About the Influence of the Magnetic Suspension on Noncontact Gyroscope Dynamics

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Abstract

Stable levitation of rotors in noncontact gyroscopes with magnetic suspension is provided by a system of automatic regulation (SAR), which converts the signal of the rotor drift from an equilibrium position into the control signal changing the currents in electromagnets to the desired direction. The absence of special transducers of rotor position is a characteristic feature in the majority of devices [1], because the rotor drift with respect to the suspension poles changes their inductance. This fact provides the appearance of the input signal of the SAR, which, in an ideal case should respond only to the translational displacements of the rotor's center of masses and should be insensitive to its angular movements.

Because of the residual unbalance of the rotor [2] and its shape deviation from the spherical one, the SAR responds not only to the translational motions, but to the angular motions as well. The phase shifts of harmonic components of the input and output signals lead to the appearance of nonconservative forces and moments, influencing the value of the angular momentum of the gyroscope and its "precession" and "nutation" stability.

It is shown in this paper that even in the case of a perfectly balanced rotor stably hanging in the suspension field, the self- modulation stipulated by the rotor surface asphericity can lead to instability of its angular motions.

1. Let us consider the motion of the ellipsoidal in shape but perfectly balanced rotor with dynamic symmetry ($I_1 = I_2 = A \neq I_3 = C$) in the force field of a magnetic suspension. (This consideration can also be applied to the gyroscopes with other types of suspension, electrostatic, for example.) Let us assume that this field is generated by N pairs of electromagnets located on the same axis but on the different sides of the rotor. In other words, they are equidistantly located on two small circles which determine the axis OZ_3 and the plane Z_1OZ_2 of suspension symmetry. In this case, for the unit vectors of the field symmetry, in the coordinate system OZ_k, we write

$$\vec{\mathbf{h}}_n = \vec{\mathbf{z}}_1 \sin\alpha\cos\beta_n + \vec{\mathbf{z}}_2 \sin\alpha\sin\beta_n + \vec{\mathbf{z}}_3\cos\alpha \; ; \quad \left(\beta_n = \frac{2\pi n}{N} \; ; \; n = 1 \div N\right) \tag{1}$$

Let us note that under $\alpha = 0$ this model corresponds to a single-axis "extension" type suspension [3,4]; the combination of the cases $\alpha = 0$ and $\alpha = \pi/2$ provides description of a three-axis suspension with two coils on each axis; the case where $\alpha \neq k\pi/2$ corresponds to some types of magnetic or cryogenic suspensions for noncontact gyroscopes [1].

If, initially, the center of masses coincides with the suspension center, then in the absence of the gravity force its position will not change under angular motions if the SAR processes in the same manner the identical clearances δ_{n1} and δ_{n2} of an $n-$th pair of electromagnets located on the same axis but on the different sides of the rotor.

Let us assume the rotor surface to be an ellipsoid of revolution with eccentricity ε and the symmetry axis $\vec{\mathbf{E}} = E\vec{\mathbf{e}}$. If the rotor interaction with any of the coil pairs is independent of the others, the take-away moment acting on the rotor and stipulated by its ellipsoidallity is represented as a sum [5]

$$\vec{\mathbf{M}} = 2\varepsilon \sum_{n=1}^{N} M_n \left(\vec{\mathbf{e}} \cdot \vec{\mathbf{h}}_n\right) \left[\vec{\mathbf{e}} \times \vec{\mathbf{h}}_n\right] \tag{2}$$

Under the identity of control channels of the source fields and under a small ratio δ_n/R_0 it can be assumed that $M_n = W(p)\delta_n$, where $W(p) = W(j\Omega) = W_0(P + jQ)$ is the suspension transfer function; δ_n is the clearance between the rotor and one (any) of the poles of an $n-$th pair of sources; R_0 is some mean radius of the rotor surface curvature so that $R_\parallel = R_0(1+\varepsilon)$, $R_\perp = R_0(1-\varepsilon)$. In the linear approximation with respect to small ε one can write

$$\delta_n = \delta_{n0} + \varepsilon R_0 \left[1 - 2\left(\vec{\mathbf{e}} \cdot \vec{\mathbf{h}}_n\right)^2\right] \tag{3}$$

