Q-parameterization Control of Magnetic Bearing Systems with Imbalance

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ABSTRACT

This paper utilizes the Q-parameterization theory to design a controller which solves the problem of imbalance in magnetic bearing systems.There are two methods to solve this problem using feedback control.The first method is to compensate for the imbalance forces by generating opposing forces on the bearing surface (imbalance compenstion).The second method is to make the rotor rotate around its axis of inertia (automatic balancing);in this case no imbalance forces will be generated. After the introduction of a mathematical model of the magnetic bearing system,the controllers which can reject the disturbances caused by imbalance on the rotor are designed,based on the Q-parameterization theory. Simulation results are presented and showed the robustness of the proposed controllers.

INTRODUCTION

This paper proposes a Q-paramrterization control for a rotating Active Magnetic Bearing(AMB) system. Imbalance in the rotor mass causes vibration phenomena in rotating machines. Since balancing is very difficult,there is often a residual imbalance in the rotor. Moreover,the rotor sometimes becomes unbalanced while the machine is in operation.But this imbalance problem can be overcome by proper contol. There are two methods to eliminate the vibration in magnetic bearing systems. The first method is to compensate for the unbalance forces by generating electromagnetic forces that cancel unbalance forces(imbalance compensation).[1]-[3] The second method is to make the rotor rotate around its axis of inetia(automatic balancing)[4]. In this case no unbalance forces will be generated. The Q-parameterization theory characterizes the set of all stabilizing controllers of a given plant in terms of a free parameter Q. The controller Q-parameter is then chosen such that design specifications are achieved.[5] In this paper we utilize the Q-parameterization theory to design a controller for a magnetic bearing system to stabilize it and solve the problem of imbalance. The design objectives are formulated as a set of linear equations in the parameter Q. The parameter Q is found by simply solving this set of linear equations.

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In imbalance compensation design,the imbalance is represented by sinusoidal disturbance forces. The controller is designed such that rejection of sinusoidal disturbance is achieved. In automatic balancing design,the imbalance in the rotor is assumed as a sinusoidal noise in the measured signal. Namely the sensor measurements should indicate the motion of the principal axis of inertia plus geometrical errors due to the difference between the geometrical axis and the inertial axis. The controller is designed such that rejection of sinusoidal noise is achieved;thus we can make the rotor rotates around its axis of inertia and hence acieve automatic balancing. In this paper,4-axis controlled horizontal shaft magnetic bearing system is employed and the axial motion is not controlled actively.

MATHEMATICAL MODEL

In imbalance compensation design,a mathematical model of a magnetic bearing system has been derived in reference [6],and the obtained results is as follows. We assume that states x_v,x_h represent the motion of the geometrical axis and imbalance is represented by sinusoidal disturbance forces p^2W.

$$\begin{bmatrix}\dot{x}_v\\ \dot{x}_h\end{bmatrix} = \begin{bmatrix}A_v & pA_{vh}\\ -pA_{vh} & A_h\end{bmatrix}\begin{bmatrix}x_v\\ x_h\end{bmatrix} + \begin{bmatrix}B_v & 0\\ 0 & B_h\end{bmatrix}\begin{bmatrix}u_v\\ u_h\end{bmatrix} + p^2\begin{bmatrix}E_v\\ E_h\end{bmatrix}W \qquad (1)$$

$$\begin{bmatrix}y_v\\ y_h\end{bmatrix} = \begin{bmatrix}C_v & 0\\ 0 & C_h\end{bmatrix}\begin{bmatrix}x_v\\ x_h\end{bmatrix} \qquad (2)$$

In automatic balancing we assume that states x_v,x_h represent the motion of the inertial axis,and the imbalance is represented by a sinusoidal sensor noise W. In this case the sensors read the motion of the inertial axis plus the sinusoidal noise W which represents the difference between the geometrical axis and the inertial axis. Then the mathmatical model of magnetic bearing system can be described by the following linear control system.

