Experimental Evaluation of Gain Scheduled H_∞ Robust Controllers to a Magnetic Bearing

Toru Namerikawa† , Kazuhiko Hagiwara† , Masayuki Fujita‡ and Fumio Matsumura†

† Dept. of Electrical and Computer Eng., Kanazawa University
2-40-20 Kodatsuno, Kanazawa, 920 Japan
tel:+81-762-34-4849; fax:+81-762-34-4870; e-mail:toru@ec.t.kanazawa-u.ac.jp

‡ School of Information Science, Japan Advanced Institute of Science and Technology, Hokuriku
15 Asahidai, Tatsunokuchi, Ishikawa 923-12, Japan
tel:+81-761-51-1291; fax:+81-761-51-1116; e-mail:fujita@jaist.ac.jp

Abstract: This paper deals with the problem of an unbalance vibration of the Magnetic Bearing system. We design a control system achieving the elimination of the unbalance vibration, using a Loop Shaping Design Procedure (LSDP). After the introduction of our experimental setup, a mathematical model of the magnetic bearing is shown. Then, the gain scheduled H_∞ robust controllers with free parameter are designed, based on the LSDP, so as to reject the disturbances caused by unbalance on the rotor asymptotically even if the rotational speed of the rotor varies. Finally, several experimental results show the effectiveness of this proposed methodology.

1 Introduction

This paper proposes a gain scheduled robust control scheme for a rotating Active Magnetic Bearing (AMB) system. By using magnetic bearings, a rotor is supported without any contact. The technique of contact-less support for rotors has become very important in a variety of industrial applications.

Imbalance in the rotor mass causes vibration in rotating machines. Balancing in the rotor is very difficult, there is often a residual imbalance. But, this imbalance problem can be conquered by active control.

This paper is a continuation of the previous research [2], [3], [5], where we have considered both the problems of the interference caused by gyroscopic effect and the problem of the vibration caused by unbalance on the rotor. In [5], the control system has been designed by using the Loop Shaping Design Procedure (LSDP) [6], and have experimentally demonstrated their attenuating effect of the unbalance vibration. The attenuation was only achieved at the fixed-regular rotational speed of the rotor in [3], however, the elimination of thevariable unbalance vibration caused by the variable rotational speed is expected in the next step. The vibrations caused by unbalance of the rigid rotor can be modeled as frequency-varying sinusoidal disturbances. Hence, in this paper, we propose the gain scheduled H_∞ controllers with the free parameter as a function of rotational speed to eliminate frequency-varying sinusoidal disturbances. This gain-scheduling approach is very simple and utilizes the free parameter of the H_∞ controller [7],[8]. The experimental results with the obtained H_∞ controllers indicate the effectiveness of this proposed approach.

2 Magnetic Bearing System

2.1 Experimental System

The magnetic bearing system employed in this research is a 4-axis controlled horizontal shaft magnetic bearing with symmetric structure, the axial motion is not controlled actively. The diameter of the rotor is 96 mm and its span is equal 660 mm. A three-phase induction motor (1kW,four poles) is located at the center of the rotor. Around a rotor, four pairs of electromagnets are arranged radially on both sides. And four pairs of eddy-current type gap sensors are located on outside of the electromagnets. Further this system employs a tachometer in order to measure the rotational speed of the rotor. The experimental machine is controlled by a digital control system that consists of a 32-bit floating point Digital Signal Processor (DSP) DSP32C(AT&T), 12-bit A/D converters and 12-bit D/A converters. Using these systems, the final discrete-time controllers including a free parameter are computed on the DSP.

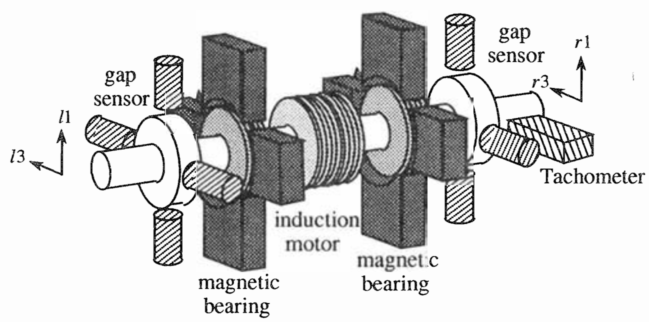

Fig. 1. Diagram of Experimental Machine

2.2 Mathematical Model of the Magnetic Bearing

A mathematical model of a magnetic bearing has been derived in reference [9], and the obtained result is as follows.

