ROBUSTNESS OF THE STABILIZING SYSTEM IN THE MAGNETIC SUSPENSION OF A ROTATING SHAFT[1]

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ABSTRACT
There is discussed robustness of the stabilizing system in the rotating shaft magnetic suspension holding one axial and two radial magnetic bearings. With respect to the criterion for obtaining a region of maximum attraction of stabilizable equilibrium state in the phase space (shaft rotation not regarded), there has been synthesized an algorithm to regulate voltages on the suspension electromagnets.
In it, its basic features of robustness are examined versus variance of parameters and nonlinear characteristics of the stabilizing system. For a nonlinear case, some family of control functions satisfying a specified criterion is revealed. For a linear case, we have found a possibility to assign an interval of parameters and a feasibility to vary regulator's structure.
The stabilizing system implementing the above algorithm is shown to possess properties of robustness. For the case of shaft not rotating, the above criterion retains valid provided the nonlinear control functions remain in the Hurwitz angle. Under the algorithm, the linearized stabilizing system also retains stable both in the case when regulator's structure varies because of errors in the speed correction of shaft departure sensor signals and at shaft rotation speeds assigned from zero to nominal.

INTRODUCTION

At present, there has become widely used a magnetic shaft suspension fabricated in the form of one axial and two radial magnetic bearings [1]. When there is a good degree of decoupling between the fluxes, electromagnetic forces in the channels of axial and radial shaft stabilization systems can be considered independent inthemutually perpendicular planes.
In our case,conductive and elastic properties of construction elements were also assumed to be neglected and the material of magnetic suspension nonsaturated, in all the operating modes of the suspension. These assumptions as well as a due regard of instability of object under control and a limited nature of control voltages on suspension electromagnets make it possible to employ general principles of constructing control algorithms described in Ref. [2]. On the basis of these principles [3,4], we have synthesized robust control algorithms for axial and radial stabilization of a nonrotating shaft. As far as in all these cases, in the capacity of a synthesizing criterion there was chosen a condition of attaining the maximum region of attraction of stabilizable equilibrium state in the phase space of the system, robustness of synthesized algorithms is shown by us with respect to initial conditions of a nonlinear stabilization system.
This study mainly aims at observing robustness of the algorithm (synthesized in Ref. [4]) for shaft radial stabilization with respect to variance of parameters and nonlinear characteristics of the system. Such statement of the problem is considered interesting from the point of view of engineering imlpementation of the synthesized algorithm; in this implementation such variance is inevitable. Besides, specificity of radial stabilization of the shaft lies in the fact that it should be effected at shaft rotation speeds varying within a wide range, from zero to nominal. And, disirably, without reconstructing the control algorithm (in [1] this reconstruction exists). Such reconstruction - free

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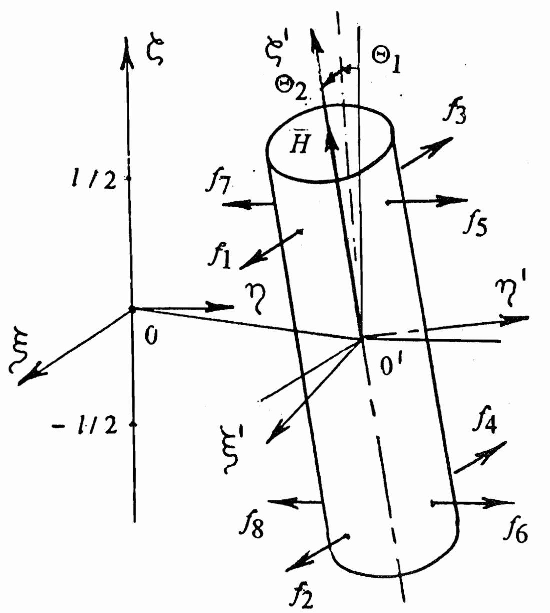

FIGURE 1: The shaft departure under action of forces from the side of radial magnetic bearing electromagnets.

approach we conconsider essential since a use of magnetic suspension in the shaft usually presupposes a necessity of its starting acceleration to very great rotation speeds not obtainable when ordinary mechanical bearings employed.