$$\begin{bmatrix}\dot{x}_v\\ \dot{x}_h\end{bmatrix} = \begin{bmatrix}A_v & pA_{vh}\\ -pA_{vh} & A_h\end{bmatrix}\begin{bmatrix}x_v\\ x_h\end{bmatrix} + \begin{bmatrix}B_v & 0\\ 0 & B_h\end{bmatrix}\begin{bmatrix}u_v\\ u_h\end{bmatrix} \qquad (3)$$

$$\begin{bmatrix}y_v\\ y_h\end{bmatrix} = \begin{bmatrix}C_v & 0\\ 0 & C_h\end{bmatrix}\begin{bmatrix}x_v\\ x_h\end{bmatrix} + W \qquad (4)$$

where the subscripts 'v' and 'h' in the vectors and the matrices stand for the vertical motion and the horizontal motion of the magnetic bearing, respectively. In addition, the subscript 'vh' stands for the interference term between the vertical motion and the horizontal motion, and p denotes the rotational speed of the rotor. Each vector in equations (1),(2),(3),(4) can be defined as

$$\begin{aligned} x_v &= [g_{l1} \quad g_{r1} \quad \dot{g}_{l1} \quad \dot{g}_{r1} \quad i_{l1} \quad i_{r1}]^T, \\ x_h &= [g_{l3} \quad g_{r3} \quad \dot{g}_{l3} \quad \dot{g}_{r3} \quad i_{l3} \quad i_{r3}]^T, \\ u_v &= [e_{l1} \quad e_{r1}]^T, \qquad u_h = [e_{l3} \quad e_{r3}]^T, \\ W &= \begin{bmatrix} \epsilon \sin(pt+\kappa) \\ \tau \cos(pt+\lambda) \\ \epsilon \cos(pt+\kappa) \\ \tau \sin(pt+\lambda) \end{bmatrix} \end{aligned} \tag{5}$$

where

g_j : deviations from the steady gap lengths between the electromagnets and the rotor
i_j : deviations from the steady currents of the electromagnets
e_j : deviations from the steady voltages of the electromagnets
ϵ, τ, κ, λ : unbalance parameters in [?]

$$(j = l1, r1, l3, r3.)$$

The subscripts 'l' and 'r' denote the left-hand side and the right-hand side of the magnetic bearing respectively, and the subscripts '1' and '3' denote one of the vertical directions and one the horizontal directions of the rotor respectively. Each matrix in equations (1),(2),(3),(4) can be defined as follows.

$$A_d := \begin{bmatrix} 0 & I & 0 \\ A_1 + A_2 A_{4d} & 0 & A_2 A_{5d} \\ 0 & 0 & -(R/L)I \end{bmatrix},$$

$$A_{vh} := \begin{bmatrix} 0 & 0 & 0 \\ 0 & A_3 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \quad B_d := \begin{bmatrix} 0 \\ 0 \\ (1/L)I \end{bmatrix},$$

$$C_d := [I \quad 0 \quad 0], \quad E_d := \begin{bmatrix} 0 \\ E_{1d} \\ 0 \end{bmatrix},$$

$$(d = v, \ h),$$

$$A_1 := \frac{\alpha}{l_l + l_r} \begin{bmatrix} (l_r - l_m)\left(\frac{1}{m} - \frac{l_l l_m}{J_y}\right) & (l_l - l_m)\left(\frac{1}{m} - \frac{l_l l_m}{J_y}\right) \\ (l_r - l_m)\left(\frac{1}{m} + \frac{l_r l_m}{J_y}\right) & (l_l - l_m)\left(\frac{1}{m} + \frac{l_r l_m}{J_y}\right) \end{bmatrix},$$