$$\begin{bmatrix} \dot{x}_v \\ \dot{x}_h \end{bmatrix} = \begin{bmatrix} A_v & pA_{vh} \\ -pA_{vh} & A_h \end{bmatrix} \begin{bmatrix} x_v \\ x_h \end{bmatrix} + \begin{bmatrix} B_v & 0 \\ 0 & B_h \end{bmatrix} \begin{bmatrix} u_v \\ u_h \end{bmatrix} + p^2 \begin{bmatrix} E_v \\ E_h \end{bmatrix} w \qquad (1)$$

$$\begin{bmatrix} y_v \\ y_h \end{bmatrix} = \begin{bmatrix} C_v & 0 \\ 0 & C_h \end{bmatrix} \begin{bmatrix} x_v \\ x_h \end{bmatrix} \tag{2}$$

where the subscripts 'v' and 'h' in the vectors and the matrices stand for the vertical motion and the horizontal motion of the magnetic bearing, respectively. In addition, the subscript 'vh' stands for the interference term between the vertical motion and the horizontal motion, and p denotes the rotational speed of the rotor. Each vector in (1) and (2) can be defined as

$$\begin{aligned} x_v &= [g_{l1} \quad g_{r1} \quad \dot{g}_{l1} \quad \dot{g}_{r1} \quad i_{l1} \quad i_{r1}]^T, \\ x_h &= [g_{l3} \quad g_{r3} \quad \dot{g}_{l3} \quad \dot{g}_{r3} \quad i_{l3} \quad i_{r3}]^T, \\ u_v &= [e_{l1} \quad e_{r1}]^T, \qquad u_h = [e_{l3} \quad e_{r3}]^T, \\ w &= \begin{bmatrix} \epsilon \sin(pt+\kappa) \\ \tau \cos(pt+\lambda) \\ \epsilon \cos(pt+\kappa) \\ \tau \sin(pt+\lambda) \end{bmatrix} \end{aligned} \tag{3}$$

where

g_j : deviations from the steady gap lengths between the electromagnets and the rotor
i_j : deviations from the steady currents of the electromagnets
e_j : deviations from the steady voltages of the electromagnets
ϵ, τ, κ, λ : unbalance parameters in [4]

$(j = l1, r1, l3, r3.)$

The subscripts 'l' and 'r' denote the left-hand side and the right-hand side of the magnetic bearing respectively, and the subscripts '1' and '3' denote one of the vertical directions and one the horizontal directions of the rotor respectively. Each matrix in (1) and (2) can be defined as follows.

$$A_d := \begin{bmatrix} 0 & I & 0 \\ A_1 + A_2 A_{4d} & 0 & A_2 A_{5d} \\ 0 & 0 & -(R/L)I \end{bmatrix},$$

$$A_{vh} := \begin{bmatrix} 0 & 0 & 0 \\ 0 & A_3 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \quad B_d := \begin{bmatrix} 0 \\ 0 \\ (1/L)I \end{bmatrix},$$

$$C_d := [I \quad 0 \quad 0], \quad E_d := \begin{bmatrix} 0 \\ E_{1d} \\ 0 \end{bmatrix},$$

$$(d = v,\ h),$$

$$A_1 := \frac{\alpha}{l_l + l_r} \begin{bmatrix} (l_r - l_m)\left(\frac{1}{m} - \frac{l_l l_m}{J_y}\right) & (l_l - l_m)\left(\frac{1}{m} - \frac{l_l l_m}{J_y}\right) \\ (l_r - l_m)\left(\frac{1}{m} + \frac{l_r l_m}{J_y}\right) & (l_l - l_m)\left(\frac{1}{m} + \frac{l_r l_m}{J_y}\right) \end{bmatrix},$$

$$\begin{aligned} A_2 &:= \begin{bmatrix} -\frac{1}{m} - \frac{l_l^2}{J_y} & -\frac{1}{m} + \frac{l_l l_r}{J_y} \\ -\frac{1}{m} + \frac{l_l l_r}{J_y} & -\frac{1}{m} - \frac{l_r^2}{J_y} \end{bmatrix}, \\ A_3 &:= \frac{J_x}{J_y(l_l + l_r)} \begin{bmatrix} -l_l & l_l \\ l_r & -l_r \end{bmatrix}, \\ A_{4v} &:= -\frac{2}{W} diag[F_{l1} + F_{l2},\ F_{r1} + F_{r2}], \\ A_{4h} &:= -\frac{2}{W} diag[F_{l3} + F_{l4},\ F_{r3} + F_{r4}], \end{aligned}$$

