Non Linear Control of Active Magnetic Bearings : Digital Implementation

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ABSTRACT

The aim of this paper is to present a non linear control scheme synthesis and its digital implementation. The process under consideration is a rotating shaft magnetically suspended by means of eight electromagnets located by pairs in order to control four degrees of freedom of the shaft. A passive thrust stabilises the motion along the main axis of the shaft. The problem is to drive actuator inputs which can be the currents or voltages of each electromagnet.

In this paper we focus on the non linear and multivariable features of the model : it is why we don't introduce premagnetisation currents and we don't reduce the model of the system to its linearized form around an equilibrium point.

The description of the control scheme synthesis is decomposed into two parts. The first one is devoted to the analysis of the continuous control scheme, while the second presents the transformation of this scheme in a digital version which can be implemented in a computer.

Then some experimental results are presented. These correspond to the behaviour of a pilot which is used to experiment the control laws. The analysis of these data raises some new questions about disturbances or modeling improvements, which are discussed in the conclusion.

KEYWORDS

Non Linear Control, Active Magnetic Bearings, Transputers, Digital Control.

I - INTRODUCTION

This paper presents a non linear control scheme of active magnetic bearings involved in the magnetic suspension of a rotating shaft and its digital implementation.

The modeling of the considered process is presented in section II. It is composed of a rotating shaft, suspended by means of two active magnetic bearings located in parallel planes orthogonal with rotation axis. Four degrees of freedom are controlled by means of electromagnetic forces. The translation along the main axis of the rotor is passively controlled by annular permanent magnets located in the rotor and in the stator. The magnetic passive thrust is radially destabilizing and these effects are compensated by control law. The rotation around the main axis is controlled by angular velocity regulation.

In section III, we present the continuous control scheme in currents and hierarchical cases. The fourth section is devoted to the implementation aspects and tests in horizontal and vertical cases are presented in section V.

II - MODELING

The process under consideration is a rotating shaft with two active magnetic bearings (AMB). The bearings control four degrees of freedom and a passive thrust stabilizes the projection of the motion of the shaft along the main axis.

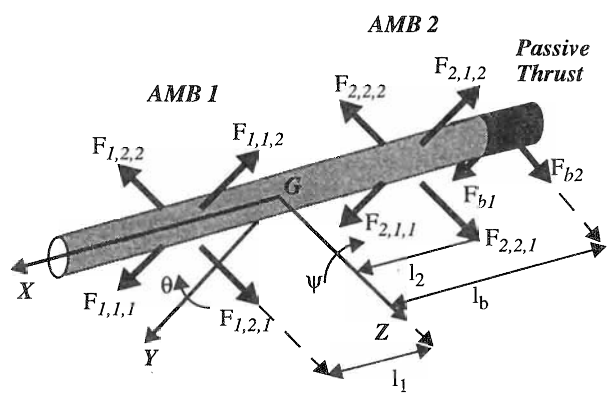

figure 1 : Active magnetic suspension

Let us denote (figure 1) :

(y,z) : the displacement of G (translations along Y and Z axes respectively),

(θ,ψ) : the rotations of the body around Y and Z axes respectively,

$F_{i,j,k}$: forces created by the electromagnets,

F_{bj} : reaction of passive thrust,

R_y, R_z, Γ_y and Γ_z : efforts applied at the centre of mass resulting from the applications of $F_{i,j,k}$ forces,

g_j : components of gravity,

ω : rotation speed of the shaft around X axis,

with $i,j,k = 1,2$ such as :

- i is the number of the AMB,
- j is the number of axes according to direction of the forces (Y axes $j=1$; Z axes $j=2$),
- k is the electromagnet number which works for an AMB and an axe given (positive electromagnet $k=1$, negative electromagnet $k=2$).