$$\begin{aligned} A_2 &:= \begin{bmatrix} -\frac{1}{m} - \frac{l_l^2}{J_y} & -\frac{1}{m} + \frac{l_l l_r}{J_y} \\ -\frac{1}{m} + \frac{l_l l_r}{J_y} & -\frac{1}{m} - \frac{l_r^2}{J_y} \end{bmatrix}, \\ A_3 &:= \frac{J_x}{J_y(l_l + l_r)} \begin{bmatrix} -l_l & l_l \\ l_r & -l_r \end{bmatrix}, \\ A_{4v} &:= -\frac{2}{W} diag[F_{l1} + F_{l2}, \ F_{r1} + F_{r2}], \\ A_{4h} &:= -\frac{2}{W} diag[F_{l3} + F_{l4}, \ F_{r3} + F_{r4}], \\ A_{5v} &:= 2diag\left[\frac{F_{l1}}{I_{l1}} + \frac{F_{l2}}{I_{l2}}, \ \frac{F_{r1}}{I_{r1}} + \frac{F_{r2}}{I_{r2}}\right], \\ A_{5h} &:= 2diag\left[\frac{F_{l3}}{I_{l3}} + \frac{F_{l4}}{I_{l4}}, \ \frac{F_{r3}}{I_{r3}} + \frac{F_{r4}}{I_{r4}}\right], \end{aligned}$$

$$\begin{aligned} E_{1v} &:= \begin{bmatrix} -1 & l_l\left(1 - \frac{J_x}{J_y}\right) & 0 & 0 \\ -1 & -l_r\left(1 - \frac{J_x}{J_y}\right) & 0 & 0 \end{bmatrix}, \\ E_{1h} &:= \begin{bmatrix} 0 & 0 & 1 & l_l\left(1 - \frac{J_x}{J_y}\right) \\ 0 & 0 & 1 & -l_r\left(1 - \frac{J_x}{J_y}\right) \end{bmatrix}. \end{aligned}$$

The parameters of a magnetic bearing system are given in Table 1. In the above equations, α denotes the coefficient of the force which occurs when the rotor eccentrically deviates, and hence we set $\alpha = 0$.

Table 1: Magnetic Bearing Parameters

Parameter	Symbol	Value	Unit
Mass of the Rotor	m	1.39×10^1	kg
Moment of Inertia about X	J_x	1.348×10^{-2}	kg · m^2
Moment of Inertia about Y	J_y	2.326×10^{-1}	kg · m^2
Distance between Center of Mass and Electromagnet	$l_{l,r}$	1.30×10^{-1}	m
Distance between Center of Mass and Motor	l_m	0	m
Steady Attractive Force	$F_{l1,r1}$	9.09×10	N
	$F_{l2\sim4,r2\sim4}$	2.20×10	N
Steady Current	$I_{l1,r1}$	6.3×10^{-1}	A
	$I_{l2\sim4,r2\sim4}$	3.1×10^{-1}	A
Steady Gap	W	5.5×10^{-4}	m
Resistance	R	1.07×10	Ω
Inductance	L	2.85×10^{-1}	H

Q-PATAMETERIZATION THEORY

The Q-parameterization theory[7]-[8] states that the set of all stabilizing controllers for a given plant can be characterized by a free parameter Q. Consider the one-parameter-control feedback system shown in Fig.1 to control the system described (1) (2) (3) (4). Where r