MATHEMATICAL MODEL OF STABILISATION

With due regard of above assumptions, a mathematical model for the electromechanical system of the type "shaft-radial electromagnet bearings (SREB)" at linearized force characteristics of pairs ($f_1 - f_3$, $f_2 - f_4$, $f_5 - f_7$, $f_6 - f_8$) of electromagnets (diametrically opposed relative to the shaft, as shown in Fig.1) can be represented in the dimensionless standard form

$$\dot{x} = A \cdot x + B \cdot u, \qquad (1)$$

where nonzero elements of matrices A (12 * 12) and B (12 * 4) will be

$$a_{1,2} = a_{4,5} = a_{7,8} = a_{10,11} = 1,$$
$$a_{2,1} = - a_{2,3} = a_{5,4} = - a_{5,6} = a_{8,7} = - a_{8,9} =$$
$$= a_{11,10} = - a_{11,12} = (1 + \upsilon) / 2,$$
$$a_{2,4} = - a_{2,6} = a_{5,1} = - a_{5,3} = a_{8,10} =$$
$$= - a_{8,12} = a_{11,7} = - a_{11,9} = (1 - \upsilon) / 2,$$
$$a_{2,8} = - a_{2,11} = - a_{5,8} = a_{5,11} = - a_{8,2} =$$
$$= a_{8,5} = a_{11,2} = -a_{11,5} = - H / 2,$$
$$a_{3,2} = a_{6,5} = a_{9,8} = a_{12,11} = h / T,$$
$$a_{3,3} = a_{6,6} = a_{9,9} = a_{12,12} = - 1 / T,$$
$$b_{3,1} = b_{6,2} = b_{9,3} = b_{12,4} = 1 / T,$$

where dimensionless variables

$$x_1 = (\delta_1 - \delta_3) / \delta_m, x_2 = \dot{x}_1 \cdot t_m,$$
$$x_3 = (I_1 - I_3) / I_m, x_4 = (\delta_2 - \delta_4) / \delta_m,$$
$$x_5 = \dot{x}_4 \cdot t_m, x_6 = (I_2 - I_4) / I_m,$$
$$x_7 = (\delta_5 - \delta_7) / \delta_m, x_8 = \dot{x}_7 \cdot t_m,$$
$$x_9 = (I_5 - I_7) / I_m, x_{10} = (\delta_6 - \delta_8) / \delta_m,$$
$$x_{11} = \dot{x}_{10} \cdot t_m, x_{12} = (I_6 - I_8) / I_m,$$
$$u_1 = (u_{e1} - u_{e3}) / u_m, u_2 = (u_{e2} - u_{e4}) / u_m,$$
$$u_3 = (u_{e5} - u_{e7}) / u_m, u_4 = (u_{e6} - u_{e8}) / u_m$$

are employed. These variables are expressed via variance of gaps δ_j in the directions of forces specified in Fig.1, through variance of currents I_j and voltages u_j of the related electromagnets (j = 1 - 8) and also via the accepted scales of variables

$$\delta_m = b \cdot I_m / a, I_m = u_m / R, t_m^2 = m / 2 \cdot a.$$

The dimensionless parameters (1) describe the following:

$\upsilon = m \cdot l^2 / 4 \cdot J$ -geometry of the shaft and its mass distribution (for a cilinder isotropic shaft we take $\upsilon = 3$);

$H = J_0 \cdot \Omega \cdot t_m / J$ - kinetic momentum of the shaft;

$h = b^2 / a \cdot R \cdot t_m$ - selfdamping of magnetic bearings;

and $T = L / R \cdot t_m$ - a time constant of the electromagnet.

These dimensionless parameters were expressed via dimension parameters of the shaft: m - mass ; l -length; J_0,J - axial and transversal inertia moments; Ω-speed of rotation. Parameters of indentical electromagnets were taken as follows: a,b - slopes of force characteristics at variance of gaps and currents; L, R - inductance and effective resistance.

If the shaft has no axial rotation ($H = 0$), the system of equations (1) is broken in two independent subsystems corresponding to shaft motions within two mutually orthogonal planes of radial bearings.

The algorithm for regulating voltages on electromagnets in radial bearings has been created [4] for each of these subsystems on the basic of the criterion of maximum region of attraction of stabilizable equilibrium state in the state space of the suspension, under specified constraints u^+ for control actions and assumption of complete observability of the object under control.