Attempting to use the averaging method for investigation of the gyroscope motion under the moment (2), let us write the equations for the phase variables [6]. For this purpose, apart from the mentioned fixed trihedron OZ_k, we introduce the trihedron OY_j with kinetic axis OY_3 connected with the angular momentum $\vec{\mathbf{K}}$ and the trihedron OX_i with an apex on the axis OX_3 connected with the rotor. We choose the axis OX_1 so that the unit vector of surface symmetry $\vec{\mathbf{e}}$ is in the plane X_1OX_3

$$\vec{\mathbf{e}} = \vec{\mathbf{x}}_1 \sin\gamma + \vec{\mathbf{x}}_3 \cos\gamma = s\vec{\mathbf{x}}_1 + c\vec{\mathbf{x}}_3 \tag{4}$$

Using the spherical angles ρ, σ and the Euler angles ϑ, ψ, and φ for description of mutual orientation of the trihedrons

$$\vec{\mathbf{x}}_i = b_{ij}(\vartheta, \psi, \varphi)\vec{\mathbf{y}}_j = b_{ij}(\vartheta, \psi, \varphi)\, a_{jk}(\rho, \sigma)\vec{\mathbf{z}}_k$$

we represent the equations of gyroscope motion in the form of the system

$$\begin{aligned} K\dot\rho &= M_1, & K\dot\sigma\sin\rho &= M_2, \\ \dot K &= M_3, & K\dot\vartheta &= M_+; \end{aligned} \tag{5}$$

$$\begin{aligned} \dot\varphi &- \frac{K}{A}\frac{A-C}{C}\cos\vartheta = -\frac{M_-}{K\sin\vartheta}, \\ \dot\psi &- \frac{K}{A} = \frac{M_-}{K}\,\mathrm{ctg}\vartheta - \frac{M_2}{K}\,\mathrm{ctg}\rho; \end{aligned} \tag{6}$$

where

$$M_j = \left(\vec{\mathbf{M}}\cdot\vec{\mathbf{y}}_j\right); \quad \begin{aligned} M_+ &= M_1\cos\psi + M_2\sin\psi; \\ M_- &= M_1\sin\psi - M_2\cos\psi. \end{aligned}$$

Introducing the characteristic values of the angular momentum K^*, the disturbing torques M^*, and moving to the nutation time scale, the right sides of the equations (5) for a high-speed rotor can be made proportional to the small parameter $\mu = (A\,M^*)/(K^*)^2$, which is necessary for use of asymptotic methods. Omitting the standard procedure of normalization, let us substitute the expression (2) for a disturbing torque allowing for the relations (1), (3), (4) into the right sides of the equations and perform averaging of the equations along the paths of the generating solution where

$$\dot\rho = \dot\sigma = \dot K = \dot\vartheta = 0;$$

$$\dot\psi = \frac{K}{A}; \quad \dot\varphi = \frac{K}{A}\frac{A-C}{C}\cos\vartheta.$$

Restricting the solution of the dynamic problem to the consideration of stability of zero values of the angles ρ and ϑ and constancy of the angular momentum K, we linearize the equations with respect to these angles, and, as a result, we obtain the following system of equations describing evolution of the slow variables ρ, σ, K, and ϑ:

$$\begin{aligned} K\dot\sigma = &- \frac{1}{2}NM_c\,(1-3c^2)(1-3\cos^2\alpha) \\ &+ \frac{1}{2}NM_{nc}\{(3c^2-1)[(1-3\cos^2\alpha)c^2 \\ &+ (1-3c^2)(3-5\cos^2\alpha)\cos^2\alpha] - \\ &- 4s^2c^2(10\cos^4\alpha - 9\cos^2\alpha + 1)P_{11} \\ &+ \frac{1}{2}s^4(1-5\cos^2\alpha)P_{22}\sin^2\alpha\}; \end{aligned}$$

$$\dot K = \frac{1}{2}NM_{nc}s^2(8c^2Q_{11}\cos^2\alpha + s^2Q_{22}\sin^2\alpha)\sin^2\alpha;$$

$$K\dot\rho = \frac{1}{4}NM_{nc}s^2[8c^2Q_{11}(3-5\cos^2\alpha)\cos^2\alpha + s^2Q_{22}(1-5\cos^2\alpha)\sin^2\alpha]\rho;$$

$$\begin{aligned} K\dot\vartheta = &- \frac{1}{2}NM_{nc}\Big\{2s^2c^2Q_{01}(3\cos^2\alpha-1)^2 \\ &- 2\Big[(3c^2-1)^2Q_{10} - 2s^2c^2Q_{11} \\ &- s^4Q_{1,2}\Big]\sin^2\alpha\cos^2\alpha \\ &- \left(2c^2Q_{21} - \frac{1}{2}s^2Q_{22}\right)\sin^4\alpha\Big\}\vartheta. \end{aligned} \tag{7}$$

