Advanced Mixed H_2/H_∞ Control Design For Active Magnetic Bearing Systems

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Abstract

Although H_∞ control design maintains good performance in frequency domain, the time domain performance is poor comparing with H_2 control design. Major aim of the mixed H_2 $/H_\infty$ control design is to combine these two performance measures in a single control objective. In this study, the mixed H_2 $/H_\infty$ control formulation is given in terms of Linear Matrix Inequality (LMI) based H_∞ control design. The designed controllers applied to an actual AMB spindle. The flexible rotor of the AMB system is modelled using finite element method and controllers are designed for this AMB system by taking into account unknown higher order flexible modes and unknown disturbances of the plant. The controllers were discretized using bilinear transformation method. Utilizing this discretized controller, rotation tests up to 45000 rpm were successfully completed. The LMI based H_∞ control and H_2/H_∞ control systems for the milling AMB spindle demonstrated good performance and robustness.

1 Introduction

Robust control is one of the most important topics in recent years and many application studies of robust control theory have been carried out succesfully [1-3]. In the last decade, H_∞ control has been widely studied for the practical interests in many areas of control engineering such as robust stability and robust performance. From a computational point of view, the known H_∞ control problems requires the solution of two Algebric Riccati Equations (ARE) with the some rank condition on system matrices in general.

Recently, Linear Matrix Inequality (LMI) based control design approaches have begun to take place in the designing of control systems because of some good advantages. Many control problems can be solved in terms of LMI such as LQG control, H_∞ control, mixed H_2 $/H_\infty$ control, etc. LMI problems are often convex and can therefore be solved very efficiently. On the other hand, LMI problems generally don't have analytical solutions. However, increasing computing power and developing efficient convex programing techniques and also availability of some good software as a toolbox make this approach a powerful tool for designing control problems. LMI based control design approach described here is a complementary or alternative approach to the known H_∞ control and mixed H_2 $/H_\infty$ control design approach and doesn't require any condition on system matrix such as rank condition.

The research monograph [5] describes the LMI problems in control and system engineering with a mathematical sense in general. The studies [6] and [7] have made great contributions to designing suboptimal controllers by means of LMI. Practical engineering design problems can be solved using the LMI Control Toolbox [8] in MATLAB.

2 LMI Based H_∞ Control Formulation

The formulation of LMI based output feedback control given here aims for practical use and for this reason the proofs of the formulation are not considered. References [5], and [6] have proved the formulation given here mathematically. Optimal controllers are generally not unique for Multple Input Multiple Output (MIMO) systems. In practice, it is often not necessary to design a strictly optimal controller, and it is usually much cheaper to obtain controllers that are very close in the norm sense to optimal ones. These are often referred as suboptimal controllers.

Consider state-space representation of a control system given by

$$\begin{aligned} \dot{x} &= Ax + B_1 w + B_2 u \\ z &= C_1 x + D_{11} w + D_{12} u \\ y &= C_2 x + D_{21} w \end{aligned} \tag{1}$$

where x, y, z denote the state vector, the controlled output vector, and the measured output vector, respectively. u, w denote the control input, and the exogenous input. Matrix dimensions are $A \in \Re^{n\times n}, B_1 \in \Re^{n\times m_1}, B_2 \in \Re^{n\times m_2}, C_1 \in \Re^{p_1\times n}, C_2 \in \Re^{p_2\times n}, D_{11} \in \Re^{p_1\times m_1}, D_{12} \in \Re^{p_1\times m_2}, D_{21} \in \Re^{p_2\times m_1}$. Given any proper real rational controller such that

$$\begin{aligned} \dot{x}_k &= A_k x_k + B_k y \\ u &= C_k x_k + D_k y \end{aligned} \tag{2}$$

The closed loop transfer matrix from w to z can be obtained by

$$\begin{bmatrix} \dot{x}_{cl} \\ z \end{bmatrix} = \begin{bmatrix} A_{cl} & B_{cl} \\ C_{cl} & D_{cl} \end{bmatrix} \begin{bmatrix} x_{cl} \\ w \end{bmatrix} \quad (3)$$

