Multivariable Identification of Active Magnetic Bearing Systems

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Abstract: Magnetic bearing systems are unstable MIMO[1] *plants. In addition, if the levitated rotors are flexible, there may be poorly damped resonances of the levitated rotors. Advanced control is a crucial issue. System identification is therefore an important prerequisite for fast and reliable commissioning.*

Reported algorithms have difficulties to estimate the real unstable poles in magnetic bearing systems. Therefore a novel algorithm for multivariable identification has been developed. One of the key ideas is to identify the system poles from the determinant of the measured frequency response function matrix.

The algorithm provides a state-space model of pre-defined order and structure, suited for controller design and verification.

Experimental results with measured data from a magnetic bearing system with flexible rotor are included.

1 Introduction

1.1 Active Magnetic Bearings

Active Magnetic Bearings (AMB's) [1] allow contactless levitation. They do not require lubrication, allow high circumferential velocities at high loads, do not have friction nor wear, and no maintainance is needed. In the domain of rotating machinery, they are used in an increasing number of high-performance applications, including high vacuum pumps, pipeline compressors/expanders, tool machines, and others.

The force of the magnetic bearing depends on current and displacement in a non-linear way. This relation can be linearised to

$$F = k_i \cdot i + k_s \cdot y \qquad (1.1)$$

The positive coefficient k_s reflects a *negative stiffness* of the bearing.

The multivariable plant considered in this paper consists of the (flexible) non-rotating rotor suspended in two magnetic bearings. The two radial planes are then not coupled. We will therefore consider only one plane. The considered plant is then a multivariable system with 2 inputs and 2 outputs (*cf.* figure 1).

1.2 Motivation and Goal

AMB systems are unstable without control. A position controller is needed to stabilise the system and to provide a sufficient stiffness w.r.t. disturbance forces, often over a large frequency range. This results then in a large controller - and system - bandwidth. There are often eigenfrequencies of the rotor within the system bandwidth. If the plant model is not precisely known, the plant uncertainty requires robustness of the controller. The requirement or robustness, however, imposes limitations to the achievable controller performance. It is therefore important for reliable and fast controller design to have an accurate plant model for a large frequency range (sometimes 0 .. 3000 Hz).

A dynamic model can also be obtained from theory (FE modelling of the rotor), modal analysis of the rotor, and static force measurements of the bearings. However, many effects, such as eddy currents, hysteresis, and sensor/amplifier dynamics, cannot be assessed with this approach. Dynamic identification can provide a more accurate plant model, and reduce the time required for modelling and controller tuning. It is thus an important prerequisite for fast and reliable commissioning of AMB systems.

1.3 Why Yet Another Identification Algorithm?

Why is it necessary to reconsider the identification problem anew especially for AMB systems?

The answer can be found looking at a typical pole/zero configuration, as shown in figure 1.2. The poles and zeros can be grouped in two sets: A set close to the imaginary axis (flexible modes) and a set on the positive and negative real axis (rigid body modes). The present paper shows that especially for multivariable AMB identification, the set with the 4 real poles is very hard to estimate. None of the

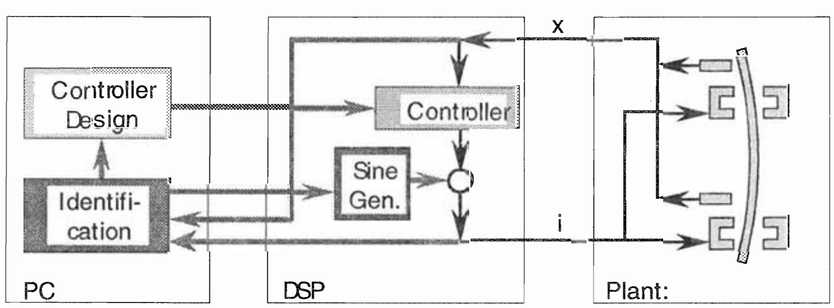

Figure 1: Measurement set-up and context of the identification procedure.