Under the assumptions that the shaft is a rigid body and of small displacements (the trigonometric functions can be replaced by their first order approximations), it is possible to describe the mechanical model by the following state space representation :

$$\begin{bmatrix} \dot{y} \\ \ddot{y} \\ \dot{z} \\ \ddot{z} \\ \dot{\theta} \\ \ddot{\theta} \\ \dot{\psi} \\ \ddot{\psi} \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ \frac{-k_r}{m} & 0 & 0 & 0 & 0 & 0 & \frac{-k_r \cdot l_b}{m} & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{-k_r}{m} & 0 & \frac{k_r \cdot l_b}{m} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & \frac{k_r \cdot l_b}{J_y} & 0 & \frac{-k_r \cdot l_b^2}{J_y} & 0 & 0 & -\omega \cdot \frac{J_x}{J_y} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ \frac{-k_r \cdot l_b}{J_y} & 0 & 0 & 0 & 0 & \omega \cdot \frac{J_x}{J_y} & \frac{-k_r \cdot l_b^2}{J_y} & 0 \end{bmatrix} \cdot \begin{bmatrix} y \\ \dot{y} \\ z \\ \dot{z} \\ \theta \\ \dot{\theta} \\ \psi \\ \dot{\psi} \end{bmatrix} + \begin{bmatrix} 0 & 0 & 0 & 0 \\ \frac{1}{m} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & \frac{1}{m} & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{J_y} & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{J_y} \end{bmatrix} \cdot \begin{bmatrix} R_y + mg_1 \\ R_z + mg_2 \\ \Gamma_y \\ \Gamma_z \end{bmatrix} \tag{1}$$

It is assumed that along radial directions, passive thrust behaves like a spring with negative stiffness k_r.

l_1, l_2, l_b, m, J_x, J_y are constant parameters describing geometrical or mechanical characteristics of the shaft. Outputs are taken as displacements of rotor axis measured in sensors planes which are parallel to magnetic bearings.

Note that the efforts applied at the centre of mass can be expressed in terms of the forces created by the electromagnets with a linear relation.

The second step concerns modeling of electromagnets : electrical behaviour is really simple and can be resumed in the following equations :

$$U_{i,j,k} = R_{i,j,k} \cdot i_{i,j,k} + L_{i,j,k} \cdot \frac{di_{i,j,k}}{dt} + i_{i,j,k} \cdot \frac{dL_{i,j,k}}{dt} . \tag{2}$$

$$L_{i,j,k} = \frac{\lambda_{i,j,k}}{e_i + (-1)^k \cdot \left(x_j + (-1)^{i+j} \cdot l_i \cdot \sin(x_{5-j})\right)} \tag{3}$$

with x_j (j=1..4) and $\begin{bmatrix} x_1 & x_2 & x_3 & x_4 \end{bmatrix}^T = \begin{bmatrix} y & z & \theta & \psi \end{bmatrix}^T$.

Last step deals with force production. A model which is often presented in the literature consists in the following static equation :

$$F_{i,j,k} = \frac{\lambda_{i,j,k} \cdot i_{i,j,k}^2}{\left[e_i + (-1)^k \cdot \left(x_j + (-1)^{i+j} \cdot l_i \cdot \sin(x_{5-j})\right)\right]^2} \tag{4}$$

where $\lambda_{i,j,k}$ is a constant with resumes physical characteristics of the coil and kernel, while e_i is another constant which describes the nominal equivalent magnetic circuit length in the AMB number i, including iron and air gap weighted by relative permeability.

The control variables may be chosen either as the input voltages $U_{i,j,k}$, or if the electric circuits are assumed much faster than the mechanical dynamics, as the currents $i_{i,j,k}$. This means, in the latter case, that the equation (2) is not considered.

III - CONTINUOUS CONTROL SCHEME

III-1 - Objectives

The two main difficulties are due to the multivariable feature of the process on the one hand, and to non linearities that exist in the electromagnetical aspects modeling on the other hand.