Here

$$\begin{aligned} M_{c(onservative)} &= \varepsilon W_0\delta_0, \\ M_{n(on)c(onservative)} &= \varepsilon^2 W_0 R_0; \\ P_{mn}, Q_{mn} &= P, Q(\Omega_{mn}), \end{aligned}$$

where $\Omega_{mn} = m\dot\psi + n\dot\varphi$ are the frequencies of the spectrum of disturbing torques.

If the generating solution under the small ϑ is characterized only by two frequencies ($\Omega_{10} = \dot\psi = K/A$ and $\Omega_{11} = \dot\psi + \dot\varphi = K/C$), then the right sides of the equations (7) depend on the values of the suspension transfer function at the frequencies Ω_{01}, Ω_{10}, Ω_{11}, Ω_{12}, Ω_{21} and Ω_{22}.

Let us note that in the absence of self-modulation we have only the conservative component of the moment $1/2\,\varepsilon NW_0\delta_0\,(1-3c^2)(1-3\cos^2\alpha)$, which induces precession of the angular momentum $\vec{\mathbf{K}}$ about the suspension axis OZ_3.

2. Let us consider some particular cases of constructive realization of the suspension mentioned above.

A. A single-axis suspension with two coils located on the axis OZ_3 on the different sides of the plane Z_1OZ_2. In this case the equations have the form

$$\begin{aligned} K\dot\sigma &= M_c(1-3c^2) \\ &+ M_{nc}[(1-3c^2)(1-2c^2) - 4s^2c^2P_{11}]; \end{aligned}$$

$$\dot K = 0, \quad K\dot\rho = -4M_{nc}s^2c^2Q_{11}\rho,$$

$$K\dot{\vartheta} = 4\,M_{nc}s^2c^2Q_{01}\vartheta\,, \tag{8}$$

from which one can see that the changes in the angles ρ, σ and ϑ occur at a constant value of the angular momentum determined by the initial conditions.

B. A three-axis suspension consisting of a pole pair on each of the axes OZ_k on the different sides of the suspension center

$$K\dot{\sigma} = M_{nc}\left[(1-3c^2)^2 - 8s^2c^2P_{11} + \frac{1}{2}s^4P_{22}\right];$$

$$\dot{K} = -M_{nc}s^4Q_{22}\,;$$

$$K\dot{\rho} = -\frac{1}{2}M_{nc}s^2\left(8c^2Q_{11} - s^2Q_{22}\right)\rho\,;$$

$$K\dot{\vartheta} = M_{nc}\left(6c^2Q_{21} + \frac{1}{2}Q_{22}\right)s^2\vartheta\,. \tag{9}$$

For such a construction and the small angles ρ and σ, the rotor is acted upon only by the moments due to the self- modulation.

C. An "equirigid" suspension, in which the source rings are located at an angle α of $\approx 54^{\circ}44'$, for which $3\cos^2\alpha = 1$. In this case, as previously, the take-away moments are stipulated only by the self-modulation of the suspension field due to rotation of an ellipsoidal rotor in the latter

$$K\dot{\sigma} = -\frac{1}{9}NM_{nc}\left[2(1-3c^2)^2 + 16s^2c^2P_{11} + s^4P_{22}\right]$$

$$K\dot{\rho} = \frac{1}{9}NM_{nc}s^2\left(8c^2Q_{11} - s^2Q_{22}\right)\rho\,;$$

$$\dot{K} = -\frac{2}{9}NM_{nc}s^2\left(4c^2Q_{11} + s^2Q_{22}\right);$$

$$K\dot{\vartheta} = -\frac{1}{9}NM_{nc}\Big[2(1-3c^2)^2Q_{10}$$

$$-4s^2c^2(Q_{11}-Q_{21}) - s^4(2Q_{12}+Q_{22})\Big]\vartheta\,. \tag{10}$$

Most interesting is the action on the rotor of nonconservative moments due to the imaginary part of transfer function $W(j\Omega)$, responsible for an asymptotic stability of the equilibrium position of the rotor's center of mass. As for the conservative part of the self-modulation moment, it changes only the precession frequency of the angular momentum influencing neither its value nor the precession (the angle ρ) and nutation (the angle ϑ) stability.