$$\begin{aligned} A_{5v} &:= 2diag\left[\frac{F_{l1}}{I_{l1}} + \frac{F_{l2}}{I_{l2}},\ \frac{F_{r1}}{I_{r1}} + \frac{F_{r2}}{I_{r2}}\right], \\ A_{5h} &:= 2diag\left[\frac{F_{l3}}{I_{l3}} + \frac{F_{l4}}{I_{l4}},\ \frac{F_{r3}}{I_{r3}} + \frac{F_{r4}}{I_{r4}}\right], \\ E_{1v} &:= \begin{bmatrix} -1 & l_l\left(1 - \frac{J_x}{J_y}\right) & 0 & 0 \\ -1 & -l_r\left(1 - \frac{J_x}{J_y}\right) & 0 & 0 \end{bmatrix}, \\ E_{1h} &:= \begin{bmatrix} 0 & 0 & 1 & l_l\left(1 - \frac{J_x}{J_y}\right) \\ 0 & 0 & 1 & -l_r\left(1 - \frac{J_x}{J_y}\right) \end{bmatrix}. \end{aligned}$$

For the notations, as well as the parameter values, see Table 1. In the above equations, α denotes the coefficient of the force which occurs when the rotor eccentrically deviates, and hence we set $\alpha = 0$.

Table 1: Parameters of Experimental Machine

Parameter	Symbol	Value	Unit
Mass of the Rotor	m	1.39×10^1	kg
Moment of Inertia about X	J_x	1.348×10^{-2}	$kg \cdot m^2$
Moment of Inertia about Y	J_y	2.326×10^{-1}	$kg \cdot m^2$
Distance between Center of Mass and Electromagnet	$l_{l,r}$	1.30×10^{-1}	m
Distance between Center of Mass and Motor	l_m	0	m
Steady Attractive Force	$F_{l1,r1}$	9.09×10	N
	$F_{l2\sim4,r2\sim4}$	2.20×10	N
Steady Current	$I_{l1,r1}$	6.3×10^{-1}	A
	$I_{l2\sim4,r2\sim4}$	3.1×10^{-1}	A
Steady Gap	W	5.5×10^{-4}	m
Resistance	R	1.07×10	Ω
Inductance	L	2.85×10^{-1}	H

3 H_∞ Gain Scheduling

3.1 Loop Shaping Design Procedure

In order to attenuate the unbalance vibration of the rotor, we design the robust H_∞ controllers which achieve the sinusoidal disturbance rejection asymptotically. For such a control system design, the LSDP based on the normalized Left Coprime Factor (LCF) robust stabilization method [6] is employed. Using the free parameter method which have been proposed in the reference [3], it is possible to obtain the gain scheduled controllers by the free parameter as the function of rotational speed.

Let (N, M) represent a normalized left coprime factorization of a plant G. Let these coprime factors be assumed to have uncertainties Δ_N, Δ_M and let G_Δ represent the plant with these uncertainties.

$$\begin{aligned} G_\Delta &= M_\Delta^{-1} N_\Delta \\ &= (M + \Delta_M)^{-1}(N + \Delta_N) \end{aligned} \tag{4}$$

where N_Δ and M_Δ represent a left coprime factorization of G_Δ, and

$$\Delta = \{[\Delta_N, \quad \Delta_M] \in RH_\infty\, ;\, \|[\Delta_N, \quad \Delta_M]\|_\infty < \varepsilon\} \tag{5}$$

G_Δ can be written in the form of an Upper Linear Fractional Transformation (ULFT) as follows.

$$\begin{aligned} G_\Delta &= F_U(P, \Delta) \\ &= P_{22} + P_{21}\Delta(I - P_{11}\Delta)^{-1}P_{12} \end{aligned} \tag{6}$$

where

$$P = \left[\begin{array}{c|c} P_{11} & P_{12} \\ \hline P_{21} & P_{22} \end{array}\right] = \left[\begin{array}{c|c} 0 & I \\ M^{-1} & G \\ \hline M^{-1} & G \end{array}\right] \tag{7}$$

