# Analysis and Evaluation of Unbalance Resonance Vibration and Bearing Reaction Force for Active Magnetic Bearing Equipped Flexible Rotor

Yoshitaka Hisanaga*, Osami Matsushita**, Shinobu Saitoh***

* Dept. of Mechanical Engineering, National Defense Academy
1-10-20 Hashirimizu, Yokosuka, 239 Japan
tel:+81-468-41-3810; fax:+81-468-44-5900
** Dept. of Mechanical Engineering, National Defense Academy
1-10-20 Hashirimizu, Yokosuka, 239 Japan
tel:+81-468-41-3810; fax:+81-468-44-5900; e-mail:osami@cc.nda.ac.jp
*** Mechanical Engineering Laboratory of IHI, 3-1-15 Toyosu, Koutou, 135 Japan
tel:+81-3-3534-3351; fax:+81-3-3534-3322

**Abstract**

Flexible Rotors supported by active magnetic bearing (AMB) must pass through critical speeds of two rigid modes and several bending modes. The AMB equipped rotor requires control means to achieve low unbalance vibrations with the low driving current during the spin-up operation and the rated rotation. For this requirement, an unbalance vibration analysis program is developed including calculation of the resultant bearing reaction force. By the use of this developed program, the optimum control technique is evaluated for flexible rotors supported by AMBs, regarding the following four means : PID, Ncross, Ncut + Ncross, Feed Forward (FF) Excitation. In addition to the numerical evaluation, a rotation test is performed to determine experimentally the best strategy for the AMB control. The obtained numerical evaluation is accepted by the experimental results gained from the rotation test. It is concluded numerically and experimentally that the technique of FF excitation is the best one among the four above methods with respect of low resonance amplitude and low control current to pass the first bending critical speed.

## 1. Introduction

Active magnetic bearing (AMB) is featured by no contact, no wear, low power loss, maintenance free and so on. These features make the AMB popular in industrial applications in relation to vacuum technology, turbocompressor, fly wheel technology and so on. The AMB was limited to be employed mainly for rigid rotor system as its application. However, it has recently extended to flexible rotor systems.

During spin-up to the operational speed range, flexible rotors supported by AMBs must pass through two rigid mode critical speeds ($N_{c1}$ and $N_{c2}$) and several bending mode critical speeds ($N_{c3}$, $N_{c4}\cdots$). At all range of the operational speed, the AMB must work by economical consumption of the actuator power. Evaluation of the control techniques requires the simulation program to calculate the unbalance vibration amplitude and the resultant bearing reaction force, that is, the AMB current power simultaneously. In this study, an unbalance vibration analysis program is developed for this purpose. It is based upon quasi-modal modeling.

Generally speaking, if we mention optimal control, we can imagine many modern control theories, i.e., the LQR theory[1], $H^{\infty}$ theory[2], Sliding-mode control theory[3] and so on. These theories are prepared for the optimal control design for the servo feedback system. However, it seems that they are too general for practical use of the high speed flexible rotor. The well damped levitation and passage of the rigid mode resonances can easily be achieved, but the passage of the bending mode resonances is generally difficult. Good techniques for passing the bending critical speed are thus required.

Optional means are, therefore, innovated for the unbalance resonance vibration control in rigid and/or bending modes. In fact many innovations are proposed for this demand. These optional control techniques operates during the rotation, but not at stop. As important innovations, this paper selected several methods called Ncross (x-y cross stiffness effect) [4], [5], Ncut (no reaction force to unbalance vibrations) [6] + Ncross and Feed Forward (FF) excitation (Open-loop balancing) [7]~[10] methods to provide a more effective means of control for the practical use.

In this paper, the unbalance vibration analysis method is initially discussed including the calculation of the bearing reaction force. By using this developed program, the comparison of the optional means are performed by numerical evaluation concerning the vibration amplitudes and bearing reaction forces to pass the first bending critical speed ($N_{c3}$). In addition to the numerical evaluation, a rotation test is performed to determine experimentally the best strategy for the AMB controller design.

