ISOTROPIC OPTIMAL CONTROL OF ACTIVE MAGNETIC BEARING SYSTEM

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ABSTRACT

As a new rotor control scheme, isotropic control of weakly anisotropic rotor bearing system in complex state space is proposed, which utilizes the concepts on the eigenstructure of the isotropic rotor system. Advantages of the proposed method are that the controlled system always retains isotropic eigen-structure, leading to circular whirling due to unbalance and that it is efficient for control of unbalance response. And the system analysis and controller design becomes simple and yet comprehensive since the order of the matrices treated in the complex domain approach is half of that in the real approach.

The control scheme is applied to a rigid rotor-active magnetic bearing system which is controlled by a digital controller and the control performance is investigated experimentally in relation to unbalance response and control energy.

INTRODUCTION

Complex notation has been commonly adopted in dynamic analysis of a rotor bearing system due to the notational convenience and clear physical interpretation, and the dynamic analysis[1,2] and control of isotropic rotor bearing system[3,4] in complex space has been well developed. In this work, an isotropic optimal control of anisotropic rotor bearing system in complex state space is proposed, which utilizes the concepts on the eigenstructure of the isotropic rotor system[5]. Isotropic optimal controller design in complex state space[6] is essentially composed of two steps. Firstly system is decomposed into isotropic and anisotropic parts, and direct canceling control of the system anisotropy is performedl. Secondly an isotropic control scheme such as the optimal control in complex domain is applied to the resulting isotropic system[3].

The proposed control method is applied to the control of rigid rotor-active magnetic bearing(AMB) which is controlled by a digital controller and the control performance is investigated experimentally in relation to unbalance response and control energy. It is shown that the proposed method is efficient for control of unbalance response.

Advantages of the proposed method are that the controlled system retains isotropic eigenstructure, leading to circular whirling due to unbalance[1] and that the system analysis and controller design becomes simple and yet comprehensive[6].

CONTROL OF ROTOR BEARING SYSTEM

Modeling of Rotor Bearing System

The equation of motion of a multi-degree-of-freedom rotor bearing system, such as rigid rotor-AMB system shown in **FIGURE 1**, may be written as

$$M\ddot{q} + C\dot{q} + Kq = f \qquad (1)$$

where

$$q = \begin{Bmatrix} y \\ z \end{Bmatrix}, \quad f = \begin{Bmatrix} f_y \\ f_z \end{Bmatrix}$$

$$M = \begin{bmatrix} M_{yy} & M_{yz} \\ M_{zy} & M_{zz} \end{bmatrix}, C = \begin{bmatrix} C_{yy} & C_{yz} \\ C_{zy} & C_{zz} \end{bmatrix}, K = \begin{bmatrix} K_{yy} & K_{yz} \\ K_{zy} & K_{zz} \end{bmatrix}.$$

Here M, C, K are the $2n \times 2n$ mass, damping including gyroscopic effect, and stiffness matrices, and y (z) and f_y (f_z) are the n-dimensional y-(z-) directional displacement and force vectors.

Assuming that the rotor is axisymmetric and introducing complex notations such that $p = y + jz$ and $g = f_y + jf_z$, we can rewrite Eq.(1) as

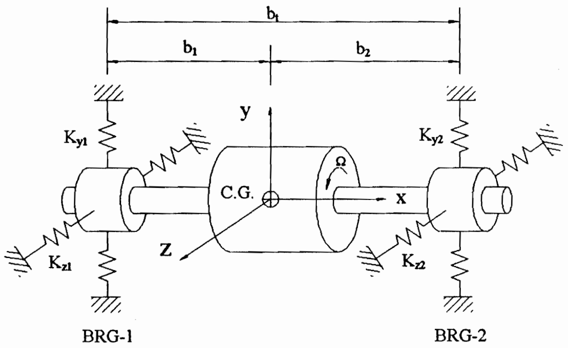

FIGURE 1 : Rigid Rotor-AMB System Model

$$M_c\ddot{p}+C_c\dot{p}+C_\Delta\dot{\bar{p}}+K_c p+K_\Delta\bar{p}=g \tag{2}$$

where

$$\begin{aligned}
&M_c=M_{yy}=M_{zz},\ \ M_{yz}=M_{zy}=0\\
&K_c=\frac{K_{yy}+K_{zz}}{2}+\mathrm{j}\frac{K_{zy}-K_{yz}}{2}\\
&K_\Delta=\frac{K_{yy}-K_{zz}}{2}+\mathrm{j}\frac{K_{zy}+K_{yz}}{2}\\
&C_c=\frac{C_{yy}+C_{zz}}{2}+\mathrm{j}\frac{C_{zy}-C_{yz}}{2}\\
&C_\Delta=\frac{C_{yy}-C_{zz}}{2}+\mathrm{j}\frac{C_{zy}+C_{yz}}{2}.
\end{aligned} \tag{3}$$