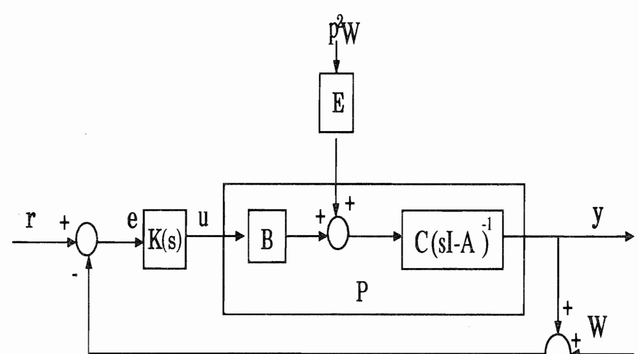


Fig. 1: One-parameter-control feedback system

is the reference input signal,W is the sensor noise,p^2W is the disfurbance force,u is the controller output, y is the plant output to be regulated,and K is the stabilizing controller for P(s). Note that W ,p^2W may also represent model uncertainties. To characterize the set of all stabilizing controllers K for P(s) using Q-parameterization theory,first we need to construct a doubly coprime factorization N,M, $\widetilde{N}$, $\widetilde{M}$,X,Y, $\widetilde{X}$, $\widetilde{Y}$ $\in$ M(s) for P(s)(M(s) denotes stable transfer function matrices).Such factorization is possible if pairs (A,B) and (C,A) are stabilizerable and detectable pairs,respectively (A,B,C are the system matrices of the state equations (1),(2),(3),(4)) To find N,M, $\widetilde{N}$, $\widetilde{M}$, X,Y, $\widetilde{X}$, $\widetilde{Y}$, first we choose real matrices F_1,F_2 such that the eigenvalues of A_0=A-BF_1, $\widetilde{A}_0$=A-F_2C have negative real parts. Then N,M, $\widetilde{N}$, $\widetilde{M}$,X,Y, $\widetilde{X}$,

$\widetilde{Y}$ can be expressed in terms A, B, C, A_0, $\widetilde{A}_0$, F_1, F_2[6]. F_1, F_2 are obtained using the algebraic Riccati equation. With these choices, the set of all stabilizing controllers for P(s) is given by

$$K(P) = \{(Y - Q\,\widetilde{N})^{-1}(X + Q\,\widetilde{M}), |Y - Q\,\widetilde{N}| \neq 0\} \quad (6)$$

CONTROLLER REQUIREMENTS

The control problem for either design(imbalance compensation or automatic balancing) can be defined as follows:Find a controller K such that the following requirements are satisfied:

1) The closed loop system is internally stable for all speeds p in a prespecified range. This constraints can be satisfied if and if only all poles of the closed loop have negative real parts.

2) There is good robustness to variation of plant parameters and there is fast transient response. This requirements can be satisfied if the following inequality holds.

$$R(s_i) + \alpha_s \; < \; 0 \quad (7)$$

where s_i denote the closed loop poles,and α_s is a positive number chosen to ensure a certain degree of stability.

3) Damping Factor: In order to prevent undesirable high frequency oscillation and to help the magnetic bearing system step through critical speeds safely, we must put a lower limit for the damping factor. To achieve this the following inequality must hold[5]

$$R(s_i) + \beta_d|I(s_i)| \; \leq \; 0 \quad (8)$$

β_d is a positive constant chosen as a lower limit for damping factor and $|I(s_i)|$ indicates the imaginary part of the complex number s_i.

4) There is asymptotic tracking to command signals. This tracking problem can be solved if we choose the controller Q-parameter Q such that

$$W_1(s = 0) = I \quad (9)$$

where W_1 is the transfer function that is from r to y

$$W_1 = N(X + Q\,\widetilde{M}) \quad (10)$$

5) Asymptotic rejection of sinusoidal disturbance (imbalance compensation): let W_2 be the transfer function from p^2W to y, then W_2 is given by

$$W_2 = (I - N(X + Q\,\widetilde{M})) \quad (11)$$

In order to achieve this requirement the following identity must hold

$$W_2(s = jp) = 0 \quad (12)$$

6) Asymptotic rejection of sinusoidal sensor noise (automatic balancing): let W_3 be the transfer function from w to y, then W_3 is given by

$$W_3 = -N(X + Q\,\widetilde{M}) \quad (13)$$

In order to achieve this requirement the following identity must hold

$$W_3(s = jp) = 0 \quad (14)$$

CONTROLLER DESIGN

In this section, we design the Q-parameterization controller. At first we assume that the speed of the nominal plant is 0. It means that there is no coupling between the vertical motion and horizontal motion.Therefore the plant model can be separated into vertical plant and horizontal plant.