## 2 Notation

| | |
|---|---|
| $C_g$ = diagonal($C_{g1}$, $C_{g2}$) | gyro matrix of rotor |
| I | unit matrix |
| i | imaginary unit |
| K = [ $K_{ij}$(i, j = 1, 2) ] | shaft stiffness matrix |
| M = diagonal ($M_1$, $M_2$) | mass matrix of rotor |
| Q = [ 0, $Q_2^t$ ]$^t$ | bearing reaction force |
| U = [ $U_1^t$, $U_2^t$ ]$^t$ | unbalance force |
| Z = X+iY = [ $Z_1^t$, $Z_2^t$ ]$^t$ | complex displacement vector |
| $Z_1$ | all displacement vector excluding bearing portions |
| $Z_2$ | displacement vector of bearing portions |
| $\Omega$ | rotational speed |
| $\omega_c$ | critical speed |

## 3 Analysis of Unbalance Vibration and Bearing Reaction Force

### 3.1 Equation of Rotor Motion for Unbalance Response

The equation of motion of the rotor-bearing system including unbalance force is written:

$$M\ddot{Z} + i\Omega C_g\dot{Z} + KZ - Q = U\Omega^2 e^{i\Omega t} \tag{1}$$

The bearing reaction force vector Q depends upon the rotational speed.

The displacement vector Z is separated by one of the bearing portions ($Z_2$) and others ($Z_1$). The former is called the boundary points and the latter the inner points. This type of separation of the displacement vector provides the equation of motion with each of the following matrices:

$$\begin{bmatrix} M_1 & 0 \\ 0 & M_2 \end{bmatrix}\begin{bmatrix} \ddot{Z}_1 \\ \ddot{Z}_2 \end{bmatrix} + i\Omega\begin{bmatrix} C_{g1} & 0 \\ 0 & C_{g2} \end{bmatrix}\begin{bmatrix} \dot{Z}_1 \\ \dot{Z}_2 \end{bmatrix} + \begin{bmatrix} K_{11} & K_{12} \\ K_{21} & K_{22} \end{bmatrix}\begin{bmatrix} Z_1 \\ Z_2 \end{bmatrix} - \begin{bmatrix} 0 \\ Q_2 \end{bmatrix} = \Omega^2\begin{bmatrix} U_1 \\ U_2 \end{bmatrix} e^{i\Omega t} \tag{2}$$

The feature of this description is the allocation of the bearing reaction force $Q_2$ acting only on the boundary portions $Z_2$. In the case of the oil film bearing, $Q_2$ is defined by bearing dynamic properties characterized by so called eight parameters. If the rotor is supported by AMBs, $Q_2$ is characterized by the transfer function of the control circuit for the AMB driving unit.

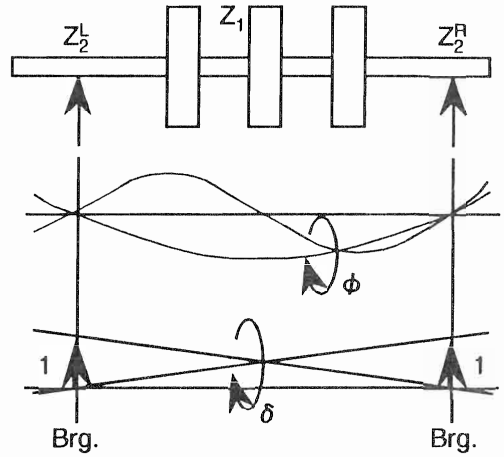


Fig.1 Quasi-modal transformation modes

### 3.2 Unbalance Vibration Analysis Method

The unbalance vibration amplitude can generally be expressed by the following forms:

$$\begin{bmatrix} Z_1 \\ Z_2 \end{bmatrix} = \begin{bmatrix} A_1 \\ A_2 \end{bmatrix} e^{i\Omega t} \tag{3}$$

In order to avoid the large scale calculation to determine the vibration amplitude of Eq.(3), a system reduction is applied by the quasi-modal transformation[11] as follows:

$$\begin{bmatrix} Z_1 \\ Z_2 \end{bmatrix} = \begin{bmatrix} \phi & \delta \\ 0 & I \end{bmatrix}\begin{bmatrix} A_s \\ A_2 \end{bmatrix} e^{i\Omega t} \tag{4}$$

The system reduction enables the determination of each amplitude for the quasi-modal transformation modes of Eq.(4) instead of Eq.(3).