Here j is the imaginary number and '-' denotes the complex conjugate. In Eq.(2), the $n\times n$ complex matrices M_c, C_c, K_c represent the isotropic properties of the rotor bearing system whereas the $n\times n$ complex matrices C_Δ, K_Δ represent the anisotropic properties of bearings.

Conventional Optimal Control

The state space form of Eq.(1) can be written, on the assumption of the positive definiteness of mass matrix M, as

$$\dot{x}=Ax+Bu \tag{4}$$

where

$$A=\begin{bmatrix}0 & I\\ -M^{-1}K & -M^{-1}C\end{bmatrix},\ B=\begin{bmatrix}0\\ M^{-1}\end{bmatrix},$$

$$x=\begin{Bmatrix}q\\ \dot{q}\end{Bmatrix},\ u=f=\begin{Bmatrix}f_y\\ f_z\end{Bmatrix}.$$

Here A is the real valued $4n\times 4n$ system matrix and x is the $4n\times 1$ state vector. Consider the quadratic performance index given by

$$J=\int_0^\infty (x^TQx+u^TRu)\,dt \tag{5}$$

where Q and R are the positive semidefinite and positive definite weighting matrices, respectively. Then the solution to minimization of J is the optimal control law given by

$$u=-R^{-1}B^TPx \tag{6}$$

where P is the solution of the algebraic matrix Riccati equation

$$PA+A^TP-PBR^{-1}B^TP+Q=0. \tag{7}$$

Here the positive definite solution matrix P always exists and the controlled system is asymptotically stable, if $[A,B]$ is controllable and $[A,D]$ is completely observable, where D is any matrix such that $DD^T=Q$ [7]. As the result, optimal control gain matrix, K_{opt}, can be written as

$$K_{opt}=\begin{bmatrix}K_{po} & K_{do}\end{bmatrix}\equiv R^{-1}B^TP \tag{8}$$

where K_{po} and K_{do} are the proportional and derivative gain matrices. From Eqs.(6) and (8), control force vector can be written using the feedback gain matrices as

$$f=u=-\left\{K_{po}q+K_{do}\dot{q}\right\} \tag{9}$$

Substituting Eq.(9) into Eq.(1), we can write the controlled system as

$$M\ddot{q}+\left[C+K_{do}\right]\dot{q}+\left[K+K_{po}\right]q=f_e \tag{10}$$

where f_e is the external force vector.

In general, conventional optimal controlled rotor bearing system (10) in real domain retains the characteristics of uncontrolled system. Therefore if the original system is anisotropic, controlled system is also likely to be anisotropic, leading to the elliptic whirling due to unbalance.

Isotropic Optimal Control in Complex State Space

The essence of the isotropic optimal control of rotor bearing system in complex domain is that the control effort is twofold. The first part of control action is solely devoted to make the system isotropic and then the second part of control action is applied to the resulting isotropic system, which utilizes an optimal control in complex domain. Complex control force g in Eq.(2) can be decomposed into two parts as

$$g=g_c+g_\Delta \tag{11}$$

where

$$g_\Delta=C_\Delta\dot{\bar{p}}+K_\Delta\bar{p} \tag{12}$$

Here g_c is determined from the equation of motion associated with the isotropic system resulting from the control action of g_Δ, i.e.

$$M_c\ddot{p}+C_c\dot{p}+K_cp=g_c \tag{13}$$

and then the optimal controller in complex state space is to be designed.