1 MIMO = Multiple Input, Multiple Output; SISO = Single Input, Single Output. SIMO, MISO: Accordingly.

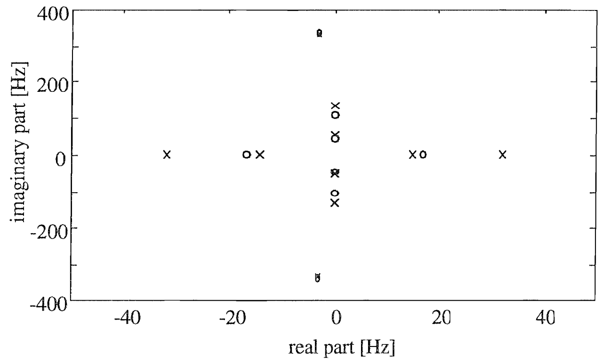

Figure 1.2: Typical pole/zero configuration

methods that we found in the literature was capable of estimating both rigid body modes and flexible modes robustly. In particular, an application of standard modal analysis methods is not successful. We therefore have developed a novel multivariable identification method capable of coping with the above-mentionned plant characteristics.

2 Plant Description and Problem Formulation

For the sake of simplicity, the proposed identification algorithm, and therefore also plant description and problem formulation, are given for a system with two actuators (system inputs) and two sensors (system outputs). However, the algorithm can be extended to a system with more than two inputs and/or outputs without modification.

2.1 Modal AMB System Description: The Theoretical Model

The plant (AMB System in one plane) has the set control currents, $\boldsymbol{i} = \begin{bmatrix} i_A & i_B \end{bmatrix}^T$, at bearings A and B as inputs and the measured displacements, $\boldsymbol{y} = \begin{bmatrix} y_A & y_B \end{bmatrix}^T$, as outputs. The plant's FRF (= frequency response function) in the Laplace domain is defined by

$$\boldsymbol{y}(s) = \boldsymbol{H}(s) \cdot \boldsymbol{i}(s), \text{ or}$$

$$\begin{bmatrix} y_A(s) \\ y_B(s) \end{bmatrix} = \begin{bmatrix} h_{11}(s) & h_{12}(s) \\ h_{21}(s) & h_{22}(s) \end{bmatrix} \cdot \begin{bmatrix} i_A(s) \\ i_B(s) \end{bmatrix} \tag{2.1}$$

The (flexible) rotor is a mass-stiffness-damper system (MKD system). The magnetic bearings act as a negative stiffness onto the rotor. This makes the rigid-body modes unstable and moves the corresponding poles from the origin onto the positive and negative real axis in the Laplace plane (see figure 1.2).
Still, our plant can be described as an MKD system with modal damping. We can therefore use the following modal description:

$$\boldsymbol{H}(s) = \Phi\left(\boldsymbol{M}s^2 + \boldsymbol{D}s + \boldsymbol{K}\right)^{-1} \Psi^T \tag{2.2}$$

Let us consider m modes of the rotor, where the first two modes are the rigid-body modes and the higher modes are the flexible modes; *i.e.*, mode 3 is the first flexible mode. Then,

$$\Phi = \begin{bmatrix} | & | & | \\ \varphi_1 & \cdots & \varphi_m \\ | & | & | \end{bmatrix}; \ \Psi = \begin{bmatrix} | & | & | \\ \psi_1 & \cdots & \psi_m \\ | & | & | \end{bmatrix} \tag{2.3}$$

$\boldsymbol{M}$, $\boldsymbol{K}$ and $\boldsymbol{D}$ are the diagonal stiffness, damping, and mass matrices of the plant. Using mass-normalised co-ordinates, we can set $\boldsymbol{M} = \boldsymbol{I}$ (identity matrix).
With theoretical modelling, $\boldsymbol{M}$, $\boldsymbol{K}$, $\boldsymbol{D}$, Φ and Ψ can be obtained from an FE model of the rotor and the parameters k_i and k_s of the magnetic bearings. Φ is a partition of the eigenvector matrix of the system: Its columns φ_r contain the displacements of the r-th eigenform of the rotor supported in uncontrolled AMBs at the sensor locations. In general, $\Phi \neq \Psi$ [2].