A decoupling feedback control law is used to avoid the first difficulty.

The second difficulty is circumvented, in many papers, by introducing premagnetisation currents. In our solution, we minimize the energy consumption in the electromagnets by using the only electromagnet which is able to produce the required effort with the right sign. In fact, only one actuator is working at one time along one direction (according to the sign of the required force).

The feedback synthesis is presented with current variables as inputs and a hierarchical control scheme is used to drive voltage inputs.

III-2 - Hierarchical control

We now consider the model made up with (1), (2), (3) and (4), with $U_{i,j,k}$ as control variables. We are confronted to a singularity problem when we want to control the bearings by the input voltages ([4]). At switching, the currents are equal to zero, and some voltages are not defined. A solution to this problem is to consider a hierarchical control scheme.

Two different dynamics appear in the complete model of the process :

- a slow dynamic in the mechanical model,
- a fast dynamic in the electromagnetic model.

So, we can control the mechanical model in efforts, and the control currents are obtained by inversion of the actuators, that is we inverse the relation (4) to express the currents as function of the electromagnetic forces such that

$$i_{i,j,k} = \sqrt{\frac{F_{i,j,k}}{\lambda_{i,j,k}}} \cdot \left[e_i + (-1)^k \cdot \left(x_j + (-1)^{i+j} \cdot l_i \sin(x_{5-j})\right)\right]$$

Let us call high-level control, the current control. We design a low-level control loop to track the current reference trajectory with stability. To be more specific, we consider the system (1) as if it were controlled by the currents $i_{i,j,k}$ and the voltages $U_{i,j,k}$ are designed in such a way that these currents are tracked at a faster rate.

The following figure represents the control scheme.

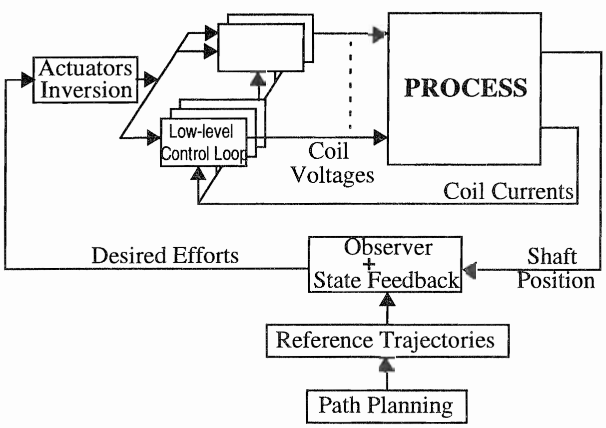

figure 2 : Control scheme

III-2-a - Low-level control

Let us denote by $i^{**}_{i,j,k}$ the high-level control variables. We want to design the voltage loops such that

- the errors between the currents $i_{i,j,k}$ and their reference $i^{**}_{i,j,k}$, locally decrease exponentially fast, the use of high-gains being allowed,
- the high-level tracking error between the rotor position and its reference is stabilized,
- without using a precise knowledge of the currents dynamics.

If we pose

$$U_{i,j,k} = R_{i,j,k} \cdot i_{i,j,k} + L_{i,j,k} \cdot \frac{di_{i,j,k}}{dt} + i_{i,j,k} \cdot \frac{dL_{i,j,k}}{dt} = -K_{i,j,k} \cdot \left(i_{i,j,k} - i^{**}_{i,j,k}\right) \quad (5)$$

the error equation satisfies

$$L_{i,j,k} \cdot \frac{d}{dt} \cdot \left(i_{i,j,k} - i^{**}_{i,j,k}\right) = -\left(\frac{dL_{i,j,k}}{dt} + R_{i,j,k} + K_{i,j,k}\right) \cdot \left(i_{i,j,k} - i^{**}_{i,j,k}\right) - U^{**}_{i,j,k} \quad (6)$$

with

$$U^{**}_{i,j,k} = R_{i,j,k} \cdot i^{**}_{i,j,k} + L_{i,j,k} \cdot \frac{di^{**}_{i,j,k}}{dt} + i^{**}_{i,j,k} \cdot \frac{dL_{i,j,k}}{dt} \quad (7)$$

Note that $U^{**}_{i,j,k}$ is not used in the low-level loop (5). It depends on the coefficients of the currents dynamics, while the feedback (5) does not. Therefore, if we can stabilize (6) by (5), the result will not be sensitive to modeling inaccuracies in (6).

