### Position Sensorless AMB in Four Degrees of Freedom

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Abstract: This paper reports on the first ever experiments to suspend a magnetic bearing rotor without position sensors in all four radial degrees of freedom. The rotor is an industrial turbo molecular pump. The position sensorless method used here, sometimes also called «self-sensing» bearings, relies on voltage control and current measurement as the only plant output-variable. The complete system is linear, i.e. there is no implicit position sensing through some HF-modulation or demodulation. The interesting properties of such bearings have been described in several earlier publications [1, 2, 3, 4, 7, 8].

The main focus of this paper is a report on the operation in all four radial degrees of freedom. Motions in the x- and yradial bearing stator plane are treated separately. Special emphasis is put on the identification procedures necessary to achieve this goal. The plant structuration is described in detail. A rotational speed of 14400 rpm has been reached.

### 1. Introduction

An important industrial application for active magnetic bearings (AMBs) today are turbo molecular pumps (TMPs). The advantages of AMBs such as frictionless operation and

avoiding lubrication prove essential in vacuum technology. There are two major reasons why operating AMBs without gap sensors is interesting for an industrial application such as a turbo molecular pump:

-**Reduction of costs:** Gap sensors and their installation are expensive. The concept of self sensing AMBs would replace these elements with a more sophisticated controller software which causes virtually no costs in mass production.

-More compact construction: Gap sensors require space. This space could be saved yielding a more compact and stiffer rotor, thus rising the critical speed, operational speed and performance.

The concept of "self sensing magnetic bearings" has been introduced by Vischer [2]. The idea is to observe the motion of the rotor - which must be known in order to stabilize it -

from the voltage input as well as the current output signal of the magnetic bearing.

Fig. 1 Turbo Molecular Pumps with control unit as produced by Koyo Seiko Co., Ltd.

### 2. Procedure of Implementation

A sensorless AMB is open loop unstable and closed loop nonminimal phase. This means a very precise model of the plant is necessary in order to find an observer-controller structure to stabilize the system.



Fig. 2 Procedure of implementation for motions in the x-z or y-z planes

In our case, we used a refined analytical model in combination with identification in frequency domain in order to obtain an accurate state space model with isolated state space variables that have a physical interpretation. This method enables us to use diagonal weighting matrices for the design of the LQR/LQE state space observers and controllers. The performance of the controllers designed on the base of the identified model is compared to that of controllers based on the analytical model only.

#### 3. Modeling of the TMP

The TMP is a five degree of freedom system. It is supported by a thrust bearing as well as two radial bearings, all of which are AMBs. The thrust AMB is operated with the analog control system using a displacement sensor. Since the force vector of the thrust bearing acts along a line through the center of gravity, it has no influence on the radial motion. For the radial AMBs a sensorless control scheme is designed.

In this chapter the model used to design the control system for the suspension of the TMP by self sensing AMBs is described. First the transfer functions for a one mechanical degree-of-freedom (MDOF) self sensing AMB is used. Considering the mechanical coupling between the upper and lower bearing a state space model for a two MDOF self sensing AMB system is derived.

### 3.1. Transfer Functions for one DOF voltage controlled AMB

The input of the system is the voltage u defined as:

$$u = \frac{u_1 - u_2}{2} \tag{1}$$

The control signals for the opposing magnets are:

 $u_1 = u_0 + u$ ;  $u_2 = u_0 - u$ 

with

*u*<sub>0</sub> voltage in working point [V]

Transfer function voltage to current:

$$G_{vi}(s) = \frac{-2k_s + m s^2}{L_{tot}m} \left( \frac{-2k_s R_{tot}}{L_{tot}m} + \left( \frac{2k_i^2}{L_{tot}m} - \frac{k_s}{m} \right) s + \frac{R_{tot}}{L_{tot}} s^2 + s^3 \right)$$
(3)

Transfer function voltage to displacement:

$$G_{vx}(s) = \frac{2k_i}{L_{tot}m} \left(\frac{-2k_sR_{tot}}{L_{tot}m} + \left(\frac{2k_i^2}{L_{tot}m} - \frac{k_s}{m}\right)s + \frac{R_{tot}}{L_{tot}}s^2 + s^3\right)$$
(4)

The inductance  $L_{tot}$  refers to the inductance of the whole load connected to the amplifier and includes the inductance of cables etc.. In the same way  $R_{tot}$  refers to the whole resistance of the load connected to the amplifier. The parameters  $k_i$  and  $k_s$  are the linearized force-current respectively force-displacement factors as defined in [1].