The robust stabilization problem for the *perturbed* plant G_Δ can be treated as the next H_∞ control problem:

$$\left\| \begin{bmatrix} K \\ I \end{bmatrix} (I - GK)^{-1} M^{-1} \right\|_\infty \le \varepsilon^{-1} := \gamma \qquad (8)$$

It is known that the solution of this problem and the largest number of ϵ ($= \epsilon_{\max} := \gamma_{\min}$) can be obtained by solving two Riccati equations without iterative procedure. All controllers K satisfying (8) are given by

$$K = F_L(K_a, \Phi) := K_{11} + K_{12}\Phi(I - K_{22}\Phi)^{-1}K_{21} \quad (9)$$

where

$$K_a = \left[\begin{array}{c|c} K_{11} & K_{12} \\ \hline K_{21} & K_{22} \end{array}\right] \qquad (10)$$

$$\|\Phi\|_\infty \le 1 \qquad (11)$$

For the calculation of K_a and $\epsilon_{\max}$, see [6]. In order to eliminate the unbalance vibration of the rotor, which can be modeled as sinusoidal disturbances [9][1], the robust controller should be designed to achieve sinusoidal disturbance rejection asymptotically. In this case, as is well known, the controller must have the imaginary poles at the frequencies corresponding to the rotational speed of the rotor [7]. Hence, for the achievement of sinusoidal disturbance rejection whose frequency is ω_0 [rad/s], $K(s)$ is required to satisfy

$$K(\pm j\omega_0) = \infty \Leftrightarrow \{I - G(\pm j\omega_0)K(\pm j\omega_0)\}^{-1} = 0 \qquad (12)$$

We then derive the conditions, by adopting the H_∞ problem with boundary constraints [8] shown in Appendix to this problem, whereby there exist the controllers satisfying both (8) and (12). The boundary constraint $\{L, \Pi, \Psi\}$ corresponding to (12) is given by

$$L = [0 \quad I], \quad \Pi = M(\pm j\omega_o), \quad \Psi = 0 \qquad (13)$$

The basic constraint $\{L_B, \Psi_B\}$ in (30) (Appendix) is described by

$$L_B = P_{12}^{\perp}(\pm j\omega_0) = [-G(\pm j\omega_0) \quad I] \qquad (14)$$

$$\Psi_B = P_{12}^{\perp}(\pm j\omega_o)P_{11}(\pm j\omega_o) = M^{-1}(\pm j\omega_o) \qquad (15)$$

It is obvious that $\{L, \Pi, \Psi\}$ is satisfying condition (b) of Theorem A, and the extended boundary constraint $\{\hat{L}, \hat{\Psi}\}$ in (31) (Appendix) is given by

$$\hat{L} = \begin{bmatrix} -G(\pm j\omega_0) & I \\ 0 & I \end{bmatrix}, \quad \hat{\Psi} = \begin{bmatrix} I \\ 0 \end{bmatrix} \qquad (16)$$

After some straightforward calculation, we have

$$\gamma\bar{\sigma}(N(\pm j\omega_0)) > 1 \qquad (17)$$

where

$$\bar{\sigma}(N(\pm j\omega_0)) = \left(\frac{\bar{\sigma}^2(G(\pm j\omega_0))}{1 + \bar{\sigma}^2(G(\pm j\omega_0))} \right)^{1/2}$$

$\bar{\sigma}(\bullet)$: the maximum singular value

from the condition (c) of Theorem A.

If we choose free parameter $\Phi(s)$ such that

$$\Phi(\pm j\omega_o) = K_{22}^{-1}(\pm j\omega_o) \qquad (18)$$

under the conditions (11) (17), it can be seen that we obtain the controller with the imaginary poles at $\pm j\omega_0$ from (9). Based on the above, we design the control system using the LSDP [6]. Thus, we can design the robust controllers achieving sinusoidal disturbance rejection asymptotically. Moreover, utilizing the free parameter for such design, it is possible to obtain the gain scheduled controllers by scheduling the free parameter as the function of rotational speed of the rotor, which achieve the elimination of the unbalance vibration even if the rotational speed of the rotor varies.