2.2 Plant Parameterisations

Various plant parameterisations will be used in the proposed algorithm. They are presented in this section, and relations between them are discussed.
Since $\boldsymbol{M}$, $\boldsymbol{K}$ and $\boldsymbol{D}$ are diagonal, equation (2.3) can be written as a *sum of second-order systems:*

$$\boldsymbol{H}(s) = \sum_{r=1}^{2} \frac{\varphi_r \cdot \psi_r^T}{s^2 + d_r s - p_r^2} + \sum_{r=3}^{m} \frac{\varphi_r \cdot \psi_r^T}{s^2 + 2\delta_r \omega_{0r} s + \omega_{0r}^2} \tag{2.4}$$

with

$$\begin{aligned} \boldsymbol{K} &= diag\left(\begin{bmatrix} -p_1^2 & -p_2^2 & \omega_{03}^2 & \cdots & \omega_{0m}^2 \end{bmatrix}\right) \\ \boldsymbol{D} &= 2 \cdot diag\left(\begin{bmatrix} d_1 & d_2 & \delta_3\omega_{03} & \cdots & \delta_m\omega_{0m} \end{bmatrix}\right) \end{aligned} \tag{2.5}$$

d_1 and d_2 are small, such that the real-valued rigid-body poles are almost symmetrical to the imaginary axis (*cf.* fig. 1.2).
Equation (2.4) can be re-written in the form

$$\boldsymbol{H}(s) = \sum_{r=1}^{2} \frac{\boldsymbol{R}_r}{s^2 + d_r s - p_r^2} + \sum_{r=3}^{m} \frac{\boldsymbol{R}_r}{s^2 + 2\delta_r \omega_{0r} s + \omega_{0r}^2} \tag{2.6}$$

The dyadic products

$$\boldsymbol{R}_r = \varphi_r \cdot \psi_r^T \tag{2.7}$$

are called *residual matrices*.
Further, (2.6) can be transformed to

$$\boldsymbol{H}(s) = \frac{\boldsymbol{N}(s)}{d(s)} = \frac{\begin{bmatrix} n_{11}(s) & n_{12}(s) \\ n_{21}(s) & n_{22}(s) \end{bmatrix}}{d(s)} \tag{2.8}$$

with a *common denominator polynomial* $d(s)$ of order $2m$, and nominator polynomials of order $2m$-2.
Last but not least, (2.2) and (2.4) can also be formulated as

$$\boldsymbol{H}(s) = \boldsymbol{C} \cdot (s\boldsymbol{I} - \boldsymbol{A})^{-1} \cdot \boldsymbol{B} \tag{2.9}$$

with the *state space description*

$$\left.\begin{aligned} s \cdot \boldsymbol{x}(s) &= \boldsymbol{A} \cdot \boldsymbol{x}(s) + \boldsymbol{B} \cdot \boldsymbol{i}(s) \\ \boldsymbol{y}(s) &= \boldsymbol{C} \cdot \boldsymbol{x}(s) \end{aligned}\right\} \tag{2.10}$$

It is straight-forward to construct $\boldsymbol{A}$, $\boldsymbol{B}$ and $\boldsymbol{C}$ from (2.2) or

2 $\Phi = \Psi$ if sensor and actuator locations coincide ("collocation") and if the parameter k_i of both bearings is equal.