Lower bounds for the gains $K_{i,j,k}$ can be easily computed by assuming that we restrict the velocities $(\dot{y}, \dot{z}, \dot{\theta}, \dot{\psi})$ to a bounded domain and that a given tracking precision on the currents and positions is desired. Note that, in practice, the gains may not be chosen too large to avoid an artificial amplification of external disturbances.

III-2-b - High-level control

Differential flatness is particularly useful to formalize the path planning and reference trajectories tracking [4].

Motion planning is very interesting with this type of process during the centring of the rotor, because we can avoid the actuators saturations.

We fix the initial (*t=0*) and final (*t=H*) conditions about the trajectory that we want to impose for :

- the positions x_j,
- the velocities $\dot{x}_j$,
- the accelerations $\ddot{x}_j$.

The final conditions correspond to the situation of the system at the end of the path planning.

Since we impose six conditions, we plan a polynomial trajectory of order 5 of the type

$$x^r_j(t) = a_o + a_1 \cdot t + a_2 \cdot t^2 + a_3 \cdot t^3 + a_4 \cdot t^4 + a_5 \cdot t^5$$

The four reference control variables $\left(R^r_y, R^r_z, \Gamma^r_y, \Gamma^r_z\right)$, required to track the reference trajectories, are expressed as function of the polynomial reference trajectories $x^r_j(t), \dot{x}^r_j(t), \ddot{x}^r_j(t)$.

The application of the reference trajectories control is not sufficient. This type of control is an open loop control. It is necessary to add a feedback to stabilize the trajectories around the reference trajectories.

Since the mechanical model is linear, we make a static state feedback on the tracking trajectories errors to stabilize the rotor. The control form is the following :

$$V_j = V_j^r + \Delta V_j \text{ with } \Delta V_j = k_{1j} \cdot \left(x_j - x_j^r\right) + k_{2j} \cdot \left(\dot{x}_j - \dot{x}_j^r\right),$$

x_j being one of the state of the system, and x_j^r the reference trajectory of $x_j(t)$.

V_j^r is the control required to track reference trajectories and ΔV_j is the control to stabilize the rotor trajectories around the reference trajectories. k_{1j} and k_{2j} must be computed to stabilize the close loop.

IV - DIGITAL IMPLEMENTATION

This section is devoted to the implementation aspects of the hierarchical control design. In fact, the low-level current control is realised by means of analog amplifiers. Only the high-level control loop (computation of the reference currents in the electromagnet) is implemented in a computer.

The knowledge of the position and velocity of the shaft, as well as the currents in the electromagnets, are necessary to compute the feedback : since the positions are only measured, an observer is needed to reconstruct the velocity informations.

Instead of using a complete observer, we design a reduced observer in order to reconstruct informations about velocities. Position of the centre of mass is deduced from sensor's measures.

The reduced observer is retained since measures are not too much corrupted by noise and it reduces the amount of computation, which allows a smaller sampling period.