3.2 Controller Design

In this section, the feedback controllers are designed with the LSDP. We assume rotational speed $p = 0$ in the nominal plant G. In this case, from (1), we see that there is no coupling between the vertical motion and horizontal motion. Therefore, the plant model can be separated into the vertical plant $G_v(s) := C_v(sI - A_v)^{-1}B_v$ and the horizontal plant $G_h(s) := C_h(sI - A_h)^{-1}B_h$, respectively.

$$G = \begin{bmatrix} G_v & 0 \\ 0 & G_h \end{bmatrix} \qquad (19)$$

Then, two controllers will be designed for the each plant, respectively. The final controller K for the entire plant G will be constructed with the combination of these controllers.

$$K = \begin{bmatrix} K_v & 0 \\ 0 & K_h \end{bmatrix} \qquad (20)$$

where K_v denotes the controller for the vertical plant, and K_h denotes the controller for the horizontal plant. The shaping functions and the design parameters are selected as follows.

(v) Design for vertical motion

$$W_{1v}(s) = \frac{1300(1 + s/(2\pi \cdot 5))(1 + s/(2\pi \cdot 35))}{(1 + s/(2\pi \cdot 0.01))(1 + s/(2\pi \cdot 700))} \cdot \times \frac{(1 + s/(2\pi \cdot 50))}{(1 + s/(2\pi \cdot 1200))} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \qquad (21)$$

$$W_{2v}(s) = 10000 \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \qquad (22)$$

$$\epsilon_{v\_\max} = 0.19944, \quad \epsilon_v^{-1} = \gamma_v = 5.25 \qquad (23)$$

(h) Design for horizontal motion

$$W_{1h}(s) = \frac{1100(1 + s/(2\pi \cdot 5))(1 + s/(2\pi \cdot 25))}{(1 + s/(2\pi \cdot 0.01))(1 + s/(2\pi \cdot 700))} \times \frac{(1 + s/(2\pi \cdot 40))}{(1 + s/(2\pi \cdot 1200))} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \qquad (24)$$

$$W_{2h}(s) = 10000 \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \qquad (25)$$

$$\epsilon_{h\_\max} = 0.27432, \quad \epsilon_h^{-1} = \gamma_h = 3.75. \qquad (26)$$

In this design, verifying the condition (17), it can be seen that it is possible to design the controllers below $\omega_0 = 324.63$ [rad/s] (3100 [rpm]) from Fig. 2. Hence we design the controllers within the above bound. In order to satisfy the conditions (18), the free parameters are selected as

$$\Phi_d(s) = C_{\Phi d}(sI - A_{\Phi d})^{-1} B_{\Phi d} \qquad (d = v,\ h) \tag{27}$$

where

$$A_{\Phi d} = \begin{bmatrix} -a_d & 0 \\ 0 & -b_d \end{bmatrix},\ B_{\Phi d} = \begin{bmatrix} I \\ I \end{bmatrix},\ C_{\Phi d} = [\, C_{\Phi 1d} \quad C_{\Phi 2d} \,]$$

$$C_{\Phi 1d} = \frac{(a_d^2 + \omega_0^2)}{\omega_0 (a_d - b_d)} \left\{ \omega_0 \Re(K_{22d}^{-1}(j\omega_0)) + b_d \Im(K_{22d}^{-1}(j\omega_0)) \right\}$$

$$C_{\Phi 2d} = \frac{(b_d^2 + \omega_0^2)}{\omega_0 (b_d - a_d)} \left\{ \omega_0 \Re(K_{22d}^{-1}(j\omega_0)) + a_d \Im(K_{22d}^{-1}(j\omega_0)) \right\}$$

Table 2: Parameters a_d , b_d

Rotational speed (rpm)	a_v	b_v	a_h	b_h
1000 ~ 1600	2800	8	2800	8
1600 ~ 2200		16		16
2200 ~ 2600	2500	25	2500	27
2600 ~ 2900		37		36
2900 ~ 3100				40

Furthermore, in order to satisfy the condition (11), the parameters a_d , b_d of $A_{\Phi d}$ and $C_{\Phi d}$ are respectively adjusted as Table 2. When we obtain the shaped plants, a model reduction technique has been employed. The procedure of the model reduction is 'The Nominal Plant Model Reduction Procedure' as shown in [6, Procedure 5.5]. The order of the each shaped plant has been reduced from 12 states to 8. As a consequence, the final controller has 36 states. For a example, we show the frequency responses of the designed controller, which is denoted by K_{1300}, with $\omega_0 = 136.14$ [rad/s](1300 [rpm]). The singular values of the shaped plants and the open loop transfer functions are shown in Fig. 3. Fig. 4 shows the singular values of the sensitivity functions. From these figures, we can see that sensitivity approaches zero at the frequency ω_0 .