Closed loop matrices $A_{cl}, B_{cl}, C_{cl}, D_{cl}$ are:

$$\begin{bmatrix} A_{cl} & B_{cl} \\ C_{cl} & D_{cl} \end{bmatrix} = \begin{bmatrix} A_0 + \overline{B}\Theta\overline{C} & B_0 + \overline{B}\Theta\overline{D}_{21} \\ C_0 + \overline{D}_{21}\Theta\overline{C} & D_{11} + \overline{D}_{12}\Theta\overline{D}_{21} \end{bmatrix} \quad (4)$$

where

$$\begin{bmatrix} A_0 & B_0 & \overline{B} \\ C_0 & D_{11} & \overline{D}_{12} \\ \overline{C} & \overline{D}_{21} & \Theta \end{bmatrix} = \left[\begin{array}{cc|c|cc} A & 0 & B_1 & 0 & B_2 \\ 0 & 0 & 0 & I_k & 0 \\ \hline C_1 & 0 & D_{11} & 0 & D_{12} \\ \hline 0 & I_k & 0 & A_k & B_k \\ C_2 & 0 & D_{21} & C_k & D_k \end{array}\right] \quad (5)$$

Note that controller matrices are collected into a single matrix Θ.

From the Bounded Real Lemma, there exists a positive definite Lyapunov function $V(x) = x^T P x, P > 0$ that satisfies

$$\frac{d}{dt}V(x) + z^T z - \gamma^2 w^T w < 0 \quad (6)$$

Substituting equations given in (3) into the inequality (6) and arranging it with the changing of the matrix variable $X_{cl} = P^{-1}$, the H_∞ suboptimal control problem is equivalent to the existence of a solution to the following inequality for $X_{cl} > 0$

$$\begin{bmatrix} A_{cl}^T X_{cl} + X_{cl} A_{cl} & X_{cl} B_{cl} & C_{cl}^T \\ B_{cl}^T X_{cl} & -\gamma I & D_{cl}^T \\ C_{cl} & D_{cl} & -\gamma I \end{bmatrix} < 0 \quad (7)$$

The LMI (7) can be reduced the following inequality

$$\Psi_{Xcl} + \tilde{Q}^T \Theta^T P_{X_{cl}} + P_{X_{cl}}^T \Theta \tilde{Q} < 0 \quad (8)$$

where

$$\Psi_{cl} = \begin{bmatrix} A_0^T X_{cl} + X_{cl} A_0 & X_{cl} B_0 & C_0^T \\ B_0^T X_{cl} & -\gamma I & D_{11}^T \\ C_0 & D_{11} & -\gamma I \end{bmatrix} \quad (9)$$

$$\tilde{Q} = \begin{bmatrix} \overline{C} & \overline{D}_{21} & 0 \end{bmatrix}, \quad P_{Xcl} = \begin{bmatrix} \overline{B} X_{cl} & 0 & \overline{D}_{12}^T \end{bmatrix} \quad (10)$$

The inequality (8) represents the typical linear algebra problem which arises in the general control problem. Here the problem is to obtain necessary and sufficient conditions for the existence of unknown controller matrix Θ satisfying the inequality (8). The solvability conditions of (8) have been given in [5] (Lemma 3.1).

Denoting W_P and $W_{\tilde{Q}}$ matrices whose columns form bases of the null bases of $P_{X_{cl}}$ and $\tilde{Q}$, respectively. Equation (8) is solvable if and only if:

$$W_P^T \Phi_{Xcl} W_P < 0, \qquad W_Q^T \Psi_{Xcl} W_Q < 0 \quad (11)$$

where

$$\Phi_{Xcl} = \begin{bmatrix} A_0 X_{cl}^{-1} + X_{cl}^{-1} A_0^T & B_0 & X_{cl}^{-1} C_0^T \\ B_0^T & -\gamma I & D_{11}^T \\ C_0 X_{cl}^{-1} & D_{11} & -\gamma I \end{bmatrix} \quad (12)$$