$$P = \begin{bmatrix} P_v & 0 \\ 0 & P_h \end{bmatrix} \quad (15)$$

Then,a controller will be designed for each plant. The final controller K for the entire plant G will be constructed with the combination of these controllers.

$$K = \begin{bmatrix} K_v & 0 \\ 0 & K_h \end{bmatrix} = \left[\begin{array}{cc|cc} A_{kv} & 0 & B_{kv} & 0 \\ 0 & A_{kh} & 0 & B_{kh} \\ \hline C_{kv} & 0 & D_{kv} & 0 \\ 0 & C_{kh} & 0 & D_{kh} \end{array}\right] \quad (16)$$

K_v denotes the controller for the vertical plant and K_h denotes the controller for the horizontal plant. In order to satisfy controller requirements 1),2),3) we choose the matrices F_1, F_2 such that eigenvaluse of A_0, $\widetilde{A}_0$ lie in the domain D shown in Fig.2. And we choose the verti-

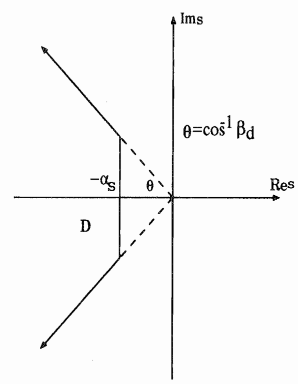


Fig. 2: Generalized Region of Stability

cal controller Q-parameter Q_v and horizontal controller Q-parameter Q_h such that the other requirements, equations (9),(12),(14), are satisfied. From (9) we have

$$N_x(0)(X_x(0) + Q_x(0)\,\widetilde{M}_x(0)) = I, x = v, h \quad (17)$$

from (12) we have

$$I - N_x(jp)(X_x(jp) + Q_x(jp)\,\widetilde{M}_x(jp)) = 0, x = v, h \quad (18)$$

from (14) we have

$$-\,N_x(jp)(X_x(jp) + Q_x(jp)\,\widetilde{M}_x(jp)) = 0, x = v, h \quad (19)$$

Equations (18),(19) are complex equations and each is in fact two equations, one for the real part and one for the imaginary part. This means that we have three equations in the unknown Q_x. Since we need to satisfy three equations,we should allow three variable coefficients. So we take the vertical controller Q-parameter Q_v and the horizontal controller Q-parameter Q_h in the form

$$q_{vij}(s) = \frac{a_{vij}s^2 + b_{vij}s + c_{vij}}{(s+p_{s1})(s+p_{s2})(s+p_{s3})} \quad (20)$$

$$q_{hij}(s) = \frac{a_{hij}s^2 + b_{hij}s + c_{hij}}{(s+p_{s1})(s+p_{s2})(s+p_{s3})} \quad (21)$$

where $a_{vij}, b_{vij}, c_{vij}, a_{hij}, b_{hij}, c_{hij}$ (i=1,2,j=1,2) are design parameters for the vertival motion and horizontal motion,and $p_{s1}, p_{s2}, p_{s3} > \alpha_s$ are fixed. Then we have the following linear equations. For imbalance compensation

$$\begin{array}{ll} A_{v1}X_{v1} = B_1, & A_{v2}X_{v2} = B_2 \\ A_{h1}X_{h1} = B_1, & A_{v2}X_{h2} = B_2 \end{array} \quad (22)$$

For automatic balancing

$$\begin{array}{ll} A_{v1}X_{v1} = B_1, & A_{v2}X_{v2} = B_3 \\ A_{h1}X_{h1} = B_1, & A_{h2}X_{h2} = B_3 \end{array} \quad (23)$$

where

$$\begin{array}{ll} A_{v1} = N_v(0), & A_{v2} = N_v(jp) \\ A_{h1} = N_h(0), & A_{h2} = N_h(jp) \\ X_{v1} = Q_v(0), & X_{v2} = Q_v(jp) \\ X_{h1} = Q_h(0), & X_{h2} = Q_h(jp) \end{array} \quad (24)$$