In this design, we ignored the interference terms, which express the gyroscopic effect, as $p = 0$. We therefore verify the robust stability of this system against changes in the rotational speed of the rotor. Let the perturbed plant $(p \neq 0)$ be denoted by G_p and the additive perturbation Δ_p of from G is as follows:

$$\Delta_p = G_p - G \tag{28}$$

Then the robust stability is guaranteed within the the following inequality (29).

$$\bar{\sigma}(\Delta_p) < \frac{1}{\bar{\sigma}(K(I - GK)^{-1})} \tag{29}$$

In Fig. 5, the singular values $1/\sigma(K(I - GK)^{-1})$ and $\sigma(\Delta_p)$ at $\omega_0 = 1675.5$ [rad/s] (16000 [rpm]) are shown. From this result, we can see the system is stable at $\omega_0 \leq 1675.5$ [rad/s].

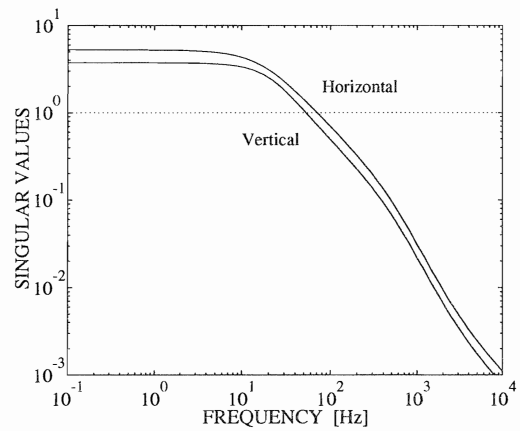

Fig. 2. Magnitude of $\gamma\sigma(N_S)$

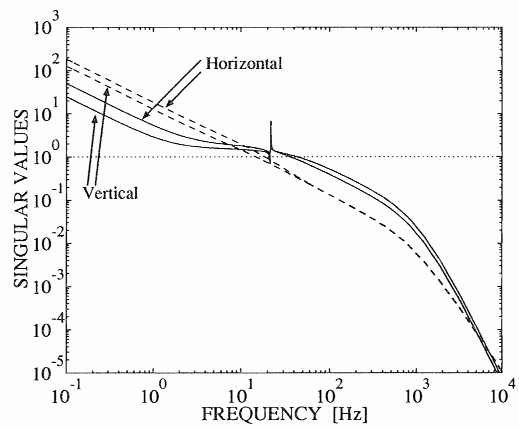

Fig. 3. GK[—] and G_S[−−]

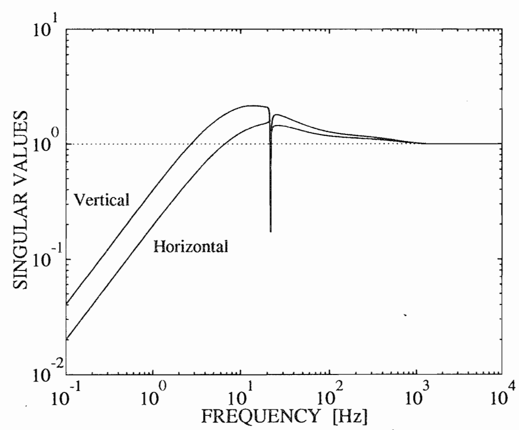

Fig. 4. $\sigma((I - GK)^{-1})$

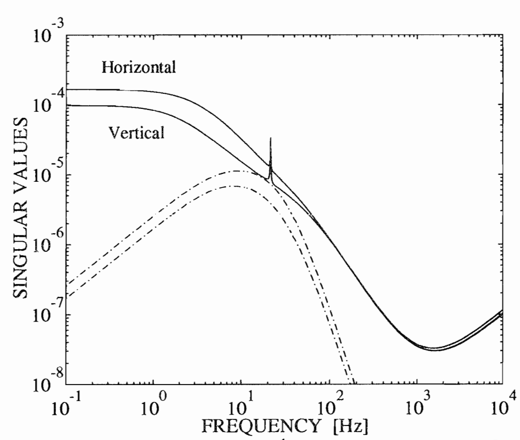

Fig. 5. $1/\sigma(K(I - GK)^{-1})$ [—] and $\sigma(\Delta_p)$[− · −]

4 Experimental Results

In order to evaluate the practical effect of this proposed approach, the experimental tests were run within the limits of the rotational speed from 1000 to 1600 rpm (see Table 2).