and $\Psi_{X_{cl}}$ is given in (9). The set of controllers of order k exists if and only if there exists some $(n+k) \times (n+k)$ positive definite matrix X_{cl} that satisfies the LMIs (11). X_{cl}^{-1} and X_{cl} can be partitioned as

$$X_{cl} = \begin{bmatrix} S & N \\ N^T & I \end{bmatrix}, \quad X_{cl}^{-1} = \begin{bmatrix} R & M \\ M^T & I \end{bmatrix} \quad (13)$$

where $R, S \in \Re^{n \times n}$ and $M, N \in \Re^{n \times k}$(in [6], Teorem 3.). First, let's consider the $W_P^T \Phi_{X_{cl}} W_P < 0$ constraint. Substituting the X_{cl}^{-1} into (12), one obtains

$$\Phi_{Xcl} = \begin{bmatrix} AR + RA^T & AM & B_1 & RC_1^T \\ M^T A^T & 0 & 0 & M^T C_1^T \\ B_1^T & 0 & -\gamma I & D_{11}^T \\ C_1 R & C_1 M & D_{11} & -\gamma I \end{bmatrix} \quad (14)$$

Here P and null space of P are

$$P = \begin{bmatrix} 0 & I_k & 0 & 0 \\ B_2^T & 0 & 0 & D_{12}^T \end{bmatrix} \quad W_P = \begin{bmatrix} W_1 & 0 \\ 0 & 0 \\ 0 & I_{m_1} \\ W_2 & 0 \end{bmatrix} \quad (15)$$

As can be seen, the second row of W_P is zero which reduces the condition $W_P^T \Phi_{Xcl} W_P < 0$ to the following inequality

$$\begin{bmatrix} W_1 & 0 \\ 0 & I_{m_1} \\ W_2 & 0 \end{bmatrix}^T \begin{bmatrix} AR + RA^T & B_1 & RC_1^T \\ B_1^T & -\gamma I & D_{11}^T \\ C_1 R & D_{11} & -\gamma I \end{bmatrix} \times \begin{bmatrix} W_1 & 0 \\ 0 & I_{m_1} \\ W_2 & 0 \end{bmatrix} < 0 \quad (16)$$

Defining

$$N_R = \begin{bmatrix} W_1 \\ W_2 \end{bmatrix} \quad (17)$$

where N_R denotes bases of the null spaces of (B_2^T, D_{12}^T). The inequality (16) can be written as

$$\begin{bmatrix} N_R & 0 \\ 0 & I \end{bmatrix}^T \begin{bmatrix} AR + RA^T & RC_1^T & B_1 \\ C_1 R & -\gamma I & D_{11} \\ B_1^T & D_{11}^T & -\gamma I \end{bmatrix} \begin{bmatrix} N_R & 0 \\ 0 & I \end{bmatrix} < 0 \quad (18)$$

If the same procedure is followed for the second constraint $W_Q^T \Psi_{Xcl} W_Q < 0$, one can obtains

$$\begin{bmatrix} N_S & 0 \\ 0 & I \end{bmatrix}^T \begin{bmatrix} A^T S + SA & SB_1 & C_1^T \\ B_1^T S & -\gamma I & D_{11}^T \\ C_1 & D_{11} & -\gamma I \end{bmatrix} \begin{bmatrix} N_S & 0 \\ 0 & I \end{bmatrix} < 0 \quad (19)$$

where N_S denotes bases of the null spaces of (C_2, D_{12}). Finally, suboptimal control problem is solvable if and only if there exist symmetric matrices R, S satisfying the LMIs (18) and (19) with the following constraint

$$\begin{bmatrix} R & I \\ I & S \end{bmatrix} \geq 0 \quad (20)$$

Supposing that LMIs (18), (19), and (20) have been solved and (R, S) have the following rank condition

$$Rank(I - RS) = k \leq n \quad (21)$$

The second step is to compute a positive definite matrix $X_{cl} \in \Re^{(n+k)\times(n+k)}$ by using the following two full column rank matrices $M, N \in \Re^{n+k}$

$$MN^T = I - RS \qquad (22)$$

Substituting M and N matrices into (13), X_{cl} can be computed easily. After computing the X_{cl}, the same inequality (8) can be used to compute the controller matrix. Since the (8) is an LMI in Θ, the controller can be obtained by the same optimization algorithms.