$$\begin{array}{l} B_1 = (B_{11} \;\; B_{12}) = (I - N_x(0)X_x(0))\ \widetilde{M}_x(0) \\ B_2 = (B_{21} \;\; B_{22}) = (I - N_x(jp)X_x(jp))\ \widetilde{M}_x(jp) \\ B_3 = (B_{31} \;\; B_{32}) = -N_x(jp)X_x(jp))\ \widetilde{M}_x(jp) \end{array} \quad (25)$$

where x=v,h. Solving equations(22),(23) for X_{v1}, X_{v2}, X_{h1}, X_{h2} we can easily find the design parameters a_{vij}, b_{vij}, c_{vij}, a_{hij}, b_{hij}, c_{hij} (i=1,2,j=1,2).

SIMULATION RESULTS

We design the Q-parameterization controller by the methods discussed in the previous section. The controller K(s) is designed for imbalance compensation and automatic balancing at speed p=2π120rad/sec. We choose $p_{s1} = p_{s2} = p_{s2} = 50, and F_1, F_2$ were obtained using the algebraic Riccati equation. The controllers Q-parameter Q_v, Q_h that can satisfy (17),(18) for imbalance compensation design was found to be

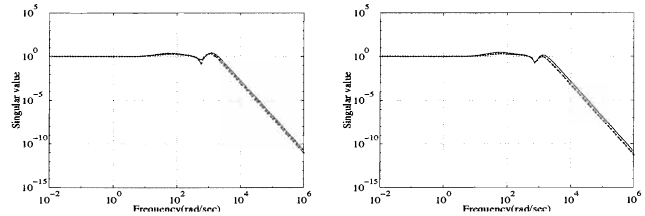
$$\begin{array}{l} Q_{v11} = \frac{2.025e+10s^2+5.385e+13s-1.257e+13}{s^3+150s^2+7500s+1.25e+05} \\ Q_{v12} = \frac{5.603e+07s^2-2.225e+10s+1.05e+09}{s^3+150s^2+7500s+1.25e+05} \\ Q_{v21} = \frac{5.603e+07s^2-2.225e+10s+1.05e+09}{s^3+150s^2+7500s+1.25e+05} \\ Q_{v22} = \frac{2.025e+10s^2+5.385e+13s-1.257e+13}{s^3+150s^2+7500s+1.25e+05} \end{array} \quad (26)$$

$$\begin{array}{l} Q_{h11} = \frac{2.511e+10s^2+5.112e+13s-1.236e+13}{s^3+150s^2+7500s+1.25e+05} \\ Q_{h12} = \frac{6.211e+07s^2-2.839e+10s+8.23e+08}{s^3+150s^2+7500s+1.25e+05} \\ Q_{h21} = \frac{6.211e+07s^2-2.839e+10s+8.23e+08}{s^3+150s^2+7500s+1.25e+05} \\ Q_{h22} = \frac{2.511e+10s^2+5.112e+13s-1.236e+13}{s^3+150s^2+7500s+1.25e+05} \end{array} \quad (27)$$

The controllers Q-parameter Q_v, Q_h that can satisfy (17),(19) for automatic balancing design was found to be

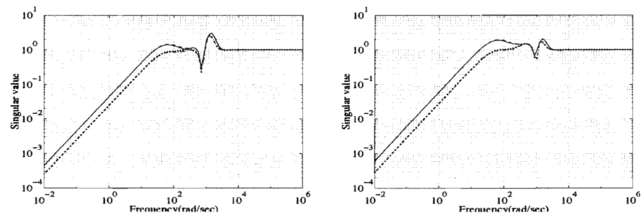
$$\begin{array}{l} Q_{v11} = \frac{4.634e+10s^2+7.323e+13s-1.257e+13}{s^3+150s^2+7500s+1.25e+05} \\ Q_{v12} = \frac{7.144e+07s^2-1.942e+11s+1.05e+09}{s^3+150s^2+7500s+1.25e+05} \\ Q_{v21} = \frac{7.144e+07s^2-1.942e+11s+1.05e+09}{s^3+150s^2+7500s+1.25e+05} \\ Q_{v22} = \frac{4.634e+10s^2+7.323e+13s-1.257e+13}{s^3+150s^2+7500s+1.25e+05} \end{array} \quad (28)$$