Both in the mentioned particular cases and under an unrestricted angle α, the action of the nonconservative moments on the gyroscope is described by the equations in the form

$$\dot{K} = F_0(K)\,,\quad K\dot{x}_i = F_i(K)x_i\,,$$
$$(x_1 = \rho,\; x_2 = \vartheta)\,. \tag{11}$$

Under unrestricted $F(K)$, this system of equations nonlinear with respect to Kcan be integrated in quadratures

$$\begin{aligned} t - t_0 &= \int_{K_0}^{K}\frac{d\xi}{F_0(\xi)}\,;\\ x_i &= x_{i0}\exp\int_{K_0}^{K}\frac{F_i(\xi)\,d\xi}{\xi F_0(\xi)}\\ &= x_{i0}\exp\int_{K_0}^{K}\frac{F_i[\xi(\tau)]\,d\xi}{\xi(\tau)}\,. \end{aligned} \tag{12}$$

Furthermore it is possible to draw some conclusions about the character of the dependencies $\rho(\tau)$, $K(\tau)$ and $\vartheta(\tau)$ without the solution of these equations. Therefore, it is possible, for example, to state that under a given $W(j\Omega)$ both the number of the equilibrium values of $K_{eq\,n}$, their stability, and the dependence $\rho(\tau)$ do not depend on the dynamic properties of the rotor or on the ratio of its moments of inertia A and C. This is associated with the fact that only Q_{11} and Q_{22} enter the equations for ρ and K, however, at any ratio of the moments of inertia we have $\Omega_{22} = 2\Omega_{11}$. Variation in the ratio A/C changes only the time scales exerting no influence on the character of the dependence of the right sides of the equations on K or t.

In this connection, evolution of the nutation angle mainly depends on the ratio A/C influencing the number, location, and stability of the roots turning to zero right side of the equation for ϑ.

3. Let us consider as an example the self-modulation influence on the equirigid suspension under

$$W(p) = W_0\frac{1+Tp}{(1+\tau_1 p)(1+\tau_2 p)}$$
$$(\tau_1,\,\tau_2 \ll T)\,, \tag{13}$$

where

$$P(\Omega) = \frac{1 + [T(\tau_1+\tau_2) - \tau_1\tau_2]\Omega^2}{(1+\tau_1^2\Omega^2)(1+\tau_2^2\Omega^2)}\,;$$

$$Q(\Omega) = \frac{T - (\tau_1+\tau_2) - \tau_1\tau_2 T\Omega^2}{(1+\tau_1^2\Omega^2)(1+\tau_2^2\Omega^2)}\,.$$

Let us turn from the variable K to the dimensionless parameter $\omega = \Omega_{11}/\Omega_0$, where

$$\Omega_0 = \sqrt{\frac{T - (\tau_1+\tau_2)}{\tau_1\tau_2 T}}$$

is the frequency of the rotor rotation, under which $Q(\Omega_0) = 0$. If we denote $\Omega_{mn} = k_{mn}\Omega_0\omega$, then

$$Q_{mn}(\omega) = \frac{k_{mn}\omega\,\Omega_0^3 T\tau_1\tau_2\left(1 - k_{mn}^2\omega^2\right)}{\left[1 + (k_{mn}\tau_1\Omega_0\omega)^2\right]\left[1 + (k_{mn}\tau_2\Omega_0\omega)^2\right]}$$

Let us take for the numerical calculation $\tau_1 = \tau_2 = 0.1T$; $C/A = 1.2$.

In the simplest case where the axis of the rotor surface symmetry and the axis of its dynamic symmetry coincide ($s = \sin\gamma = 0$), the values of ω and ρ do not change in time but the value of ω influences variation of the nutation angle

$$\dot{\vartheta} = -a^2\left(1 - k_{10}^2\omega^2\right)\vartheta\,; \tag{14}$$

ϑ asymptotically tends to zero under $k_{10}\omega < 1 (C\Omega_{11} < A\Omega_0)$ and increases in time under $k_{10}\omega > 1$.