3 Mixed H_2 / H_∞ Control

If there are uncertainities in the system model, H_∞ control maintains good robust performance. On the other hand, H_∞ control design is mainly concerned with frequency-domain performance and does not guarantee good transient behaviors for the closed-loop system. H_2 control gives more suitable performance on system transient behaviors. Combining H_2 and H_∞ control objectives in a controller is one further step in robust control theory. Mixed problems can be solved adding the H_2 control objective to known H_∞ control design. The mixed control may be described by the control objective: Find a controller $\Theta(s)$ that minimizes $||F(s)||_2$ subject to $||F(s)||_\infty < \gamma$

It is well known that H_2 norm of the closed loop system is finite if and only if $D_{21} = 0$ and there exists two symetric matrices X_2 and M such that

$$\begin{bmatrix} A_{cl}X_2 + X_2A_{cl}^T & B_{cl} \\ B_{cl}^T & -I \end{bmatrix} < 0 \qquad (23)$$

$$\begin{bmatrix} M & C_{cl}X_2 \\ X_2C_{cl}^T & X_2 \end{bmatrix} > 0 \qquad (24)$$

$$Trace(M) < \xi \qquad (25)$$

where ξ is any number such that $\xi > 0$. Here the following condition should be satisfied for mixed problems so that the LMI problem is tractable:

$$X = X_{cl} = X_2 \qquad (26)$$

Finally mixed problems can be solved with constraints (24) and (25) plus H_∞ constraints (18), (19), and (20).

4 Modelling of Flexible Rotor Magnetic Bearing System

The schematic drawing of rotor-bearing system is shown in Fig.1. The dynamics of the flexible rotor-magnetic bearing system will be described using Fig.2. For simplicity, the analysis is given in the x direction and all the coupling effects among the different axes and noncollocation are ignored. The discrete model of flexible rotor system given in Fig.2 is obtained using finite element method. The rotor dynamical equations can be written as follows:

$$M_0\ddot{q} + K_0 q = 0 \qquad (27)$$

where

$$q = \begin{bmatrix} x_1 & \theta_1 & x_2 & \theta_2 & x_3 & \theta_3 & x_4 & \theta_4 & x_5 & \theta_5 \end{bmatrix}^T$$

and $x_i, \theta_i (i = 1, ..., 5)$ are displacement and angle of the rotor, respectively. x_2 and $x_4$represent the positions where the electromagnets are located, M_0 is the mass matrix, K_0 is the stiffness matrix. The mass distrubution in the mathematical model was adjusted to agree with the experimental natural frequencies up to third flexible mode. The magnetic force due to the electromagnet along the radial direction x can be modeled by the following equation:

$$P = P_l - P_r = -2k_1x + 2k_2i \qquad (28)$$

where P is actuator total force on each direction. P_l and P_r are the left and right magnet forces, respectively. The flexible rotor is controlled by the attractive forces given in Eq.(28). The rotor-magnetic bearing system parameters used for modelling is shown in Table 1. The dynamical equation can be rewritten by

$$M_0\ddot{q} + K_0 q = Fp + D \qquad (29)$$

where

$$F = \begin{bmatrix} 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \end{bmatrix}^T$$

$$p = \begin{bmatrix} P_l \\ P_r \end{bmatrix} \qquad \begin{matrix} P_l = 2k_1x_2 - 2k_2i_l \\ P_r = 2k_1x_4 - 2k_2i_r \end{matrix}$$

and D represents the parameter uncertainty and external disturbance.