IV-1 - Reduced observer design

The information given by the sensors concern the position of the rotor relatively to the stator. If we call ζ_q (q=1,2 in the plane xGy and q=3,4 in the xGz plane) the corresponding measurements, we can directly recover the positions :

$$x_1 = \frac{l_{c_1} \cdot \zeta_2 + l_{c_2} \cdot \zeta_1}{l_{c_1} + l_{c_2}} \qquad x_2 = \frac{l_{c_1} \cdot \zeta_4 + l_{c_2} \cdot \zeta_3}{l_{c_1} + l_{c_2}}$$
$$x_3 = \operatorname{asin}\left(\frac{\zeta_4 - \zeta_3}{l_{c_1} + l_{c_2}}\right) \qquad x_4 = \operatorname{asin}\left(\frac{\zeta_2 - \zeta_1}{l_{c_1} + l_{c_2}}\right) \tag{8}$$

Note that, since the shaft rotates around its main axis at an angular speed ω, we aim at designing the observer such that its behaviour is independent of ω.

If we use the inverted relations (8), the mechanical system described by the equation (1) can be expressed in three independent subsystems, two subsystems of dimension 2 for the translations along Y (relation (9)) and Z (relation (10)), and one subsystem of dimension 4 for the rotations (relation (11)), such as :

$$\begin{bmatrix} \dot{x}_1 \\ \ddot{x}_1 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ \frac{-k_r}{m} & 0 \end{bmatrix} \cdot \begin{bmatrix} x_1 \\ \dot{x}_1 \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{1}{m} \end{bmatrix} \cdot \tilde{R}_y \tag{9}$$

with $\tilde{R}_y = R_y + mg_1 - (k_r \cdot l_b) \cdot x_4$

$$\begin{bmatrix} \dot{x}_2 \\ \ddot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ \frac{-k_r}{m} & 0 \end{bmatrix} \cdot \begin{bmatrix} x_2 \\ \dot{x}_2 \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{1}{m} \end{bmatrix} \cdot \tilde{R}_z \tag{10}$$

with $\tilde{R}_z = R_z + mg_2 + (k_r \cdot l_b) \cdot x_3$

$$\begin{bmatrix} \dot{x}_3 \\ \ddot{x}_3 \\ \dot{x}_4 \\ \ddot{x}_4 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ \frac{-k_r \cdot l_b^2}{J_y} & 0 & -\omega \cdot \frac{J_x}{J_y} & 0 \\ 0 & 0 & 0 & 1 \\ \omega \cdot \frac{J_x}{J_y} & 0 & \frac{-k_r \cdot l_b^2}{J_y} & 0 \end{bmatrix} \cdot \begin{bmatrix} x_3 \\ \dot{x}_3 \\ x_4 \\ \dot{x}_4 \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ \frac{1}{J_y} & 0 \\ 0 & 0 \\ 0 & \frac{1}{J_y} \end{bmatrix} \cdot \begin{bmatrix} \tilde{\Gamma}_y \\ \tilde{\Gamma}_z \end{bmatrix} \tag{11}$$

with $\tilde{\Gamma}_y = \Gamma_y + (k_r \cdot l_b) \cdot x_2$ and $\tilde{\Gamma}_z = \Gamma_z - (k_r \cdot l_b) \cdot x_1$

Estimation of the velocities thus reduces to the construction of three independent observers for linear systems of the form :

$$\begin{bmatrix} \dot{x}_a(t) \\ \dot{x}_b(t) \end{bmatrix} = \begin{bmatrix} A_{aa} & A_{ab} \\ A_{ba} & A_{bb} \end{bmatrix} \cdot \begin{bmatrix} x_a(t) \\ x_b(t) \end{bmatrix} + \begin{bmatrix} B_a \\ B_b \end{bmatrix} \cdot \tilde{u}(t) \tag{12}$$

with x_a the measured state part, and x_b the state part to estimate.

A reduced observer for this system is given by the following equation :

$$\begin{aligned} \dot{\hat{x}}_b(t) = {} & (A_{bb} - l\ A_{ab}) \cdot \hat{x}_b(t) + (A_{ba} - l\ A_{aa}) \cdot x_a(t) \\ & + l \cdot \dot{x}_a(t) + (B_b - l\ B_a) \cdot u(t) \end{aligned} \tag{13}$$

such as $(A_{bb} - l\ A_{ab})$ is stable.