Two types of modes, the inner eigen bending modes $\phi$ and the rigid modes $\delta$, are shown in Fig.1. The inner bending modes defines the undamped critical modes $\phi$ having the corresponding eigen frequency $\omega_c$ which is obtained under the pin-pin boundary condition at bearing portions. These rigid modes are straight lines $\delta$ having unity displacement at one of the bearing portions. Since the combination of these two types of modes provides the system reduction with a small scale of equation of motion, the unbalance vibration amplitudes are determined by the following equation:

$$\begin{bmatrix} -\Omega^2 M_1^* + \omega_c^2 M_1^* & -\Omega^2 M_c \\ -\Omega^2 M_c^t & -\Omega^2 M_2^* + K_{eq} \end{bmatrix}\begin{bmatrix} A_s \\ A_2 \end{bmatrix} - \begin{bmatrix} 0 \\ \hat{Q}_2 \end{bmatrix} = \Omega^2\begin{bmatrix} \phi^t U_1 \\ \delta^t U_1 + U_2 \end{bmatrix} \tag{5}$$

where

$$M_1^* = \phi^t(M_1 + C_{g1})\phi \ , \ M_c = \phi^t(M_1 + C_{g1})\delta$$

$$M_2^* = \delta^t(M_1 + C_{g1})\delta + M_2 + C_{g2} \ ,$$

$$K_1^* = \phi^t K_{11}\phi = \omega_c^2 M_1^* \ , \ K_{eq} = K_{22} - K_{21}K_{11}^{-1}K_{12}$$

and $Q_2 = \hat{Q}_2 e^{i\Omega t}$.

If the bearing reaction force is given by AMBs, $Q_2$ is determined by the transfer function G(s) from the displacement sensor to the AMB force including the controller:

$$Q_2 = -G(s)Z_2 \tag{6}$$

The reaction force $Q_2$ is thus replaced by the transfer function as follows:

$$\begin{bmatrix} -\Omega^2 M_1^* + \omega_c^2 M_1^* & -\Omega^2 M_c \\ -\Omega^2 M_c^t & -\Omega^2 M_2^* + K_{eq} + G(i\Omega) \end{bmatrix} \begin{bmatrix} A_s \\ A_2 \end{bmatrix} = \Omega^2 \begin{bmatrix} \phi^t U_1 \\ \delta^t U_1 + U_2 \end{bmatrix} \tag{7}$$

By solving the above equation at each rotational speed, the unbalance vibration calculation is obtained. Once the unbalance vibration amplitude is obtained, the bearing reaction force is calculated from the modified formula of Eq.(5):

$$\hat{Q}_2 = -\Omega^2(\delta^t U_1 + U_2) + (-\Omega^2 M_c^t A_s - \Omega^2 M_2^* A_2 + K_{eq} A_2) \tag{8}$$

## 4 Bending Vibration Control Methods

In this paper, several optional methods to pass the bending critical speeds within small vibrations and low AMB forces are proposed and they are compared with each other. PID is standard, Ncross, Ncut + Ncross and FF Excitation are optional methods.

### 4.1 PID Control

With regard to AMB placed at one side for the two axis (X and Y) controls, the control network is laid out in the X and Y directions independently, as shown in (a) of Fig.2. Each X and Y displacement signal is connected to the controllers and AMB coils to generate the magnetic force in similar specification, respectively. In this controller layout the PID control law is commonly used, having a large phase lead at the frequencies of rigid and bending critical speeds to provide much damping. Instead of the conventional PID control law, modern control laws mentioned above are also available for this purpose. Since the high gain at high frequency domain potentially induces high frequency mode instability, known as the spill over problem, it is even harder to tune well damping effect for the bending modes.

### 4.2 Ncross

One of the innovations is called Ncross as shown in (b) of Fig.2. The detected X and Y directional displacement signals are fed to the tracking filter synchronizing with the rotational speed which can only select the unbalance vibration. The output signals from the tracking filter are connected with each power amplifier in a cross manner. It is a well known fact that the cross stiffness induces the system instability called oil whip in the case of the oil film bearing. This idea is reversely applied to the AMB control in order to increase system stabilization by connecting the X and Y cross cables to generate the opposite sign of the cross stiffness.