# 3.2. Mechanical Coupling of the Upper and Lower Bearing

The rotor is assumed to be rigid. The influence of gyroscopic effects is assumed to be small. The rotation of the shaft is thus not modeled. Motions in the x-z-plane and the y-z-plane can be modeled separately. The dynamics are reduced to the dynamics of a two degree of freedom rigid body (fig.3).



Fig. 3 Two degree of freedom rigid body

### 3.3. Two MDOF State Space Model

The state space model of the two MDOF self sensing magnetic bearing is:

$$\dot{x} = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ \frac{2k_{s1}}{m_1} & 0 & \frac{2k_{i1}}{m_1} & \frac{2k_{s2}}{m_{12}} & 0 & \frac{2k_{i2}}{m_{12}} \\ 0 & -\frac{k_{b1}}{L_1} & -\frac{R_1}{L_1} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ \frac{2k_{s1}}{m_{12}} & 0 & \frac{2k_{i1}}{m_{12}} & \frac{2k_{s2}}{m_{2}} & 0 & \frac{2k_{i2}}{m_{2}} \\ 0 & 0 & 0 & 0 & -\frac{k_{b2}}{L_2} - \frac{R_2}{L_2} \end{bmatrix} x + \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ \frac{1}{L_1} & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & \frac{1}{L_2} \end{bmatrix} u$$
(5)

with

(2)

$$\frac{1}{m_1} = \frac{1}{m} + \frac{l_1^2}{J_z} \; ; \; \frac{1}{m_2} = \frac{1}{m} + \frac{l_2^2}{J_z} \; ; \; \frac{1}{m_2} = \frac{1}{m} - \frac{l_1 l_2}{J_z} \; (6)$$

The terms  $m_1$  and  $m_2$  are here called the equivalent mass at the upper and lower bearing respectively,  $m_{12}$  is the coupling term. Forces at the upper bearing cause accelerations at the lower bearing and vice versa. The term  $k_b$  (back EMF coefficient) is theoretically equal to  $k_s$ .

The state vector is  $\mathbf{x} = [x_1, v_1, i_1, x_2, v_2, i_2]^{T}$ . Subscripts 1 and 2 refer to upper and lower bearing respectively. The input is  $\mathbf{u} = [u_1, u_2]^{T}$ . The plant's output is:

$$\mathbf{y} = \begin{bmatrix} 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \mathbf{x}$$
(7)

The definitions of equations (1) and (2) are used for currents and voltages respectively.

#### 4. Identification of the TMP-System

For the identification, the system is stabilized with voltage control and feedback of current and displacement. The bias current of all radial bearings is 0.5 A. Due to a very good noise rejection during the measurement, the identification was carried out in frequency domain.

The transfer function  $G_i$  from voltage to current as well as the transfer function  $G_x$  from voltage to displacement of the open loop system is then measured. An identification procedure [3] based on a priori knowledge yields the physical parameters used in equation (3) for a one MDOF system.

The open loop systems of the upper and lower bearings are identified separately using this procedure. It is thus identified as a third order SIMO system or as if there were no other bearings in the system.

### 4.1. Extention of the Identified Model to a Model with Two Mechanical D.o.F.

The following definitions of the state space matrices A and B are used:

$$A = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ a_1 & 0 & a_2 & a_5 & 0 & a_6 \\ 0 & -a_3 & -a_4 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ a_7 & 0 & a_8 & a_9 & 0 & a_{10} \\ 0 & 0 & 0 & 0 & -a_{11} & -a_{12} \end{bmatrix}$$

$$B = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ b_1 & 0 \\ 0 & 0 \\ 0 & b_2 \end{bmatrix}$$

$$(8)$$

The terms  $m_1$ ,  $m_2$ ,  $m_{12}$  are the equivalent masses and the coupling term respectively as defined in equation (6). These terms are calculated analytically from the design data. It is thus considered that the mass and geometry of the rotor are well known parameters.

### 4.2. Results of Identification

The identification in x-direction yields similar results as the identification in y-direction. For the controller design the model in x-direction is used for both directions.

The eigenvalues and bode plots are shown in fig.4 through fig.5.



Fig. 4 Poles and zeros of identified model



Fig. 5 Bode Plot for upper bearing of open loop identified system. Solid line: Transfer function from voltage  $u_1$  to current  $i_1$ . Dotted line: Measurement. Dashed line: Transfer function from voltage  $u_1$  to current  $i_2$ .