$$\begin{array}{l} Q_{h11} = \frac{5.035e+10s^2+5.798e+13s-1.236e+13}{s^3+150s^2+7500s+1.25e+05} \\ Q_{h12} = \frac{1.549e+07s^2-1.644e+11s+8.23e+08}{s^3+150s^2+7500s+1.25e+05} \\ Q_{h21} = \frac{1.549e+07s^2-1.644e+11s+8.23e+08}{s^3+150s^2+7500s+1.25e+05} \\ Q_{h22} = \frac{5.035e+10s^2+5.798e+13s-1.236e+13}{s^3+150s^2+7500s+1.25e+05} \end{array} \quad (29)$$



(a) (b)

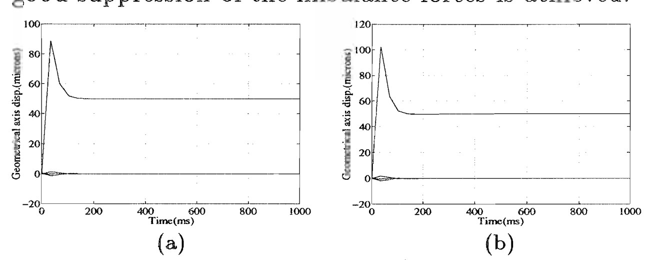
Fig3:Complementary sensitivity fuction σ(T)(p=2π120)
(a)imbalance compensation,(b)automatic balancing



(a) (b)

Fig4:Sensitivity fuction σ(S)(p=2π120)
(a)imbalance compensation,(b)automatic balancing

Fig3,Fig.4 show the singular values of complementary sensitivity function σ(T) and sensitivity function σ(S) for the imbalance compensation and automatic balancing when the speed of the plant is 2π120. Fig.3 shows that the σ(T) equals one in the low frequency range for imbalance compensation design and automatic balancing design which indicates good tracking. Fig.3(b) shows that σ(T) is very small at the imbalance frequency for the automatic balancing design. This means good suppression of the imbalance noise. Fig.4 shows that σ(S) is very small at the low frequency for imbalance compensation design and automatic balancing design. This means that good disturbance rejection is achieved. Fig.4(a) also shows that σ(S) is very small at the rotational frequency for imbalance compensation design. This means that good suppression of the imbalance forces is achieved.



(a) (b)

Fig5:Step response at the ref. input of 50μm(p=2π120)
(a)imbalance compensation,(b)automatic balancing

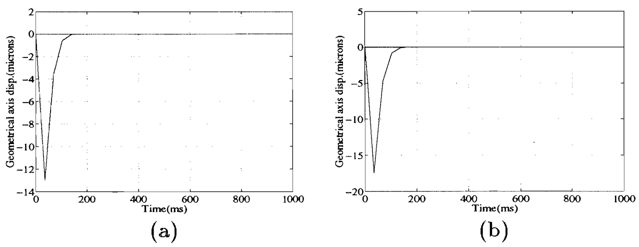


Fig6:Response of a step disturbance force of (1N)($p=2\pi120$)
(a)imbalance compensation,(b)automatic balancing

Fig.5 shows the system response for a step at the refrence input of 50 μm. The steady-state error is zero which shows that aysmptotic tracking is achieved. Fig.6 shows the system response for a step disturbance force of 1N. In Fig.5,Fig.6 the steady-state error is zero and the coupling between the vertical and horizontal motions is minimized.