The state space form of Eq.(13) is

$$\dot{x}_c = A_c x_c + B_c u_c \tag{14}$$

where

$$A_c = \begin{bmatrix} 0 & I \\ -M_c^{-1}K & -M_c^{-1}C \end{bmatrix}, \; B_c = \begin{bmatrix} 0 \\ M_c^{-1} \end{bmatrix},$$

$$x_c = \begin{Bmatrix} p \\ \dot{p} \end{Bmatrix}, \; u_c = g_c$$

Here A_c is the $2n\times 2n$ complex system matrix and x_c is the $2n\times 1$ complex state vector. Consider the quadratic performance index in complex domain given by

$$J_c = \int_0^\infty (x_c^* Q_c x_c + u_c^* R_c u_c)\, dt \tag{15}$$

where Q_c and R_c are the positive semidefinite and positive definite Hermitian matrices, respectively, and '*' denotes the conjugate transpose. Then the solution to minimization of J_c is the complex optimal control law given by

$$u_c = -R_c^{-1} B_c^* P_c x_c = -\left[K_{pc}\; K_{dc}\right] x_c \tag{16}$$

where the positive definite Hermitian matrix P_c is the solution of the complex valued algebraic Riccati equation

$$P_c A_c + A_c^* P_c - P_c B_c R_c^{-1} B_c^* P_c + Q_c = 0. \tag{17}$$

From Eq.(16), the complex control force vector becomes, using feedback gain matrices,

$$g_c = u_c = -\left\{K_{pc} p + K_{dc} \dot{p}\right\} \tag{18}$$

and the final control law is the superposition of the two control actions given in Eqs. (12) and (18), i.e.

$$\begin{aligned} g &= g_c + g_\Delta \\ &= -\left\{K_{pc} p + C_{dc}\dot{p}\right\} + \left\{K_\Delta \bar{p} + C_\Delta \dot{\bar{p}}\right\} \end{aligned} \tag{19}$$

The controlled system becomes then

$$M_c \ddot{p} + \left[C_c + K_{dc}\right]\dot{p} + \left[K_c + K_{pc}\right] p = g_e \tag{20}$$

where g_e is the external force.

Unlike the conventional optimal control, the isotropic optimal control ensures that the controlled system remains always isotropic. Since the order of the matrices treated in the complex domain approach is half of that in the real approach, the system analysis and controller design is far simpler and more comprehensive. In practical application of this control scheme, however, some cautions are necessary for good performance such that the total control gain matrix in real domain must be at least positive semidefinite, if not, some control energy may be consumed to degrade the control performance.

CONTROL OF RIGID ROTOR-AMB SYSTEM

The equation of motion for an axisymmetric rigid rotor-active magnetic bearing system shown in **FIGURE 1** can be written, using complex notations, as

$$M_c \ddot{p} + C_c \dot{p} + K_c p + K_\Delta \bar{p} = K_i i + g_e \tag{21}$$

where

$$M_c = \begin{bmatrix} ml_2^2 + i_d & ml_1 l_2 - i_d \\ ml_1 l_2 - i_d & ml_1^2 + i_d \end{bmatrix}$$

$$C_c = -j\Omega i_p \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$$

$$K_c = \begin{bmatrix} K_{10} & 0 \\ 0 & K_{20} \end{bmatrix}, \; K_\Delta = \begin{bmatrix} K_{1\Delta} & 0 \\ 0 & K_{2\Delta} \end{bmatrix}$$

$$K_i = \begin{bmatrix} K_{i1} & 0 \\ 0 & K_{i2} \end{bmatrix}$$

$$q = \begin{Bmatrix} y_1 + jz_1 \\ y_2 + jz_2 \end{Bmatrix}, \; i = \begin{Bmatrix} i_{y1} + ji_{z1} \\ i_{y2} + ji_{z2} \end{Bmatrix}, \; g_e = \Omega^2 \begin{Bmatrix} u_1 \\ u_2 \end{Bmatrix}$$

and

$$i_d = I_d / b_t^2, \; i_p = I_p / b_t^2,$$

$$l_1 = b_1 / b_t, \; l_2 = b_2 / b_t, \; b_t = b_1 + b_2$$

$$K_{10} = \frac{1}{2}(K_{y1} + K_{z1}), \; K_{1\Delta} = \frac{1}{2}(K_{y1} - K_{z1})$$

$$K_{20} = \frac{1}{2}(K_{y2} + K_{z2}), \; K_{2\Delta} = \frac{1}{2}(K_{y2} - K_{z2})$$

Here m, I_d and I_p denote the mass, the diametrical and the polar mass moment of inertia of the rotor respectively, b_1 and b_2 are the distances of two magnetic bearings from the mass center of the rotor, Ω is the rotating speed of the rotor, K_{i1} and K_{i2} are the current stiffnesses of magnetic bearings, K_q and i_q are the negative stiffness of the uncontrolled magnetic bearings and the control current of each magnet, respectively, and g_e is the unbalance force due to mass unbalances of u_1 and u_2 in AMB 1 and 2.