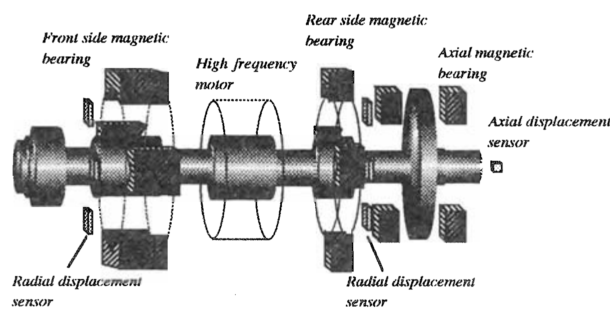


Fig.1 AMB system

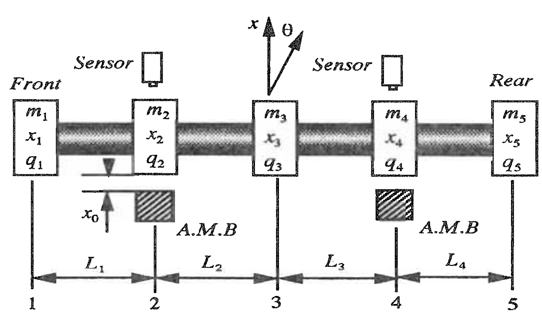


Fig.2 Equivalent flexible rotor-bearing model

Table 1. Parameters of AMB spindle

Parameter	Symbol	Value	Unit
Mass	m_1	1.22	kg
	m_2	2.29	kg
	m_3	1.55	kg
	m_4	1.8	kg
	m_5	0.98457	kg
Lenght	L_1	76.0	mm
	L_2	90.0	mm
	L_3	132.0	mm
	L_4	28.0	mm
Diameter	d	59.4	mm
Damping constant	ζ_i	0.002	
Gap	h_0	0.3	mm
Bias current	i_{0_l}	0.3	A
	i_{0_r}	0.3	A
Bias attractive force	p_{0_l}	23.54	N
	p_{0_r}	21.939	N
Sensor	G_s	10000	V/m

The bias attractive forces and the control forces of Eq.(29) are separated as follows:

$$M_0\ddot{q} + Kq = F_i i + D \tag{30}$$

where

$$i = [\ i_l \quad i_r\ ]^T \qquad K = K_0 + K_i$$
$$K_i = diag(0,0,-2k_1,0,0,0,-2k_1,0,0,0)$$

$$F_i = \begin{bmatrix} 0 & 0 & -2k_2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -2k_2 & 0 & 0 & 0 \end{bmatrix}^T$$

Using the modal analysis technique, it can be chosen the following normalized modal matrix,

$$q = \Psi\xi \tag{31}$$

Equation (30) is transformed into modal coordinates as follows:

$$\ddot{\xi} + \Lambda\dot{\xi} + \Omega^2\xi = f_i i + d \tag{32}$$

where

$$I = \Psi^T M\Psi \qquad \Omega^2 = \Psi^T K\Psi \qquad \Lambda = 2\xi\Omega$$
$$f_i = \Psi^T F_i \qquad d = \Psi^T D$$

and Λ is the damping matrix. The damping ratio is determined experimentally. The state equation of the electromagnetic-mechanical system is given by

$$\dot{x}_f = A_f x_f + B_f u + D_f \tag{33}$$

where

$$x_f = [\ \xi \quad \dot{\xi}\ ]^T \qquad u = [\ i_l \quad i_r\ ]^T$$
$$A_f = \begin{bmatrix} 0 & I \\ -\Omega^2 & -\Lambda \end{bmatrix} \quad B_f = \begin{bmatrix} 0 \\ f_i \end{bmatrix} \quad D_f = \begin{bmatrix} 0 \\ d \end{bmatrix}$$

If the rotor displacement at the magnetic bearings is measured, the output equation is

$$y = C_f x_f = [\ x_2 \quad x_4\ ]^T \tag{34}$$

where

$$C_f = [\ F^T\Psi \quad 0\ ]$$

The control objective is to levitate the rotor and to maintain the stability because control system is originally unstable in open loop. There are only two unstable rigid modes, and the flexible modes are essentially stable. To design a controller for this high order flexible system is quite complicated. Therefore, the construction of the reduced order model is considered for the aim of stabilizing the two rigid modes and controlling the vibration of flexible modes. The reduced order model is constructed by truncation of the higher order modes in modal coordinates. The state equation and the output equation including up to the i-th order mode can be written as follows:

$$\begin{aligned} \dot{x}_r &= A_r x_r + B_r u + D_r \\ y &= C_r x_r = [\ x_2 \quad x_4\ ]^T \end{aligned} \tag{35}$$

where

$$x_r = [\ \xi_1 \quad \xi_2 \quad \ldots \quad \xi_i \quad \dot{\xi}_1 \quad \dot{\xi}_2 \quad \ldots \quad \dot{\xi}_i\ ]^T$$