Let us denote $\tilde{A} = A_{bb} - l\ A_{ab}$ and $\tilde{B} = B_b - l\ B_a$.

Because $\dot{x}_a$ is unknown at the time t, we use a change of variables defined by $x_c = x_b - l\ x_a$.

Finally, the reduced observer is described by the system

$$\begin{cases} \dot{\hat{x}}_c(t) = \tilde{A} \cdot \hat{x}_c(t) + [\tilde{A} \cdot l + (A_{ba} - l\ A_{aa})\ ] \cdot x_a(t) + \tilde{B} \cdot u(t) \\ \hat{x}_b(t) = \hat{x}_c(t) + l \cdot x_a(t) \end{cases} \tag{14}$$

IV-2 - Digital control

There are several approaches for the construction of a digital control scheme. One consists in the use of a discrete model of the system followed by a discrete state feedback synthesis. This method was not interesting in this case because of the difficulty in tuning the parameters since the decomposition in four subsystems is less clear.

Another approach consists in discretizing the control law that is built using the continuous model of the system.

The problem is then to choose the appropriate discretization method that brings a good compromise between accuracy and computational burden. This selection has been done by means of simulation.

Finally, a zero order approximation is used for state feedback law :

$$v_d(t) = v_c(kTe)\,,\ \forall t \in [kTe,\ (k+1)\,Te[\quad ,$$

where Te is the sampling period and v_d, v_c are discrete and continuous control variables respectively.

And a first order approximation with zero order hold is chosen for the dynamic part, that is the observer :

$$\begin{cases} \hat{x}_c\,((k+1)\,Te) = Exp(\tilde{A}\cdot Te)\cdot\ \hat{x}_c\,(kTe) \\ \quad + \int_0^{Te} (Exp(\tilde{A}\cdot\eta))\cdot(\,[\tilde{A}\cdot l + (A_{ba} - l\,A_{aa})\,]\cdot x_a(kTe) \\ \qquad + \tilde{B}\cdot v(kTe)\,)\,)\,d\eta \\ \hat{x}_b\,(kTe) = \hat{x}_c\,(kTe) + l\cdot x_a(kTe) \end{cases} \qquad (15)$$

V - TESTS

V-1 - Implementation

The pilot of the Laboratory is a magnetic suspension unit made of a rotor and a stator. The rotor turns around its main axis by means of an electric motor and its position, with respect to the stator, is controlled by four magnetic bearings. Four electromagnetic sensors give a differential position measurement from which the displacement is computed by a devoted circuit. The computer architecture, supervised by a personal computer, is composed by two transputers (INMOS T800 and T222) and a conversion card (4xA/N & 4xN/A). One transputer is used for the analog/digital converters and the other for the numerical treatment. The numerical program is presently written in C with respect to real time aspects.

In the following tests, we don't take the rotating speed into account in the control law. In fact, the effect of gyroscopic coupling is not significant for small rotating speeds. So, it is possible to consider four independent reduced observers because the system (11) can be easily decoupled.

Two kinds of tests are presented, in the horizontal and vertical cases respectively.

V-2 - Horizontal case

In this case, we present the rising of the shaft according to a reference trajectory. We start from the landing position (all the bearings switched off) to a centred equilibrium position. The interest of motion planning is to have two different dynamics : one imposed by the reference trajectory (during the motion planning) and the other, defined by the characteristics of the state feedback (during the regulation around the equilibrium position). By using trajectory planning, we can avoid that control inputs reach saturation.

The following figures illustrate this contribution. The same dynamics are used for the static state feedback in all tests.