Note that the open loop system (21) is inherently unstable by the negative stiffness K_q which is generated by attractive magnetic force so that the stabilization control is always required, i.e..

$$f_q = -K_q q + K_{i_q} i_q, \quad q = y_1, y_2, z_1, z_2 \tag{22}$$

where

$$K_{i_q} = \frac{\alpha\mu_0 N^2 A (I_{q1} + I_{q2})}{2g_0^2}, \; K_q = \frac{c\alpha\mu_0 N^2 A (I_{q1}^2 + I_{q2}^2)}{2g_0^3}$$

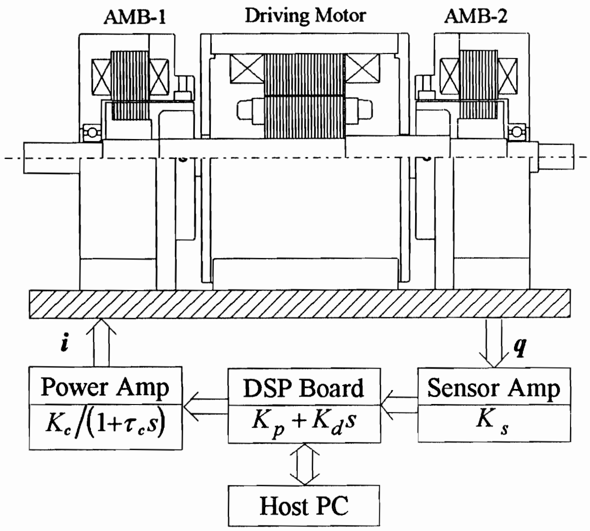

FIGURE 2 : Experimental Setup for AMB system

Here K_q, K_{iq} and i_q are the negative stiffness of the uncontrolled magnetic bearings, the current stiffness and the control current of each magnet, respectively, c and α denote the shape factors of magnet, N is the number of coil turns, A is the pole face area of magnet, g_0 denote the nominal air gap, and I_{q1} and I_{q2} are the bias currents of each magnet pairs. **FIGURE 2** shows the experimental setup for AMB system, which consists of two AMBs, displacement sensors, digital controller using DSP(TMS320C30), and PWM power amplifiers. The specifications of the AMB system are listed in **TABLE 1**.

At first, the stabilization using four single-axis PD-control is applied with

$$K_p = [3.0, 3.0, 3.0, 3.0]$$

$$K_d = [.002, .002, .002, .002].$$

The stiffness and damping of stabilized AMB, including gyroscopic effect, are estimated theoretically in complex space,

$$C_c = \begin{bmatrix} 200 & 0 \\ 0 & 197 \end{bmatrix} - j \begin{bmatrix} 85.4 & -85.4 \\ -85.4 & 85.4 \end{bmatrix},$$

$$C_\Delta = \begin{bmatrix} 20 & 0 \\ 0 & 14 \end{bmatrix} \quad \mathrm{N \cdot s/m} \tag{23}$$

$$K_c = \begin{bmatrix} 859 & 0 \\ 0 & 845 \end{bmatrix}, \; K_\Delta = \begin{bmatrix} 60 & 0 \\ 0 & 40 \end{bmatrix} \; \mathrm{kN/m}.$$

And then the isotropic and conventional optimal control using coupled 4-d.o.f. digital controller is performed to the stabilized system with the complex weighting matrices

$$Q_c = \mathrm{diag}[8\times10^6, 8\times10^6, 10, 10]$$

$$R_c = \mathrm{diag}[1, 1].$$

TABLE 1 : Specifications of AMB System

$m = 8.280$ [kg]	$A = 450$ [mm^2]
$I_p = .00788$ [kg m^2]	$N = 400$ [turns]
$I_d = .09830$ [kg m^2]	$g_0 = 0.88$ [mm]
$b_1 = 0.139$ [m]	$c = \alpha = 0.92$
$b_2 = 0.138$ [m]	$K_s = 5000$ [V/m]
$\mu_0 = 4\pi \times 10^{-7}$ [H/m]	$K_c = 0.420$ [A/V]