Designing a controller using reduced order model may seems at first insufficient from the point of stability and robustness of the magnetic bearing system. One can think that higher order flexible modes may exicite system via noise, disturbance, etc. The main idea of H_∞ control theory is to maintain stability and robustness even if control system has modelling uncertainty or parametric uncertainity. In our control system, neglected system dynamics is known because of reduction and the construction of frequency shape filters are formed using this neglected dynamics.

5 Simulations

The simulation results are obtained using LMI Control Toolbox in MATLAB [7]. In this control system, there are two control inputs and two measured outputs. The order of the reduced-order system is four. Using the frequency shaping filter W_1 and W_2, the order of the augmented plant given in Fig.4 became eight. The frequency characteristics of the weighting functions are shown in Fig.5.

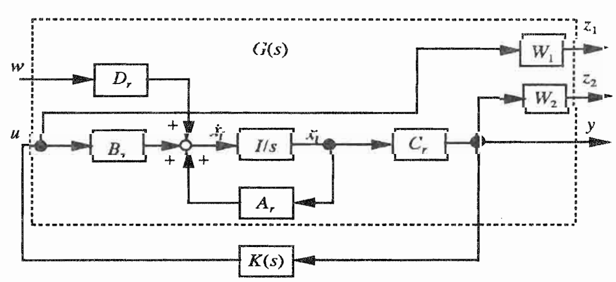


Fig.3 Augmented plant

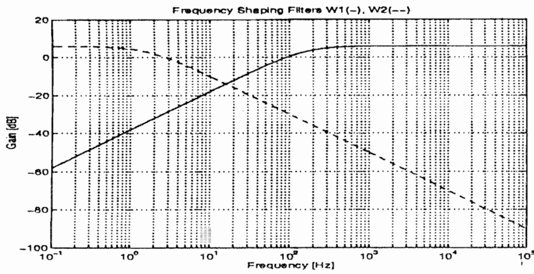


Fig.4 Frequency shaping filters

6 H_∞ and Mixed H_2/H_∞ Controller Design

The controller design process has two stages. In the first stage, the parameter γ is optimized using the function *hinflmi*. The second step is to put the a certain γ value into the function *hinflmi* again and to run it. If there exists a solution , the convex optimization program solver will find it. From a computational point of view, finding a solution to an inequality is easier than finding a solution to an equality. This makes the LMI approach powerful comparing with the ARE solutions. On the other hand, it is not possible to get a solution for the exact value of γ which is why this design opproach is called suboptimal control. The order of the suboptimal H_∞ controller is eight. The H_∞ controller bode plot and impulse response are shown in Fig.6 and Fig 7(b), respectively.

The aim of this control design study is to improve performance in time domain using H_2 control constraint. In LMI Control Tollbox, it is possible to give H_2 constraint as a design objective using the function *hinfmix*. Designer should make some trade-off between H_2 and H_∞ control performance because both performance measures is given in different domain. The resulting controller order is the same as the H_∞ controller. Fig.8 shows the LMI based mixed H_2/H_∞ controller characteristics and the impulse response of the closed loop system is given in Fig.11.

