/N, approximately 30% better than that obtained with the optimal PD controller, 26.7 μm/N. The best performance that can be achieved (as determined by H∞ theory) was 12.0 μm/N. This controller is not practical as it has infinite gain and no robustness. However, this does establish a fundamental limit on achievable performance. We also examined (via μ-synthesis) the level of performance that could be achieved for the nominal system when the controller gain was limited in the same manner as in the robust μ controller. In this case, a peak compliance of 14.3 μm/N was achieved. While this is most likely not the best performance that can be achieved for the nominal system given the restriction on controller gain, it is interesting to note that the nominal performance of the robust μ controller is not very close to this value. This indicates that a significant degree of nominal performance is sacrificed to achieve the desired robustness. Furthermore, it indicates that the important restriction on the performance of our candidate design is the required degree of robustness, not the limitation on controller gain. Thus, to achieve higher performance from our test rig, a more accurate plant model would be of greater benefit than higher controller gain. It is also interesting to compare the above performance figures to the static compliance achieved when the bearings are pinned (obtained with PID control), 10.3 μm/N.

Since the actual plant is never known perfectly, the experimental performance is likely to be worse for both controllers than these theoretical values for the nominal system. Therefore, a better comparison of performance of the two control designs is obtained by examining their worst case performance when the nominal model is perturbed. In this case the μ controller outperforms the optimal PD controller significantly as shown in Figure 5. The PD control system results in a worst case maximum point compliance of 38.2 μm/N whereas the μ-synthesis controller has a worst case maximum point compliance of 24.2 μm/N, a 36% improvement. Note that the worst case performance of the μ controller is better than the nominal performance of the PD controller.

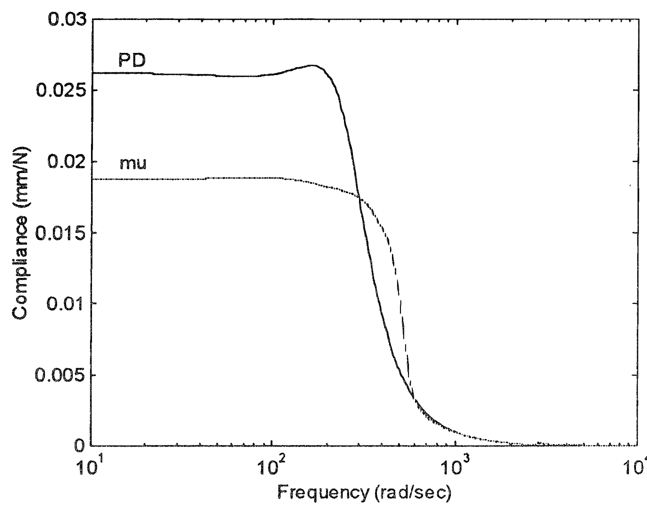

Figure 4: Nominal point compliance with PD and μ controllers

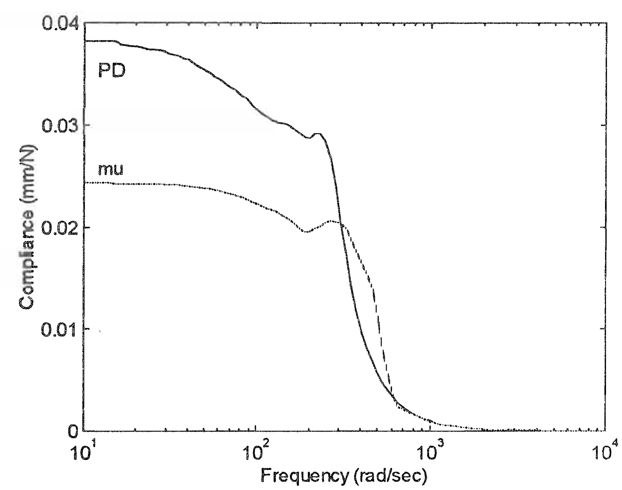

Figure 5: Worst case point compliance at each frequency with PD and μ controllers

7 Experimental Results

Since the coupling between the vertical and horizontal axes of the rotor is negligible (gyroscopics are not significant for this rotor), the benchmark PD and μ controllers may be tested using only the horizontal axis. This simplifies the task of model calibration and reduces the amount of controller computation required. For the experiments, the vertical axis was controlled via a separate digital controller programmed with a PD algorithm.

The controller used for the horizontal axes was a TMS320C40 DSP-based digital controller designed and built at the University of Virginia [5]. It was programmed in assembly language to implement the control algorithm as a discrete time, state space system. Since the μ controller cannot levitate the rotor alone, the program first brings the rotor into support under a low performance, robust PD control algorithm. After 10 seconds, the program switches to the μ control algorithm. Both the μ and PD control algorithms are implemented with a sampling rate of 10 kHz.