### 4.3 Ncut + Ncross

In the case of Ncross, damping effect is provided from PID control and Ncross. When the bending modes are considered, the damping function of the PID controller can be eliminated as shown in (c) of Fig.2. It is called Ncut + Ncross. The displacement signal excluding the unbalance vibration

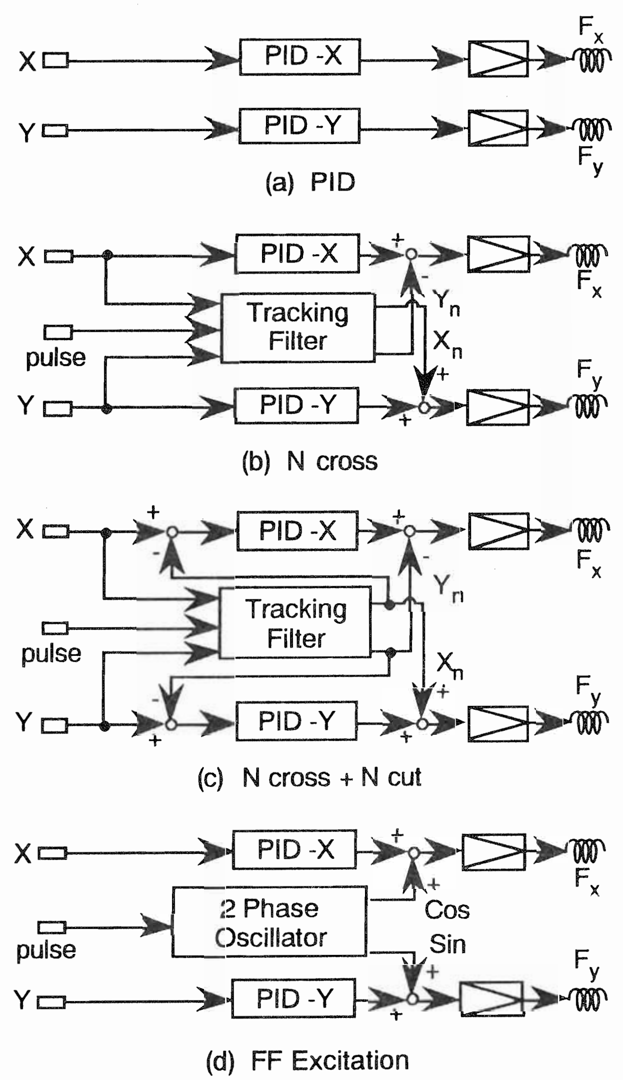


Fig.2 AMB control method for unbalance vibration

component is fed to the PID controller which reacts only to the fluctuation of the rotor motion so as to levitate the rotor. No contribution of the PID is provided for unbalance vibration control. The damping is generated only by the Ncross.

### 4.4 Feed Forward (FF) Excitation

In addition to the feedback loop network as mentioned above, the feed forward excitation is considered as an option, called FF excitation. The corresponding network is described in (d) of Fig.2. The two phase oscillator generates cosine and sine waves synchronized with the rotational speed. If the cosine and sine waves are connected with each directional power amplifier, the forward force can be generated to excite the rotor at the bearing portions for canceling the unbalance force and thus for reducing the unbalance vibration. The opposite phase of unknown unbalance is created manually by shifting the phase of the oscillator and the gain can also be adjusted by directing the unbalance vibration vector to the origin.

In this paper, these methods of Fig.2 are compared concerning the vibration amplitude and the bearing reaction force of the magnetic bearings to pass the first bending critical speeds. The criteria of the best control is definite: to achieve significant vibration reduction at resonance speeds and reducing the required current as much as possible.

## 5 Test Rotor and Results

### 5.1 Test Rotor

A simple rotor is selected for discussing the best optional technique, as shown in Fig.3. This rotor is about 800mm in length and 6.9Kg in Weight, having three unbalance weight at a mid point and both ends. AMBs support the rotor on both sides with the bearing span of 560mm. The diameter of the AMB journal is 40mm.