In the general case where $s \neq 0$ and $c \neq 0$, apart from the asymptotically stable equilibrium value $\omega_{eq\,1} = 0$ (which corresponds to rotor deceleration), there is one more unstable (under the above given $W(p)$) value of $\omega_{eq\,2}$, which is the solution of the equation

$$\cos 2\gamma = \frac{Q_{22}(\omega) + 4Q_{11}(\omega)}{Q_{22}(\omega) - 4Q_{11}(\omega)} \tag{15}$$

and it exists only in the range $0.5 \leq \omega_{eq\,2} \leq 1.0$. The bifurcation curve $\gamma(\omega)$, is shown in the Fig.1 where the instability of this solution is depicted by diverging arrows.

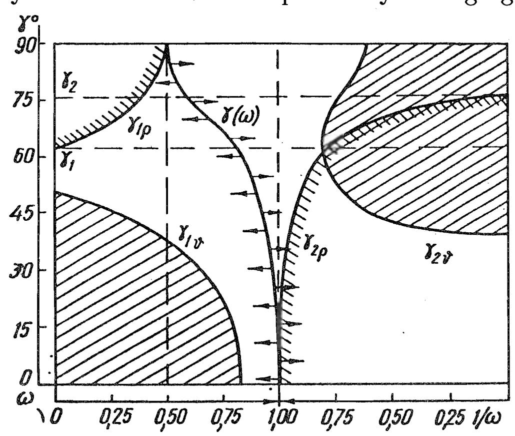

Fig.1. The bifurcation diagram

Therefore, if the initial conditions are such that under the given γ $\omega_0 > \omega_{eq\,2}$, then we can observe an increase in the kinetic energy accumulated by the rotor in the suspension or the rotor twist due to the suspension field energy rather than energy dissipation.

The function $F_\rho(\omega) = 8c^2Q_{11} - s^2Q_{22}$ determining the stability of the zero value of the angle ρ, vanishes on the bifurcation curve determined by the equation

$$\cos 2\gamma = \frac{Q_{22}(\omega) - 8Q_{11}(\omega)}{Q_{22}(\omega) + 8Q_{11}(\omega)} \tag{16}$$

which has one solution with respect to each of the domains $\omega_1 \leq 0.5$ and $\omega_2 \geq 1.0$. The bifurcation curves $\gamma_{1\rho}$ and $\gamma_{2\rho}$ shaded from the side of stability domain are also given in the Fig.1, from which it is evident that there are two critical values of the angle $\gamma : \gamma_1 = \operatorname{arctg}2$ and $\gamma_2 = \operatorname{arctg}4$ determining the presence of one or two stability domains as ω varies from 0 to ∞.

For the angle ϑ the bifurcation dependence $\gamma_\vartheta(\omega)$, when $M_+(\gamma, \omega) = 0$, is found from the equation

$$\begin{aligned}\left[18Q_{10} + 4(Q_{11} - Q_{21}) - (Q_{12} + Q_{22})\right]\cos^2 2\gamma \\ +2\left[6Q_{10} + (2Q_{12} + Q_{22})\right]\cos 2\gamma \\ +2Q_{10} - 4(Q_{11} - Q_{21}) - (2Q_{12} + Q_{22}) = 0\end{aligned}$$

and corresponds to the boundaries of the shaded domains inside which the value $\vartheta = 0$ is stable. The presence of the unshaded domains testifies that even for rotors with the flattened ellipsoid of inertia ($C > A$) the self-modulation moments can lead to an increase in the nutation angle during operation.

Summary

The conducted investigation allows us to make the following conclusions:

To provide the asymptotic stability of the rotor equilibrium position with respect to a noncontact suspension for any transfer function $W(p)$, the condition $\mathrm{Im}W(p) \not\equiv 0$ is fulfilled. This leads to the fact that even in the constructions excluding the effect of the conservative moment on the rotor stipulated by its asphericity, the rotor rotation induces the nonconservative moments influencing both the gyroscope angular momentum value (braking or twist) and its precession and nutation stability.

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