To determine the compliance of the rotor midspan, we rapped the midspan disk with an instrument hammer. The resulting vibration was measured using both the midspan displacement sensor and an accelerometer attached to the midspan disk. While the displacement sensor is located 35.6 mm from the impact location, a comparison of the frequency responses obtained with the two different sensors indicates that this non-collocation has little effect over the frequency range of interest. Since the displacement sensor gives a much better measurement of the low frequency portion of the compliance transfer function, only the results obtained with it are presented.

Figure 6 shows the midspan compliance of the test rig with the benchmark PD controller. Also shown is the theoretical midspan compliance determined from the system model. Figure 7 shows the experimentally determined and theoretical compliances for the μ controller. For both the PD and μ controllers, there is a good agreement between theory and experiment (note that the magnitude scale is linear). The peak compliance for the μ controller is 20 μm/N, a 30% improvement over that of the PD controller: 28.5 μm/N. This performance should be compared to that theoretically achievable for the nominal system without any limitation on controller gain, 12.0 μm/N. Clearly, the μ controller offers a significant improvement when one considers what is theoretically possible.

To test the robustness of the μ and PD controllers, we varied the amplifier gain in the outboard bearing from the nominal value. This is equivalent to a variation in K_i for this actuator. For the PD controller, the system was stable with variations from -48% to over +100%. The μ controller's allowable variations were from -28% to over +100%.

The rotor with μ controller was successfully operated over its entire speed range, 0 to 5000 rpm. The spectral content of the midspan sensor vibration is shown in Figure 8. Note that the amplitude of the 1X component is quite flat over the operating speed range, an indication of the H^{∞} specification used in synthesis. The harmonic components of the vibration are most likely due to sensor run-out.

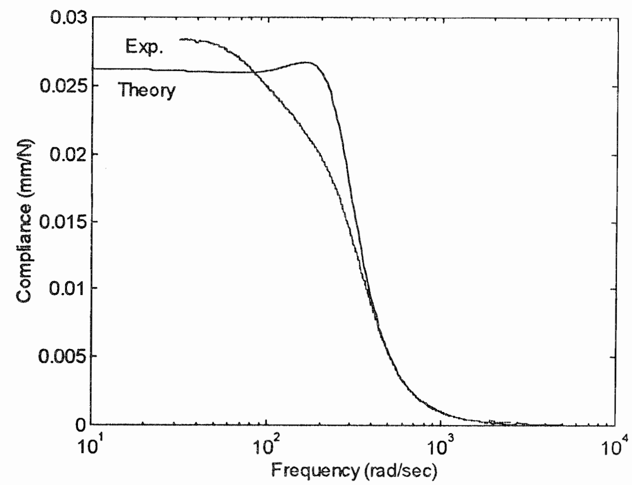

Figure 6: Point compliance of benchmark PD controller

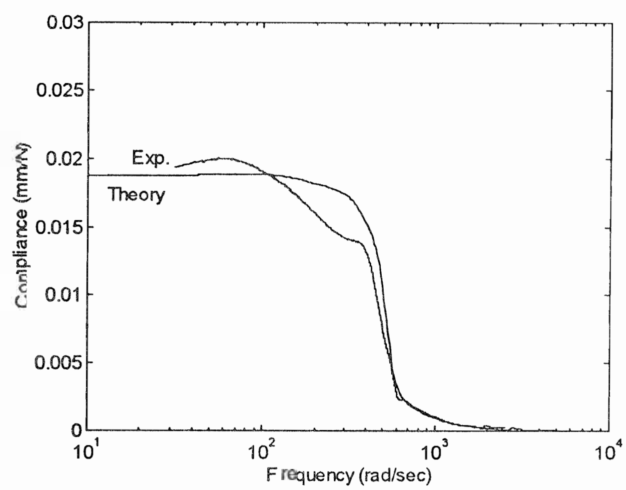

Figure 7: Point compliance of μ controller

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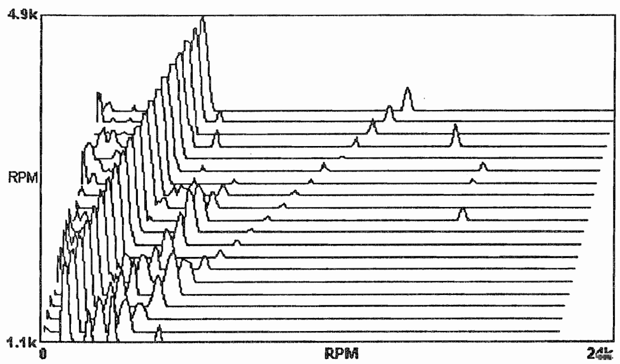

Figure 8: RPM spectral map for μ controller

8 Conclusions

As the experimental results presented demonstrate, μ-synthesis can be employed to provide very good control of rotor point compliance. A significant improvement in performance was obtained over that of an optimal benchmark PD controller. The nominal performance with the μ controller is much closer than the PD controller to the limit on achievable performance caused by the plant's RHP zeros. The primary impediment to closing this gap further appears to be the required robustness rather than the constraint on controller gain.

Interestingly, the μ controller was open loop unstable. Presently, we have neither a theoretical or physical interpretation of this result.

References

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