(2.4):

$$A=\begin{bmatrix}0 & I\\ -K & -D\end{bmatrix};\quad B=\begin{bmatrix}0\\ \Psi^T\end{bmatrix};\quad C=\begin{bmatrix}\Phi & 0\end{bmatrix} \qquad (2.11)$$

3 The Identification Problem

3.1 The Goal: A State-Space Model of Order 2*m*

Multivariable controller design can best be done in state space. A state space model of the plant is therefore needed. If this model is allowed to have a higher order than necessary, the identification algorithm will produce a model with a poorly observable and/or controllable part. If this part is unstable, the whole model becomes unstabilisable, although the true plant can be stabilised. To reduce the identified unstable model to the desired degree is then not at all a trivial problem. It is therefore of paramount importance to control the model order during the identification process.

3.2 The Rank 1 Condition

We use the terms "model order" and "system order" in the sense of *the minimal order of a model's state space representation.*
With SISO systems, the system order is equal to to the degree of $d(s)$ in (2.8). Unfortunately, this is not true with MIMO systems [7].
From (2.7) it follows for the residual matrices $\boldsymbol{R}_r$ in representation (2.6) of our plant that the *rank condition*

$$rank(\boldsymbol{R}_r)=1 \qquad \forall r \qquad (3.1)$$

holds. Conversely, a term

$$\boldsymbol{H}_r(s) = \frac{\boldsymbol{R}_r}{s^2+2\delta_r\omega_{0r}s+\omega_{0r}{}^2}$$

is a second-order system if and only if (3.1) is satisfied, but it is a 4th order system if the rank of the residue matrix is 2.
It is obvious that the rank condition (3.1) has a counterpart in representation (2.8). A horrible non-linear relation between the coefficients of all polynomials results.

3.3 The Identification Criterion

Let $\tilde{\boldsymbol{H}}(s)$ be the FRF measured at a number of discrete frequencies $s=j\cdot\omega_k$. FRFs computed by evaluation of some parametric model (*e.g.*, 2.2 or 2.9) at these frequencies will be denoted by $\hat{\boldsymbol{H}}(s)$.
The identification problem can be stated as follows:

> Find a state-space model of order $2m$ such that (2.9) evaluated at the measurement frequencies $s=j\cdot\omega_k$ fits the measured FRF data $\tilde{\boldsymbol{H}}(j\omega_k)$ in an optimal way.

The identification performance criterion that is to be minimised can be defined in different ways. We have chosen the following relative criterion:

$$J = \sqrt{\sum_k\left(\sum_i\sum_j\left(e_{ij}(j\omega_k)\right)^2\right)}, \qquad (3.2)$$

where

$$e_{ij}(j\omega_k) = w_{ij}(j\omega_k)\cdot\frac{\hat{h}_{ij}(j\omega_k)-\tilde{h}_{ij}(j\omega_k)}{\tilde{h}_{ij}(j\omega_k)} \qquad (3.3)$$

or in matrix formulation, adopting MATLAB notation,

$$J=\sqrt{\sum_k\left\|\mathbf{W}(\omega_k).*\left(\hat{\mathbf{H}}(\omega_k)-\tilde{\mathbf{H}}(\omega_k)\right)./\tilde{\mathbf{H}}(\omega_k)\right\|_F^2} \qquad (3.4)$$

$W(s)$ is a weighting function that can be used for tuning the algorithm.

3.4 Some Standard Identification Algorithms

Non-linear parameter optimisation in (2.4) Model (2.4) satisfies the rank 1 condition and therefore can be directly converted to a state-space model of correct order. However, the optimisation problem is strongly non-linear in the parameters. It converges very slowly and only if the starting values for the parameters are good.

Modal analysis [5]. In modal analysis, the system poles are estimated with good accuracy based on the resonances of the FRF data. Because the poles are accurate, rank 1 residual matrices (dyadic products of eigenvectors) can be estimated after that. This procedure breaks up the large non-linear problem into several smaller and more tractable ones.
However, this approach cannot cope with the real rigid-body poles of an AMB system.