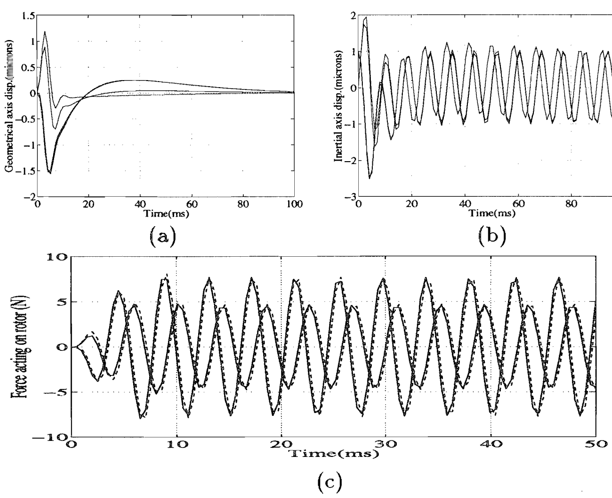


Fig7:Motion of (a)the geometrical axis,(b)the inertial axis,
(c)the bearing forces. ($p=2\pi120$,imbalance compensation)

Fig.7 shows the motion of geometrical axis,inertial axis, and magnetic forces acting on the bearing due to the imbalance for the imbalance compensation control design. In this case the rotor rotates around its geometrical axis with suppressed vibration because of the magnetic forces generated, but inertial axis is vibrating.

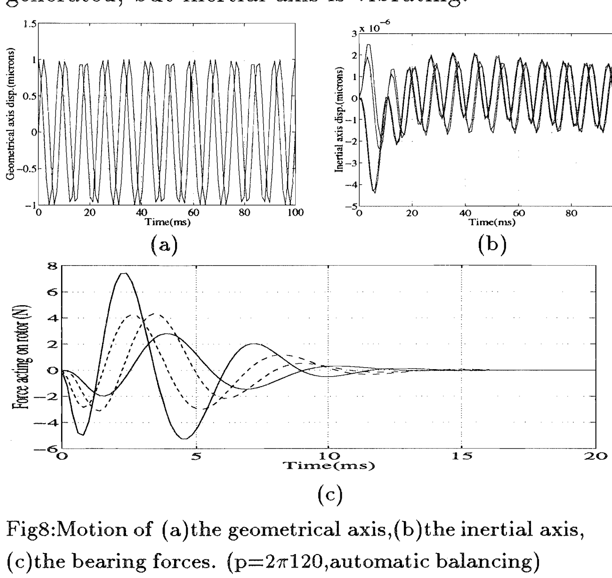


Fig8:Motion of (a)the geometrical axis,(b)the inertial axis,
(c)the bearing forces. ($p=2\pi120$,automatic balancing)

Fig.8 shows the motion of geometrical axis,inertial axis, and magnetic forces acting on the bearing due to the imbalance for the automatic balancing control design. In this case the rotor rotates around its inertial axis with suppressed vibration. But geometrical axis is vibrating. Note also that no imbalance force is generated.

Since the controllers are designed for the nominal plant with $p=0$, the results shown Fig.(3)-(8) for $p=2\pi120$ also indicate that robust stability and robust performance are achieved.

CONCLUTIONS

This paper deals with a method to control the vibration caused by imbalance in the rotor of 4-axis magnetic bearing systems. To overcome the imbalance in the rotor of the magneic bearing system, we used two diferents ways. One approach is that the imbalance is modeled as a sinusoidal disturbance forces. The other is that the imbalance is modeled as a sinusoidal sensor noise in the measured signal. The Q-parameterization theory has been employed to design a controller whice stabilizes the system and achieves the desired goals. The controller Q-parameter can be found simply by solving a set of linear equation.The controllers that were obtained have 24 stastes, with 4 inputs and 4 outputs. Simulation was done at speed $p=2\pi120$ with the controller designed at speed $p=0$(nominal plant). This results show good robustness to model uncertainties and show that the magnetic bearing systems can be used to control vibrations in rotating machinery in two different ways,by compensating for the imbalace forces (imbalance compensation) or by making the rotor rotate around its axis of inertia(automatic balancing).

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