The designed continuous-time controllers , K_{1300} and Gain Scheduled H_∞ Controller are discretized via the well known Tustin transform at the sampling rates of $252\mu s$ and $415\mu s$, respectively.

The controller K_{1300} is linear invariant dynamical controller, hence the computing burden for real-time calculation of control input is only matrix multiplication and addition. On the other hand, for the implementation of the gain scheduled H_∞ controller $K(\Phi)$, however, we have to renew $K(\Phi)$ every sampling period by using (9) . After it has obtained, the control input u is calculated. It takes longer time for the implementation of $K(\Phi)$. All through the experiments, a small weight(20[g]) is attached at the left side of the rotor in Fig. 1 so as to increase the residual unbalance.

We have measured the orbits of the center of the rotor for a period of 0.5s under several conditions. Fig. 6(a), 6(b) and 6(c) show the results with K_{1300}, and Fig. 6(d), 6(e) and 6(f) show the results with Gain Scheduled H_∞ Controller, at 1100, 1300 and 1500 rpm, respectively. Compared the Gain Scheduled H_∞ Controller K with K_{1300}, the results with Gain Scheduled H_∞ Controller K indicate better performance than the one with K_{1300} in the elimination of the unbalance vibration except at 1300 rpm. However, it is well known that direct switching and interpolation between the controllers does not capture the dynamic effects and may lead to instability, even if the controllers can stabilize the closed-loop system for each frozen value in the parameter space. This is especially true if the scheduled parameter changes rapidly.

By the numerical simulation, we have confirmed that the closed-loop system is stable when the rotational speed changes at the rate of 2 rpm/s or less(see Fig. 8 and Fig. 9). If the rotational speed changes more than 2 rpm/s, the system becomes unstable.

While the rotor speed should be able to vary, for many applications it does not need to vary quickly. For this rotor, limited power and the safety of the induction motor dictate that the rotational speed can not be changed rapidly. From a theoretical point of view, Gain Scheduled H_∞ Controller should completely attenuate the unbalance vibration even if the rotational speed of the rotor varies. However, this level of performance has not been achieved experimentally. This performance deterioration may be due to the measurement precision of the rotating speed. Gain Scheduled H_∞ Controller very strongly relies on the accuracy of the rotational speed. Since the notch in the sensitivity function is very narrow, error in the measurement of rotational speed may significantly deteriorate performance.

Further investigation and experiments examining the effects of rotational speed and the scheduled parameter's changing rate, will be made in the future.

5 Conslusion

In this paper, we proposed the gain scheduled H_∞ robust control scheme with the free parameter for a magnetic bearing to eliminate the unbalance vibration. We treated the changing unbalance vibration caused by varying rotational speed as the known frequency-varying disturbance, and adjusted the controller gain according to the rotational speed of the rotor using the free parameter of the H_∞ controller. The obtained controller has high gain at the operating frequency. First, the dynamics of the AMB system is considered and a nominal mathematical model for the system is derived. Next, the conditions for the existing of controllers are derived, and, we designed the gain scheduled H_∞ robust controllers using LSDP. It rejects the sinusoidal disturbance of the varying rotor speed. Finally, we presented experimental results with the obtained controllers. And then, we compared with the results of the former controllers that can eliminate the unbalance vibration at the fixed-regular rotational speed only, indicated the effectiveness of this proposed approach.

Appendix

- **Definition A.** " H_∞ **problem with boundary constraints"**

Find the $K(s)$ satisfying

(s1) $K(s)$ stabilizes $F_U(P,0)$,

(s2) $\|P_{zw}\|_\infty \leq \varepsilon^{-1} := \gamma$,

(s3) $LP_{zw}(j\omega)\Pi = \Psi$,

where $P_{zw} = F_L(P,\ K)$

- **Definition B.** " **Basic constraints** "

$$L_B := P_{12}^{\perp}(j\omega), \quad \Psi_B := P_{12}^{\perp}(j\omega)P_{11}(j\omega) \tag{30}$$

where $P_{12}^{\perp}(s)P_{12}(s) = 0$

- **Definition C.** " **Extended constraints** "

$$\hat{L} := \begin{bmatrix} L_B \\ L \end{bmatrix}, \quad \hat{\Psi} := \begin{bmatrix} \Psi_B \Pi \\ \Psi \end{bmatrix} \tag{31}$$

where $\hat{L}$ and $\hat{\Psi}$ are row full rank.