The Sanathanan-Koerner-Algorithm: Identification of the polynomial parameters in (2.8). Although this is a non-linear problem as well, it can be solved using an iterative linear Least Squares procedure proposed by Sanathanan and Koerner [2]. This algorithm has already been successfully applied to single degree-of-freedom AMB systems (SISO- and SIMO problem) [3,4]. The problem with extension to MIMO systems is that the rank 1 condition (or, respectively, its counterpart) appears as a crude non-linear constraint which cannot be included into the problem formulation in a tractable way. Therefore, the identified system will in general have order $4m$ if the denominator polynomial was assigned the correct order $2m$. The FRF data can then be matched well with completely wrong rigid-body poles. Because the system is unstable, model reduction is not easily possible.
The example shown in figure 3.1 might illustrate the problem. Consider an AMB system with a rigid rotor with a mass distribution such that it can be modelled by two mass points with mass m each, located at the bearing locations. In parameterisation (2.6) and (2.7), this system's transfer function is given by

$$\mathbf{H}(s) = \frac{\begin{bmatrix}1 & 0\\ 0 & 0\end{bmatrix}}{s^2-p^2} + \frac{\begin{bmatrix}0 & 0\\ 0 & 1\end{bmatrix}}{s^2-p^2} = \frac{\begin{bmatrix}s^2-p^2 & 0\\ 0 & s^2-p^2\end{bmatrix}}{\left(s^2-p^2\right)^2}$$

Both parameterisations can be simplified to

$$\mathbf{H}(s)=\frac{\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}}{s^2-p^2} \qquad (3.5)$$

In (3.5), only the fact that the residual matrix has rank 2 tells us that H(s) represents a fourth-order system: All individual transfer functions have a common second-order denominator polynomial. Identification in polynomial form (2.6) with a second order denominator polynomial would then work well. It is intuitively clear that with fourth order polynomial and independent numerator polynomials, it would fail: The unnecessary pole pair would become arbitrarily wrong, attempting to fit measurement noise.
However, as soon as the mass distribution is slightly changed, a model with fourth order nominator polynomial is necessary to reflect the system's behaviour correctly, because the double pole pair now splits. However, such a model represents a system of order 8. Unless the system is "very" different from (3.5), completely wrong poles are still likely to be the result of this over-parameterisation.

4 The Novel MIMO Identification Algorithm

The new algorithm should
- yield a minimal state-space model
- cope with particular pole/zero configuration of AMB systems (real unstable poles)
- convert the large non-linear problem into a sequence of tractable problems.

To achieve this, the poles are estimated in a first step from the *determinant* of the FRF. In a second step, the residual matrices (2.6) are estimated. Since the poles are estimated well, the resulting residuals are close to rank 1 matrices. Their approximation by rank 1 matrices (which is necessary to reduce the system order to $2m$) does therefore not deteriorate the model accuracy by much.

4.1 Identification of the Poles from the Determinant of the FRF

We make use of the following fact:

$$\det\left(\mathbf{C}(s\mathbf{I}-s\mathbf{A})^{-1}\mathbf{B}\right)=\det(\mathbf{H}(s))=\det\left(\frac{\mathbf{N}(s)}{d(s)}\right) = \frac{\det(\mathbf{N}(s))}{(d(s))^q}=\frac{n_{det}(s)}{d(s)} \qquad (4.1)$$

In words: Let us consider a MIMO system (**C**, **A**, **B**) with q inputs and q outputs (in our case: q=2). The determinant of the FRF of this system is a SISO polynomial fraction. The denominator polynomial is $d(s)$ in power one *(not power q!)*.
The system poles can therefore be computed using the following algorithm:
- Compute the determinant $\det\left(\tilde{H}(s)\right)$ from the measured FRF data.
- Estimate the coefficients of $d(s)$ from $\det\left(\tilde{H}(s)\right)$. Apply the iterative linear Least Squares estimation scheme [2,3,4].
- Compute the poles as the roots of $d(s)$.