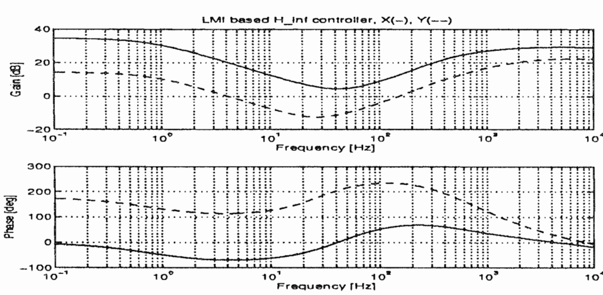


Fig.5 Bode plot of H_∞ controller

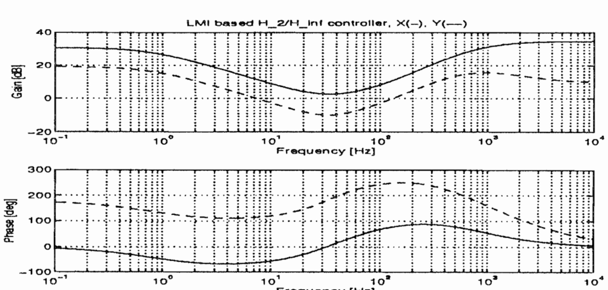
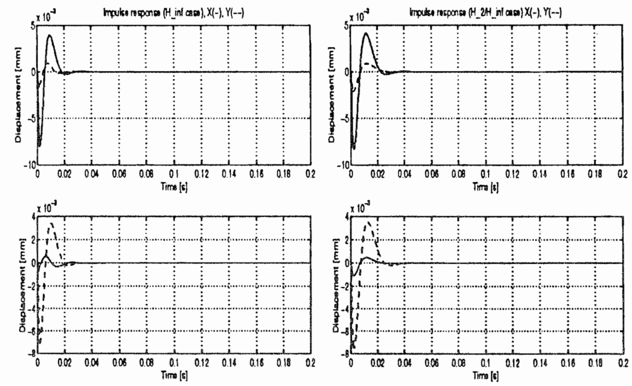


Fig.6 Bode plot of mixed H_2/H_∞ controller



(a) (b)

Fig.7 Impulse responses (a)H_∞ case , (b) H_2/H_∞ case

7 Experiments

The actual milling AMB spindle is used for experiments. Figure 8 shows the schematic block diagram of the digital control system. Two displacements measured by two sensors in the radial direction go to DSP (TMS320C40) through A/D converter and two control inputs are supplied to two electromagnets through D/A converter and power amplifiers. The sampling time was 0.125 msec (sampling frequency 8 KHz) . Figure 9 shows the measured controllers by FFT analyzer after implementation on DSP. Figure 10 shows the step responses at lift off. The performance of LMI based controllers are very good with significant damping. The performance in the case of the LMI based mixed H_2/H_∞ control is the best because the overshoot is the smallest. One can even easily improve on the overshoot using the LMI based mixed H_2/H_∞ control due to its unique capabilities. Figures 11 shows the impulse responses in levitation for the above mentioned controllers. High speed rotation test up to 45000 rpm have been successfully completed. Figure 12 shows the trajectories of the shaft center at 30000 rpm comparing with three cases. The maximum amplitudes are approximately $2\mu m$.

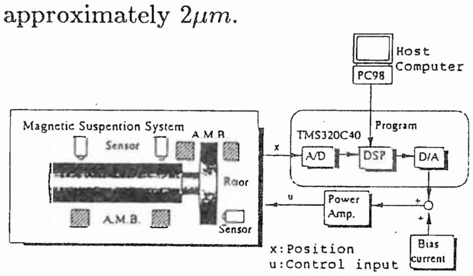
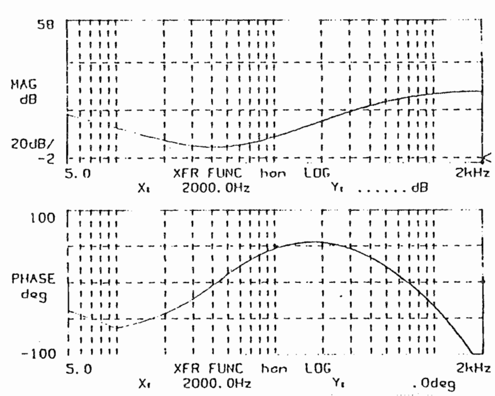
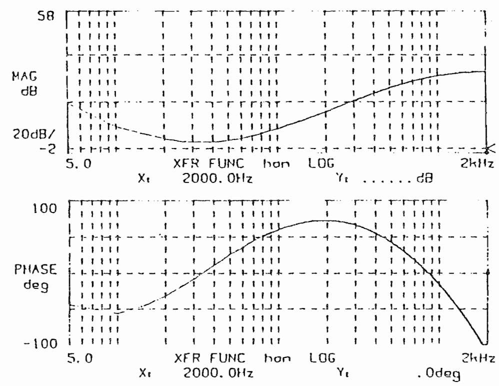