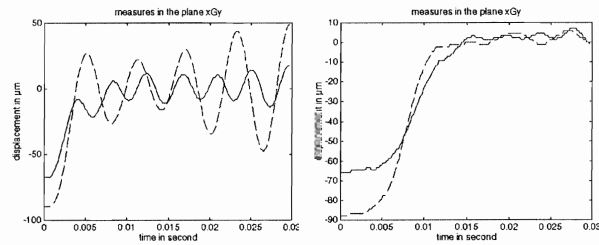

figure 3 : Trajectory of the rotor without and with path planning

In the left test, there is no path planning, when in the right test, there is motion planning with the 100 first samples. The reference trajectories used are 5th degree polynomials for y, z, θ and ψ. The period sampling is 150 µsec. We can see that without path planning, the rotor does not stabilize.

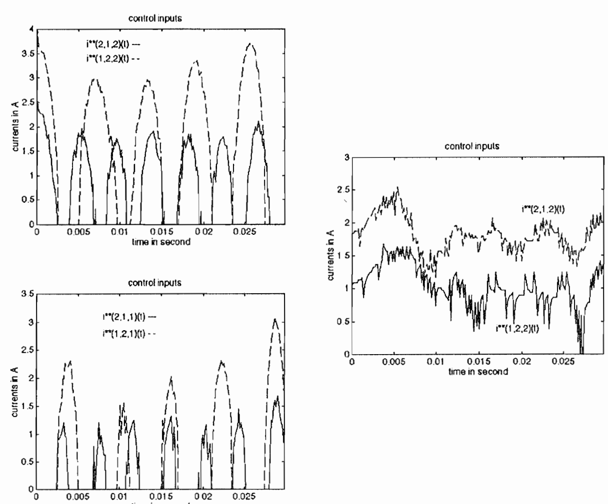

figure 4 : Currents control without and with path planning

We can see that with path planning, only one electromagnet works. Without motion planning, the oscillations of the trajectory of the rotor involve actuator commutations.

V-3 - Vertical case

In this case, we present some tests realised with a rotating speed of 530 Hz (31800 tr/mn), without information on the rotation speed of the shaft, and without any other considerations on the model. The interest of these tests are that a good behaviour is obtained without bias currents, which are harmful in some industrial applications, because they cause temperature increase.

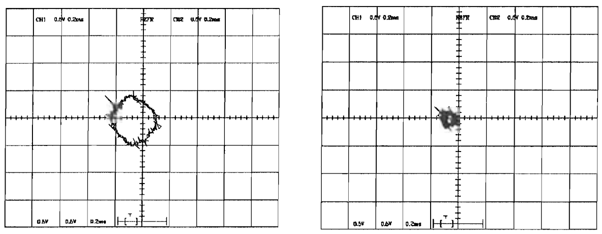

figure 5 : Trajectory of the rotor in the both AMB planes (2 div (1 Volt) = 15 μm on the left and 30 μm on the right)

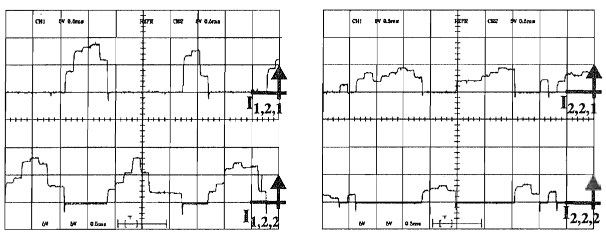

figure 6 : Currents control (1 div = 1.25 A)

We can see (figure 6) that only one electromagnet works at a time. Thermal losses reduction is visible with only two amperes for the AMB 1 and one ampere for the second AMB consumption, when the maximum is five amperes.

VI - CONCLUSIONS

A non linear control scheme has been presented and tested in a real process. In order to reduce thermal losses, there is no polarization current, which roughly means that only one electromagnet is working at one time in each direction. Path planning has been implemented to avoid actuators saturation.

Tests have been done in horizontal and vertical cases. They show a good behaviour at 31800 tr/mn, even without taking account the gyroscopic coupling in the control scheme.

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