Reconsider the example AMB system with rigid rotor from section 3.4 to illustrate the idea: It is not visible from either of the individual transfer functions that $\mathbf{H}(s)$ is a 4th order system. However, it is clearly visible from the determinant:

$$\det(\boldsymbol{H}(s))=\frac{1}{\left(s^2-p^2\right)^2}$$

4.2 Identification of Rank 1 matrices Using SVD

In a first step, the rank 1 constraint is neglected. Estimation of the elements of the rank 2 residual matrices $\boldsymbol{R}_r^{(2)}$ from (2.6) is a linear Least Squares problem. Every $\boldsymbol{R}_r^{(2)}$ can then be approximated by a rank 1 matrix using Singular-value decomposition [5,6]:
- Decompose $\boldsymbol{R}_r^{(2)}$ as $\boldsymbol{R}_r^{(2)}=\boldsymbol{U}_r\cdot\Sigma_r\cdot\boldsymbol{V}_r^T$ [6]. $\boldsymbol{U}_r$ and $\boldsymbol{V}_r$ are orthonormal matrices with columns $\boldsymbol{u}_{r,i}$, $\boldsymbol{v}_{r,i}$. Σ_r is diagonal and contains the singular values $\sigma_{r,i}$ with descending magnitudes.
- The best possible rank 1 approximation of $\boldsymbol{R}_r^{(2)}$ is then

$$\boldsymbol{R}_r^{(2)}\approx\boldsymbol{R}_r=\boldsymbol{u}_{r,1}\cdot\sigma_{r,1}\cdot\boldsymbol{v}_{r,1}^T=\varphi_r\cdot\psi_r^T \qquad (4.2)[3]$$

- Construct a state-space model of order $2m$ using (2.3), (2.5) and (2.11).

4.3 Insertion of Proportional Feedback for Identification

Sections 4.1-4.2 already describe an algorithm superior to those of section 3. However, the following procedure further improves its performance w.r.t. the real poles:
- Modify the measured FRF data $\tilde{H}(s)$ with a proportional feedback matrix $\boldsymbol{K}$, *i.e.*,

$$\tilde{H}_K(s)=\left(I+\tilde{H}(s)\cdot K\right)^{-1}\tilde{H}(s) \qquad (4.3)$$

  $\boldsymbol{K}$ should be chosen such that it overcompensates the negative magnetic bearing stiffness.

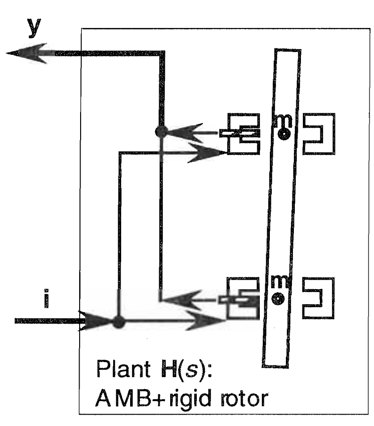

Figure 3.1: Example plant

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3 More precisely, (4.2) minimises the 2-norm of the approximation error:

$$R_r=\arg\min\left(\left\|R_r-R_r^{(2)}\right\|\right)$$
$$=\arg\min\left(\max\left(x^T\cdot\left(R_r-R_r^{(2)}\right)\cdot x\right)\right) \quad \text{where } \|x\|=1$$

- Identify a model $\hat{H}_K(s)$ from the modified data $\tilde{H}_K(s)$
- Remove the proportional feedback from the model $\hat{H}_K(s)$ to get the model $\hat{H}(s)$ of the true plant.

There are two advantages of this procedure:

- The real poles are moved to near to the imaginary axis. Their effect on the FRF is then more clearly visible, and therefore they can be identified more precisely.
- Every controller will contain a proportional feedback part to overcompensate the negative AMB stiffnes. With the described procedure, the identified model becomes more accurate in the vicinity of potential closed-loop poles and therefore more relevant for predicting the closed loop performance of the controlled plant.

Note that inserting or removing proportional feedback does not affect the system order.