Fig.8 Configuration of DSP-based control system

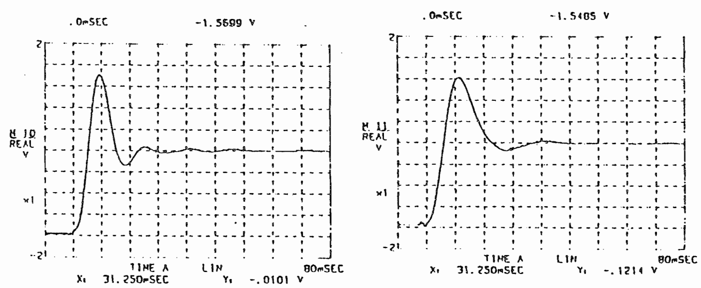


(a)



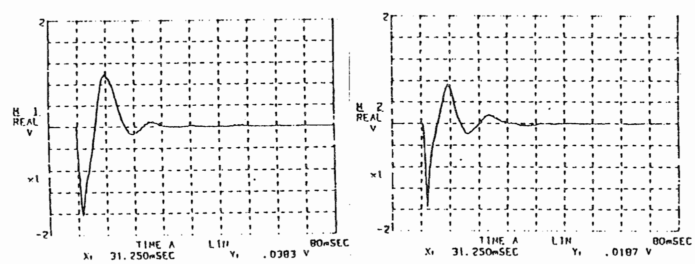
(b)

Fig.9 Measured discretized controller by FFT analyzer
(a) LMI based H_∞ (b)LMI based H_2/H_∞controller



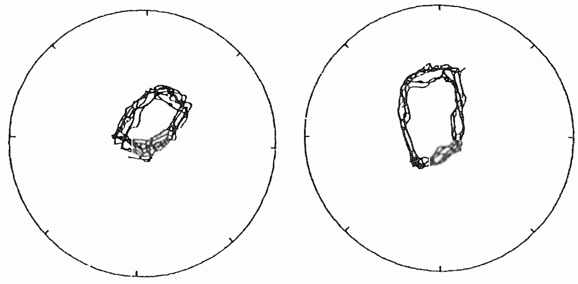
(a) (b)

Fig.10 Step responses at lift off
(a) H_∞ case (b) H_2/H_∞ case



(a) (b)

Fig.11 Impulse responses at lavitation
(a) H_∞ case (b) H_2/H_∞ case



(a) (b)

Fig.12 Orbits for high speed rotation test at 30000rpm
(The diameter is 10μm)
(a) H_∞ case (b) H_2/H_∞ case

8 Conclusions

Rotation tests up to 45000 rpm have been succesfully accomplished using LMI controllers. The LMI based control system for the milling AMB spindle has good performance. The designed controllers given in this study is also compared with PID controllers, Riccati based H_∞ controllers and μ controllers. The improvement of performance in time domain is clear in the mixed H_2/H_∞ control case. Simulation results and experimental results of the controllers frequency charecteristics look like PID controllers. This doesn't mean the controllers don't have any dynamics. The controllers given here maintain robustness to unknown system dynamics and uncertainites and may have more dynamic response increasing of uncertainty in a real control system operation.

From the computational poin of view, the inequality conditions produce a set of solution which are convex and this makes LMI based control system design attractive computationally. The design constraints can also be added easily, especially for mixed H_2/H_∞ control problem. We belive that control system design for active magnetic bearing systems is still a challenging field for control engineers and requires alternative or complementary design approaches.

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