### 5.2 Numerical Simulation

By the use of developed program, unbalance vibration calculations including the bearing reaction force were done for the PID controller and optional control methods. In the case of the PID control (a) and the FF excitation (d), the response curves with respect to the unbalance vibration ampli-

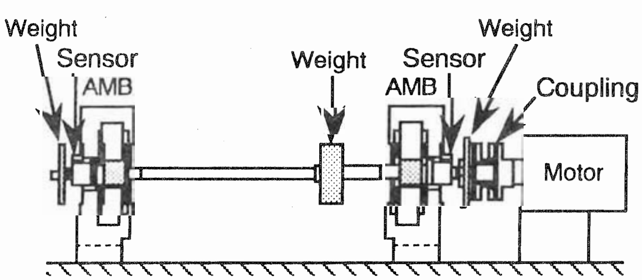


Fig.3 A test rotor

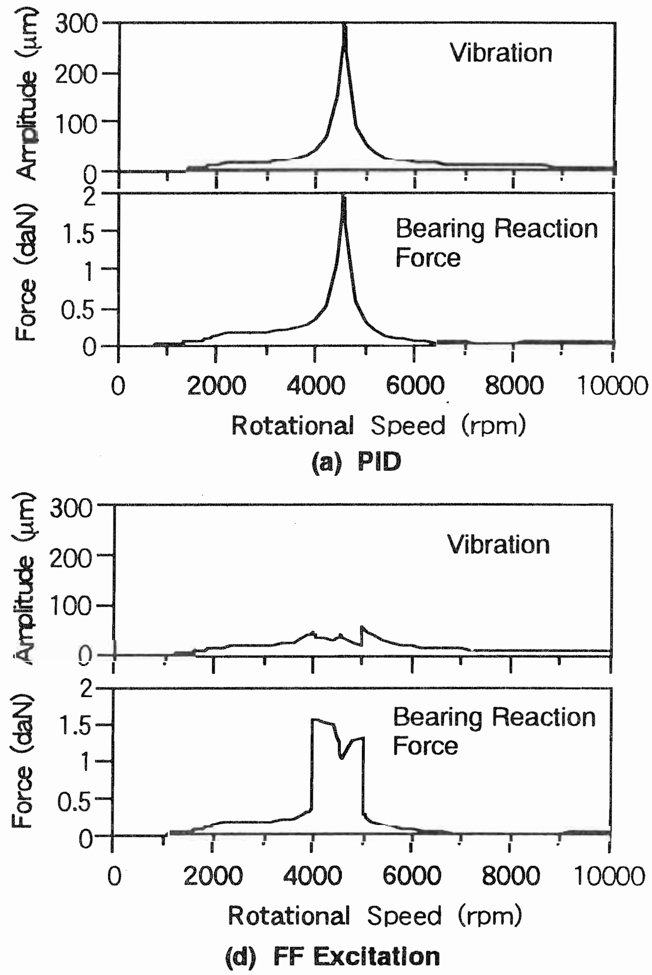


Fig.4 Simulation of unbalance vibration control

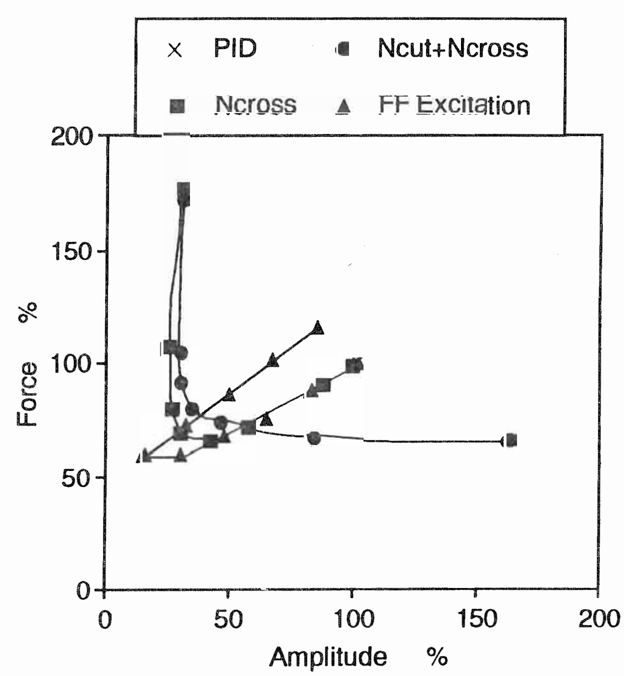


Fig.5 Evaluation of Control Methods (Calculation)