The separate identification of the upper and lower bearing as described above yields the sub matrices  $A_{11}$  and  $A_{22}$  as well as  $b_1$  and  $b_2$ . The coupling terms  $a_5$  through  $a_8$  are calculated as follows:

$$a_{5} = a_{9} \frac{m_{2}}{m_{12}}$$
$$a_{6} = a_{10} \frac{m_{2}}{m_{12}}$$
$$a_{7} = a_{1} \frac{m_{1}}{m_{12}}$$
$$a_{8} = a_{2} \frac{m_{1}}{m_{12}}$$

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Fig. 6 Bode plot for lower bearing of open loop identified system. Solid line: Transfer function from u2 to i2. Dotted line: Measurement. Dashed line: Transfer function from u2 to i1.

### 5. Controller Design

A 6th order controller stabilizing upper and lower bearing together is designed using the LQE/LQR method [5] for continuos systems. The controller matrices are then scaled and discretized using the Tustin algorithm [6]. The *x*-direction and *y*-direction are controlled separately. However, the same controller matrices are used.

Both the analytical state space model as well as the model based on the identified data are used for the controller design.

### 5.1 Closed Loop System for Self Sensing Control

The signal flow chart of the closed loop system is as follows:



## Fig. 7 Signal flow chart of the self sensing closed loop system in x-direction

For the design of the observer and controller, special emphasis was put on finding a feedback which is open loop stable. If this is not the case, numerical instabilities during the startup procedure of the nonlinear AMB system will make stable operation impossible [9].

The controllers based on the identified model data yield the following sensitivity functions:



Fig. 8 Sensitivity function of upper (solid line) and lower (dashed line) bearing

The sensitivity function is defined as:

$$S(s) = \frac{1}{1 + G_c(s)G_s(s)}$$

with

**G**<sub>**S**</sub>(s)

 $G_{\mathbf{C}}(\mathbf{s})$  transfer function of the controller

transfer function of the plant (identified)

### 6. Results and Discussion

Two controllers were designed and stabilized the system. Both can start up the system without the use of gap sensors, simply by turning the power on. To rotate the rotor the TMP is first evacuated using the pre vacuum pump. The rotor is then driven by the induction motor. This motor causes a radial disturbance force.

The system becomes abruptly instable when the rotation speed exceeds 14400 rpm.

The results can be compared as follows:

	Controller 1	Controller 2
Model Data	identified data	analytical Model
used:	and analytical model	
Performance	100Hz vibration of	100Hz vibration of
at	ca 30 µm	ca 100 µm
Zero		
Rotation		
Maximum	240 Hz	100 Hz
Rotation		
Speed		

Fig. 9 Comparison of controllers based on identified and analytical models

Figures 10-17 show some measurement results.



Fig. 10 Residual vibrations at upper bearing at standstill with controller 1



Fig. 11 Residual vibrations at lower bearing with controller 1 at standstill.



Fig. 12 Bode plot of closed loop transfer function from u1 to sensor signal of upper bearing.



Fig. 13 Impulse response in x-direction at lower bearing using controller 1



Fig. 14 Residual vibrations at upper bearing using controller 1 at 9000 rpm (Inductance motor on)



Fig. 15 Residual vibrations at lower bearing using controller 1 at 9000 rpm (Inductance motor on)



Fig. 16 Residual vibrations at upper bearing using controller 1 at 14400 rpm (inductance motor off)



Fig. 17 Residual vibrations at lower bearing using controller 1 at 14400 rpm

#### 7. Conclusions

The system was fully stabilized in sensorless operation of all four radial DOFs using a controller that couples the upper and lower bearing. Startup is possible without position sensor, simply by turning the power on.

Both models based on analytical data and the identified data can be used to design controllers that stabilize the system at standstill. However, higher rotational speeds could be achieved only using the model based on the identification carried out in this work. The highest rotation speed achieved is 240 Hz (14400 rpm).

In order to rotate the system a controller with high sensitivity has to be used. At zero rotation such controllers have a residual vibration of about 30  $\mu$ m amplitude and 100 Hz.

Two suggestions to improve the performance of self sensing operation of all four radial degrees-of-freedom are:

1) Further improvement of the identification of system parameters. If the system is identified as a 6th order system, the influence of the controller will have to be considered. This is difficult because the voltage controller is a SIMO system. The identification might be easier when the system is operated in current control. A two MDOF current controlled system has 4th order. The controller is a SISO system and thus easier to be included in the identification procedure. A closed loop current controlled system could be modeled as a mass-springdamper system as shown in fig.18. The parameters  $k_{\rm S}$  and  $k_{\rm i}$ could be identified more easily.



Figure 18 Identification of a two MDOF current controlled AMB system by modeling one bearing as a mass-springdamper-system

2) Higher order controllers using a design method such as e.g. the *H*-infinity method.

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