ROBUSTNESS OF THE SUGGESTED ALGORITHM

Basic features of robustness in the suggested algorithm with respect to variance of parameters and nonlinear characteristics of the stabilizing system are studied with regard of particularities of this algorithm. These particularities lie in variance of regulator structure caused by dynamic errors in real differential section of the sensor signals. With regard of these errors, the regulator, implementing the algorithm [4], is described by below nonlinear equations

$$\dot{y}_1 = (x_1 + x_4 - y_1)/\varepsilon \cdot \tau_1 + (x_2 + x_5)/\varepsilon,$$
$$\dot{y}_2 = (x_1 - x_4 - y_2)/\varepsilon \cdot \tau_2 + (x_2 - x_5)/\varepsilon,$$
$$\dot{y}_3 = (x_7 + x_{10} - y_3)/\varepsilon \cdot \tau_1 + (x_8 + x_{11})/\varepsilon,$$
$$\dot{y}_4 = (x_7 - x_{10} - y_4)/\varepsilon \cdot \tau_2 + (x_8 - x_{11})/\varepsilon,$$
$$\sigma_{1,2} = T \cdot (k\hat{}_{1,2} \cdot y_{1,2} - (x_3 \pm x_6)), \qquad (2)$$
$$u_{1,2} = \begin{cases} u^+, & \beta \cdot (\sigma_1 \pm \sigma_2) \geq u^+, \\ \beta \cdot (\sigma_1 \pm \sigma_2), & -u^- \leq \beta \cdot (\sigma_1 \pm \sigma_2) \leq u^+, \\ -u^-, & -u^- \geq \beta \cdot (\sigma_1 \pm \sigma_2), \end{cases}$$
$$\sigma_{3,4} = T \cdot (k\hat{}_{1,2} \cdot y_{3,4} - (x_9 \pm x_{12})),$$
$$u_{3,4} = \begin{cases} u^+, & \beta \cdot (\sigma_3 \pm \sigma_4) \geq u^+, \\ \beta \cdot (\sigma_3 \pm \sigma_4), & -u^- \leq \beta \cdot (\sigma_3 \pm \sigma_4) \leq u^+, \\ -u^-, & -u^- \geq \beta \cdot (\sigma_3 \pm \sigma_4), \end{cases}$$

where the upper sign relates to the first of underindices written through a comma, and the low to the second; and $\varepsilon < 1$ denotes the parameter describing an error of speed correction of departure sensor signals.
According to [4], optimal parameters of the regulator (2) in the shaft radial stabilization system have the bellow analytical expressions:

$$\varepsilon^* = 0, \qquad (3)$$
$$\beta^* > 0.5, \qquad (4)$$
$$k\hat{}^*_{1,2} = (T \cdot \lambda_{1,2} + 1)/T \cdot \lambda_{1,2}, \qquad (5)$$
$$\tau^*_{1,2} = \lambda_{1,2}^{-1} - h/(T \cdot \lambda_{1,2} + 1), \qquad (6)$$

where for $H = 0$ and small h the positive roots of characteristic equation for the noncontrollable object (1) wiil be approximately to

$$\lambda_1 \cong 1, \lambda_2 \cong \upsilon^{1/2}. \qquad (7)$$

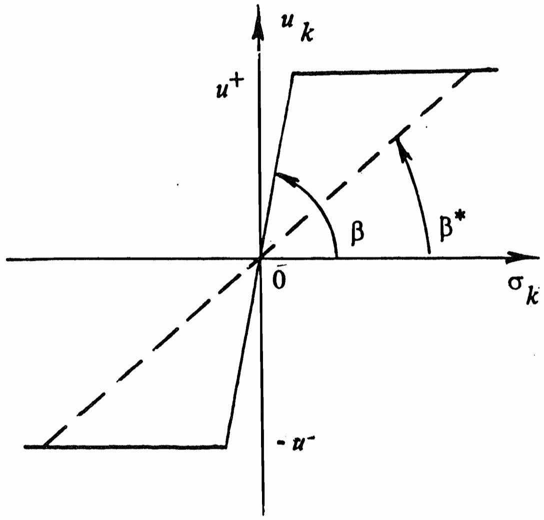

FIGURE 2: A family of nonlinear control functions satisfying a criterion of maximal attraction region.

In the given case, the robustness with respect to nonlinear characteristics of the regulator is assigned via the condition (4) from which it follows that all piecewise-linear functions located in the Hurwitz angle will (as shown in Fig.2) provide not only stability but also a maximum region of attraction of the stabilizable equilibrium state in the state space of the system, the shaft not rotating.
Robustness with respect to variance of stabilizing system parameters is observed through studying its stability by the D - decomposition method [5] in the plane of the most essential parameters:
a) k_1 - 1, τ_1 that characterize stiffness and damping of magnetic bearings at translational departures of the shaft, and
b) H, k_2 - 1 that characterize a speed of shaft rotation and stiffness of magnetic bearings, at angle departures of the shaft. All the other parameters of the stabilizing system are assumed to be fixed on their optimal values or to be near to real.
Studying stability is effected on the basis of characteristic equations of the closed-loop system (1), (2). The equations can be represented as a product of the below three characteristic polynomials:

$$\chi(\lambda) = \chi_1^2(\lambda) \cdot \chi_2(\lambda) = 0.$$

The first two similar characteristic equations

$$\chi_1(\lambda) = [(\lambda^2 - 1) \cdot (\mu \cdot T \cdot \lambda + 1) + \mu \cdot h \cdot \lambda] \cdot$$
$$\cdot (\varepsilon \cdot \tau_1 \cdot \lambda + 1) + k_1 \cdot (\tau_1 \cdot \lambda + 1) = 0 \qquad (8)$$