tude and the bearing reaction force are shown in Fig.4, respectively. The rigid mode resonances around 2000 r/min are well damped with no peaks. A peak amplitude appears at the first bending critical speed around 4500 r/min.

The response curves of the PID control observe the peak amplitude in the vibration and the bearing reaction force. However, they are suppressed adequately by the addition of the option of the FF excitation around the bending critical speed. The peak values of the bearing reaction force in the case of the FF excitation are smaller than only using the PID controller as well as the vibration amplitude.

The peak amplitude and the corresponding required bearing reaction force are picked up in each case, in relation to the parameter of the strength of the addition of each optional means. These values are plotted on an X - Y chart indicating the peak amplitude on the X axis and the peak bearing reaction force on the Y-axis as shown in Fig.5. Each axis is normalized by the values of the PID control, and expressed in a percentage form; × for PID, ■ for Ncross, ● for Ncut + Ncross and ▲ for FF excitation.

The best control strategy is indicated by low bearing reaction force and low resonance amplitude. The closer the plot-

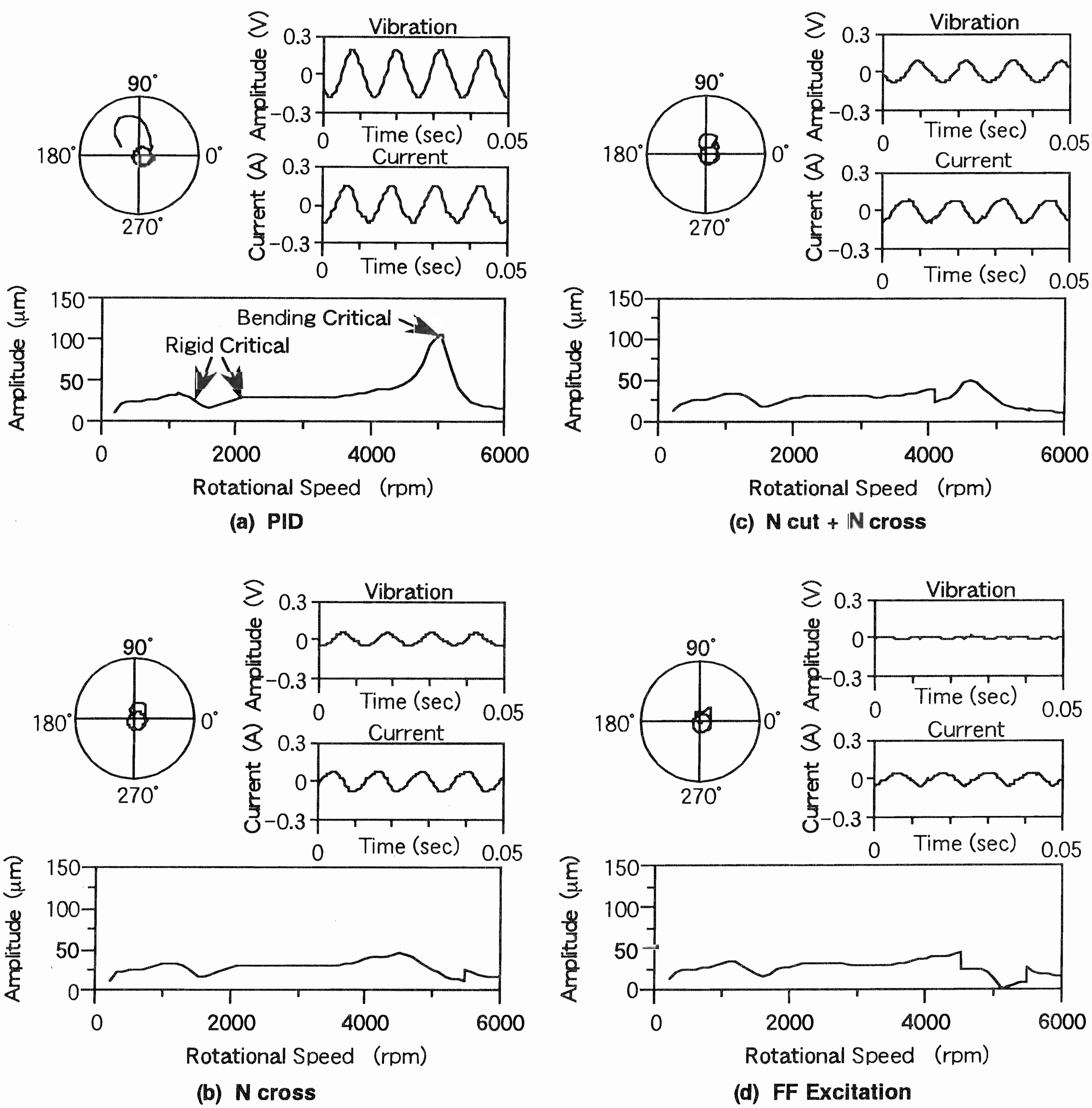


Fig.6 Rotation Test Results

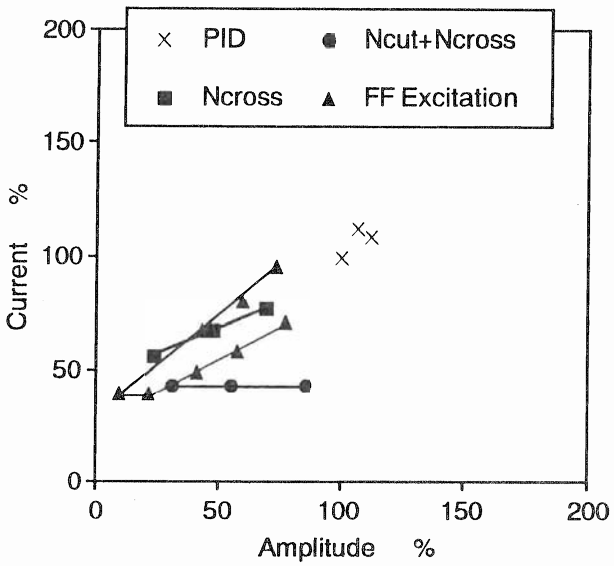


Fig.7 Evaluation of Control Methods (Experiment)

ted points of values come to the origin, the better the estimation score becomes. If the best condition is selected by the FF excitation, it is possible to pass the bending critical speed with 20% of the amplitude by about half the power.