4.4 Summary of the Algorithm

To summarise, the proposed algorithm consists of the following steps:

1) Modify the measured multivariable FRF matrix with proportional feedback.
2) Compute the determinant $\det\left(\tilde{H}_K(s)\right)$ from the modified measured FRF data. Compute the system poles using the iterative lin. LS algorithm [2,3,4].
3) Estimate the rank 2 residual matrices associated with each mode (lin. LS).
4) Approximate these by rank 1 matrices using Singular Value Decomposition.
5) Construct a minimal state space plant description.
6) Remove the proportional feedback to get back to a model of the original system.

The proposed identification algorithm solves the strongly non-linear identification problem with a sequence of iterative linear Least Squares, ordinary Linear Least Squares, and Singular-value decomposition steps.

5 Results with Experimental Data

The algorithm has been tested with FRF measurement data from an AMB system. This AMB system is described in detail in [8]. The rotor was highly flexible, with a heavy disk at one end. Its first four eigenfrequencies were at 58, 136, 332 and 508Hz. A photograph of this system is shown in figure 5.1.

The identified model includes the two rigid-body modes and four flexible modes. Figure 5.2 shows the result of identification with the proposed identification algorithm. Magnitude and phase of all four transfer functions agree very well with the measured data. This holds for both the rigid body modes (low frequency range) and for the flexible modes, for both the resonances and the antiresonances of the individual transfer functions, and for both diagonal and off-diagonal transfer functions.

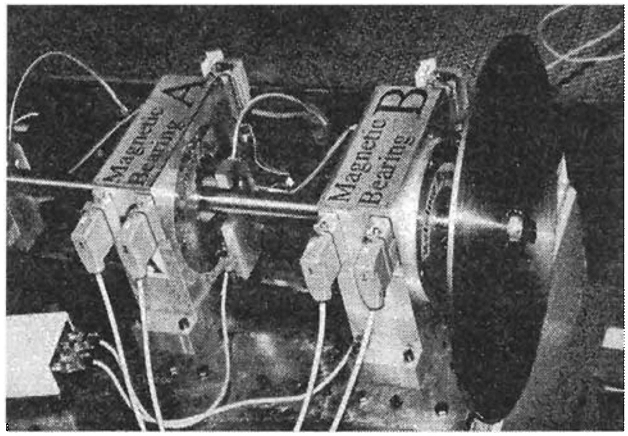

Figure 5.1: The AMB test stand used for verification of the algorithm

6 Generalisations

6.1 Systems With more than 2 Inputs and/or Outputs

The algorithm can be applied to systems with more than two inputs and/or outputs as well. If the number of inputs equals the number of outputs, no change at all is necessary. If these numbers are not equal, a square submatrix must be used to determine the system poles.

6.2 Application to Other Systems

Standard modal analysis problem. The algorithm's potential can be applied to the standard modal analysis problem (FRF from force to displacement).

Systems with different structure. Furthermore, the basic idea of the algorithm is by no means restricted to systems with the structure (2.2). The estimation of the system poles is applicable to any type of systems. The complete algorithm can also be applied to systems of the form

$$\mathbf{H}(s) = \sum_{r=1}^{n} \frac{\mathbf{R}_r}{s - p_r} \tag{6.1}$$

(sum of first order systems), and others.

7 Conclusions and Outlook

A fast, reliable algorithm for multivariable identification of AMB systems has been developed.

The algorithm can be a valuable tool for fast commissioning of AMB systems, and for increasing the achievable controller performance.

AMB systems inherently have the capability to measure the machine's transfer functions in operation. Along with the presented analysis method, this could be used for on-line monitoring and early diagnosis of upcoming faults in rotating machinery.

The presented algorithm has also the potential to solve identification problems with other multivariable plants.