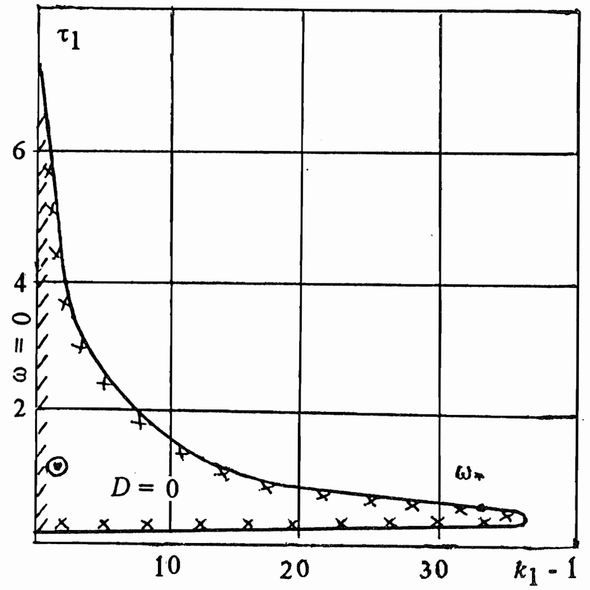

FIGURE 3: Performing the D-decomposition of the parameter plane "Stiffness - damping translative shaft departures" at the following fixed parameters of stabilizing system in active magnetic bearings: $h = 0.4$; $T = 2$; $\mu = 0.09$; $\varepsilon = 0.1$.

describe stability of the stabilizing system in the magnetic suspension in the course of translational departures of the shaft in two mutually perpendicular planes. These equations, if parameters properly chosen, describe an axial stabilization system as well. The third equation

$$\chi_2(\lambda) = \{ [(\lambda^2 - \upsilon) \cdot (\mu \cdot T \cdot \lambda + 1) + \upsilon \cdot \mu \cdot h \cdot \lambda] \cdot (\varepsilon \cdot \tau_2 \cdot \lambda + 1) + \upsilon \cdot k_2 \cdot (\tau_2 \cdot \lambda + 1) \}^2 + \{ H \cdot \lambda \cdot (\mu \cdot T \cdot \lambda + 1) \cdot (\varepsilon \cdot \tau_2 \cdot \lambda + 1) \}^2 = 0 \quad (9)$$

describes stability of radial stabilization system for a rotating shaft during its motion through angles Θ_1, Θ_2. The parameters $\mu = (\cdot 1 + \beta \cdot T)^{-1} << 1$ and $k_{1,2} = = \mu \cdot \beta \cdot T \cdot \hat{k}_{1,2}$ in (8), (9) represent a combination of initial parameters caused by the electromagnet current feedback available in the synthesized regulator.

The region of system stability in the parameter plane $k_1 - 1$, τ_1 (stiffness-translational departures damping) is shown in Fig.3. This region boundary related to stability loss at zero frequency is a special straight line of the D-decomposition $k_1 - 1 = 0$ ($\omega = 0$, a single dashing). For a general case (denoted by double dashing), a stability region boundary for $0 \leq \omega^2 \leq \omega_*^2$ is assigned parametrically, as follows:

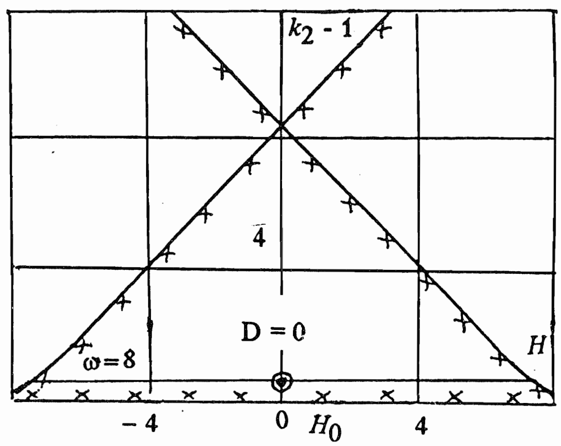

FIGURE 4: Performing the D-decomposition of the parameter plane "Shaft's kinetic momentum - angular stiffness" at the following fixed parameters of a stabilizing system in active magnetic bearings: $\upsilon = 3$; $h = 0.4$; $T = 2$; $\mu = 0.09$; $\varepsilon = 0.1$. To the shaft's nominal rotation speed $\omega_0 = 60000$ rev / min there corresponds dimensionless value of the kinetic momentum $H_0 = 0.7$ ($J_0 = 1.7 \cdot 10^{-4}$ kgm^2; $J = 4.1 \cdot 10^{-3}$ kgm^2, $t_m = 3 \cdot 10^{-3}$ sec).

$$k_1 = (1 - \mu \cdot T \cdot \varepsilon \cdot \tau \cdot \omega^2) \cdot (1 + \omega^2) + \mu \cdot h \cdot \varepsilon \cdot \tau \cdot \omega^2$$

$$\tau_1 = \frac{(1 - \varepsilon) \cdot (1 + \omega^2) \pm \sqrt{DS}}{2 \cdot \varepsilon \cdot \omega^2 \cdot [\mu \cdot T \cdot (1 + \omega^2) - \mu \cdot h]} \quad (10)$$

$$DS = (1 - \varepsilon)^2 \cdot (1 + \omega^2)^2 - 4 \cdot \varepsilon \cdot \omega^2 \cdot \mu^2 \cdot [T \cdot (1 + \omega) - h]^2 \geq 0$$

The limit frequency of a stability loss (this frequency describes a limit stiffness of a magnetic bearing) is determined through the condition $DS = 0$, and at small selfdamping (h << 1) it wiil be approximately similated in the following way

$$\omega_*^2 = (1 - \varepsilon) / 4 \cdot \varepsilon \cdot \mu^2 \cdot T^2 \quad (11)$$

Optimal parameters of the synthesized regulator (4), (5) fall into the stability region and in Fig.3 are designated by a circled point. Admissible departures of parameters from optimal ones are declared through stability region boundaries though, as shown in [6], at smaller departures the region of attraction of the stabilizable equilibrium state in the state space is reduced.

The system stability region in the parameter plane H, $k_2 - 1$ (kinetic momentum of the shaft-angular stiffness of the magnetic bearings) is shown in Fig.4. Its boundary, on which at zero frequency there occurs a stability loss, will be a special straight line of the D - decomposition $k_2 - 1 = 0$ ($\omega = 0$, a double dashing). For the general case (double dashing), a stability region boundary $0 < \omega^2 < \omega_*^2$ as follows

$$H = \left[\frac{\upsilon \cdot \mu \cdot h \cdot (1 + \varepsilon \cdot \tau_2^2 \cdot \omega^2)}{\tau_2 \cdot (1 - \varepsilon) - \mu \cdot T \cdot (1 + \varepsilon^2 \cdot \tau_2^2 \cdot \omega^2)} + \omega^2 + \upsilon \right] / \omega \qquad (12)$$

$$k_2 = - \frac{\mu \cdot h \cdot (1 + \varepsilon \cdot \tau_2^2 \cdot \omega^2)}{\tau_2 \cdot (1 - \varepsilon) - \mu \cdot T \cdot (1 + \varepsilon^2 \cdot \tau_2^2 \cdot \omega^2)}$$

A range for admissible shaft rotation speeds is determined through points where the boundary (12) intersects with the straight line $k_2 = k_2^*$. If a needed operating range of shaft rotations exceeds an obtained estimate, then system parameters have to be modified in the way as to make the stability region in Fig.4 extending. To this lead, in particular, a reduction of constant time (T) of electromagnets and that of errors of speed correction (ε), that define regulator's error and limit capabilities of magnetic bearings in stiffness and in admissible rotation speeds of the shaft.

CONCLUSION

This study has revealed that suggested algorithm [4] for regulating voltages on electromagnets in radial magnetic bearings ensures not only maximum region of attraction of stabilizable equilibrium state in the state space but also satisfies properties of robustness with respect to variance of parameters and nonlinear characteristics within the stabilizing system. In contrast to the algorithm in Ref.[1], our reconstruction-free algorithm is capable of ensuring a suspension stability without reconstructing the suspension within the entire range of shaft rotation speeds. The software developed makes it possible to calculate estimates of admissible stiffness for magnetic bearings and of shaft rotation speeds in accordance with a choice of remaining parameters of the system.

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