### 5.3 Rotation Test Results

In the case of the PID control only, the unbalance vibration is measured, as shown in (a) of Fig.6. The resonance vibration sharply appears at the bending critical speed around 5000 r/min. Rigid mode resonances may exist around 2000 r/min, but are not seen clearly due to being well-damped. As shown in these vibration response curves, it can be said that the PID control only provides enough damping for rigid mode vibration, but not sufficient for the bending mode.

By the use of these optional methods, the resonance vibration can be damped in each case. While passing the bending critical speed, the vibration amplitude and the corresponding current activating magnetic coils are monitored for each options and the each maximum values are measured. According to the optional methods, the optimum cross stiffness of Ncross or the adequate addition of the FF excitation can completely reduce the resonance amplitude with comparatively low current (bearing force), as shown in Fig.6. These measured values are plotted in Fig.7 with respect to the maximum values of the amplitude vs the current in the same manner of Fig.5.

## 6 Conclusions

(1) An unbalance vibration analysis method to simulate vibration amplitude and the bearing reaction force is developed for generating the program code.

(2) By using this program, the estimation of the best AMB control technique for passing the resonance of the first bending critical speed is performed. According to the evaluation with respect to small amplitude and low required bearing reaction force, several optional methods of the FF excitation, Ncross and/or Ncut are the best compared to the conventional PID.

(3) This numerical evaluation of the best control is accepted by the rotation test results of an AMB equipped flexible rotor.

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