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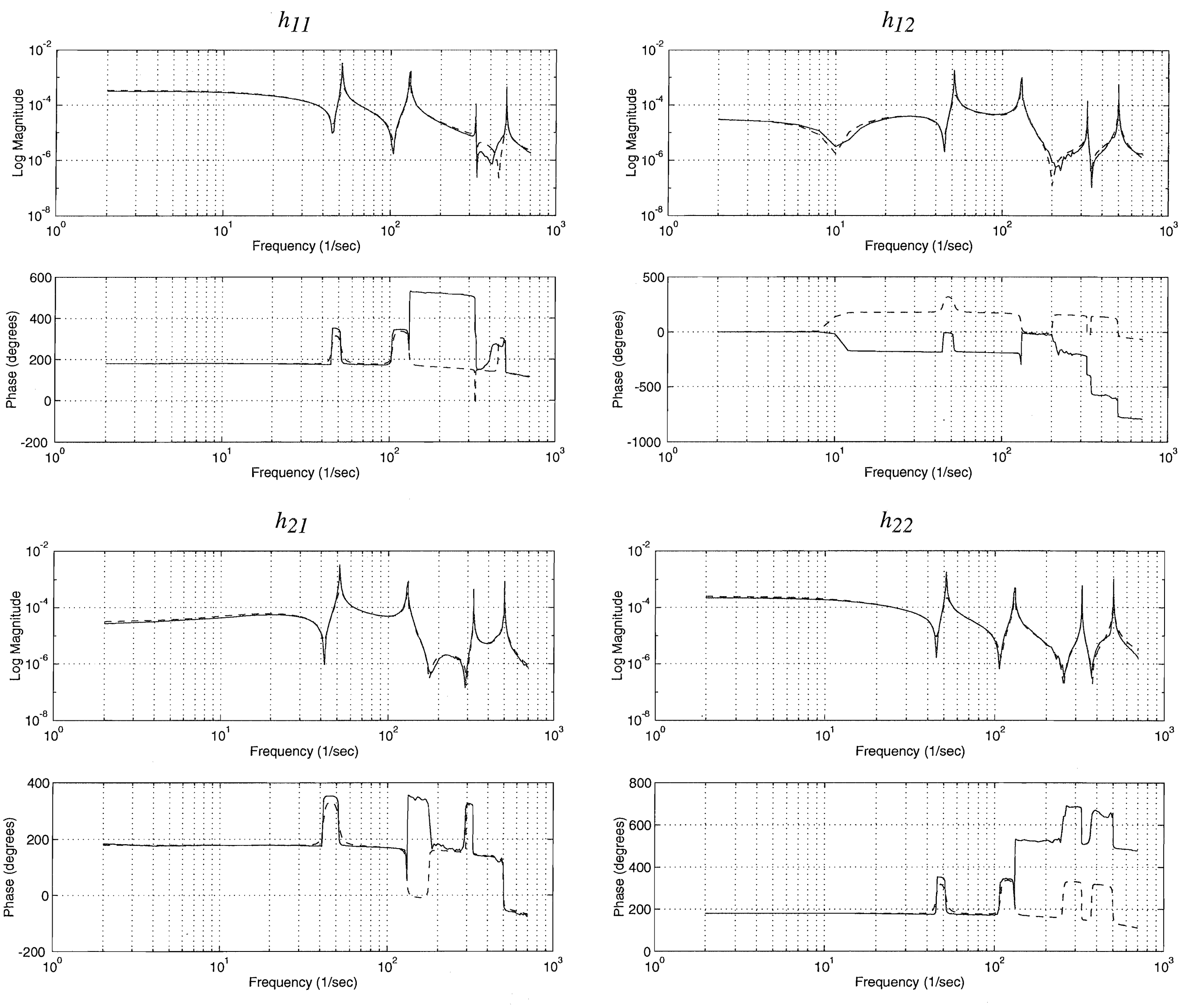

Figure 5.2: *Identification results. Solid: measured FRF; dashed: identified model. Note that the larger differences in the phase are multiples of 360 degrees.*