Theorem A.

H_∞ problem with boundary constraints $\{L, \Pi, \Psi\}$ is solvable, if the following three conditions hold:

(a) The H_∞ problem is solvable.

(b) $rank \begin{bmatrix} L_B & \Psi_B\Pi \\ L & \Psi \end{bmatrix} = rank \begin{bmatrix} L_B \\ L \end{bmatrix}$.

(c) $\hat{L}\hat{L}^* > \gamma^2\hat{\Psi}(\Pi^*\Pi)^{-1}\ \hat{\Psi}^*$.

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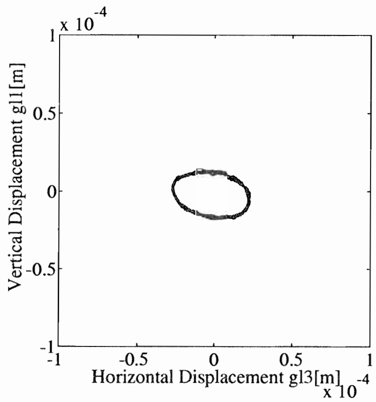
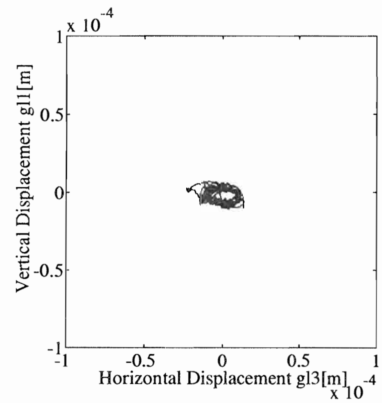
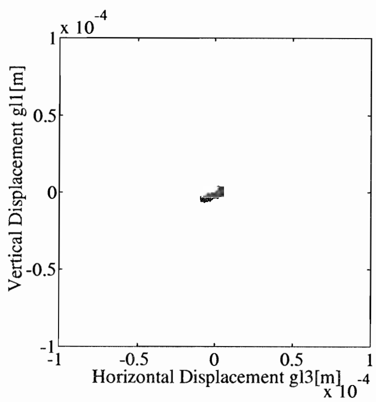


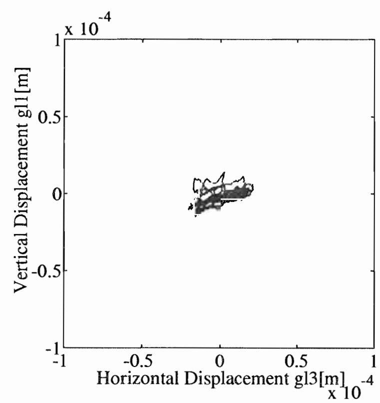
(a) The fixed H_∞ controller K_{1300} rotational speed: 1100 [rpm]



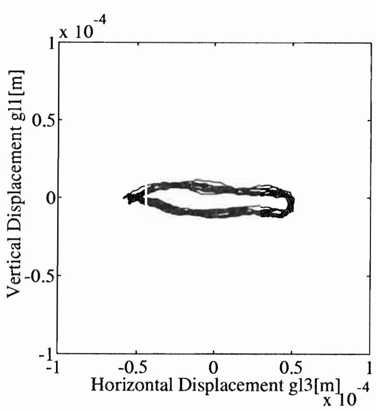
(d) The gain scheduled H_∞ controller rotational speed: 1100 [rpm]



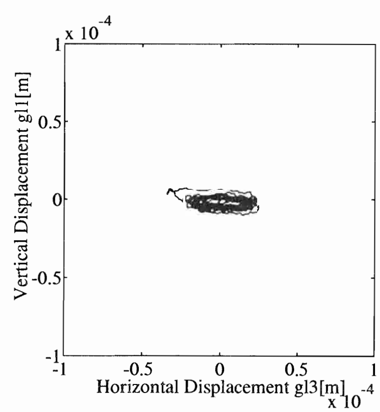
(b) The fixed H_∞ controller K_{1300} rotational speed: 1300 [rpm]



(e) The gain scheduled H_∞ controller rotational speed: 1300 [rpm]



(c) The fixed H_∞ controller K_{1300} rotational speed: 1500 [rpm]



(f) The gain scheduled H_∞ controller rotational speed: 1500 [rpm]

Fig. 6: Orbits of the center of the rotor