TABLE 2 : Modal Parameters of AMB System.

mode	1B	1F	2B	2F
	Conventional Optimal Control			
λ	-138±452j	-156±479j	-204±579j	-224±658j
$\begin{Bmatrix} y_1 \\ y_2 \\ z_1 \\ z_2 \end{Bmatrix}$	$\begin{Bmatrix} 1 \\ -1.33 \mp j.04 \\ -169 \pm j387 \\ -161 \mp j378 \end{Bmatrix}$	$\begin{Bmatrix} 1 \\ 1.07 \pm j.01 \\ -.003 \mp j.01 \\ .003 \pm j.01 \end{Bmatrix}$	$\begin{Bmatrix} 1 \\ -.912 \pm j.01 \\ -.43 \pm j1.45 \\ .45 \mp j1.49 \end{Bmatrix}$	$\begin{Bmatrix} 1 \\ -.947 \pm j.01 \\ -.16 \mp j.542 \\ .16 \pm j.552 \end{Bmatrix}$
	Isotropic Optimal Control			
λ	-146-465j	-147+466j	-210-582j	-219+656j
$\begin{Bmatrix} y_1 \\ y_2 \\ z_1 \\ z_2 \end{Bmatrix}$	$\begin{Bmatrix} 1 \\ 1.03 - j.01 \\ -j \\ -.01 - j1.03 \end{Bmatrix}$	$\begin{Bmatrix} 1 \\ 1.02 + j.01 \\ -j \\ .01 - j1.02 \end{Bmatrix}$	$\begin{Bmatrix} 1 \\ -0.974 \\ -j \\ j0.974 \end{Bmatrix}$	$\begin{Bmatrix} 1 \\ -0.983 \\ -j \\ j0.983 \end{Bmatrix}$

The modal parameters of the conventional and isotropic optimal controlled AMB system are listed in **TABLE 2**. It shows that both controlled systems have similar eigenvalues and the control forces act mainly for increasing damping. And that the eigenstructure of the conventional optimal controlled system is not isotropic since the relations, such as z_1=-jy_1 and z_2=-jy_2, do not hold, unlike the isotropic optimal controlled system.

FIGURE 3 shows measured unbalance responses and estimated control forces of each AMB at Ω=4600rpm when unbalances of u_1=61g·mm and u_2=37g· mm. In many practical rotor bearing systems, the major whirl radius and maximum control force are the important factors to be minimized. In isotropic optimal controlled AMB system, unbalance responses are characterized by a forward synchronous circular whirl; there exists no backward whirling component [3,6]. Thus the major whirl radii of isotropic optimal controlled systems tend to be smaller than those of conventional optimal controlled systems. **FIGURE 4** shows the measured major whirl radius and the maximum control force at each AMB for unbalance of

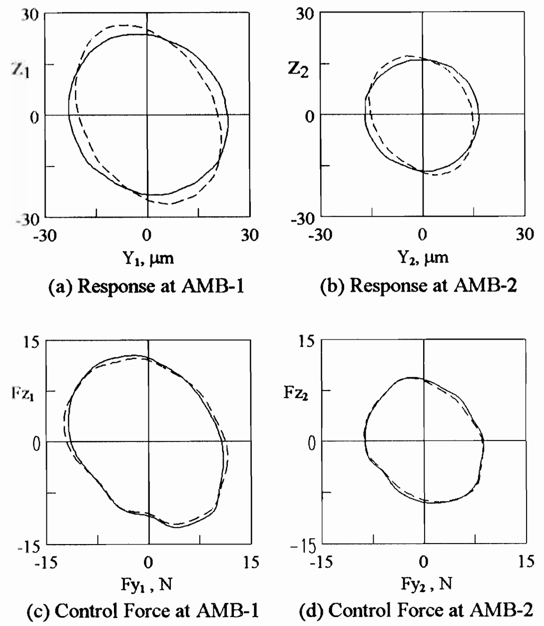

FIGURE 3 : Unbalance Responses and Control forces of AMBs at 4600rpm ; ——— iso., - - - - - con.

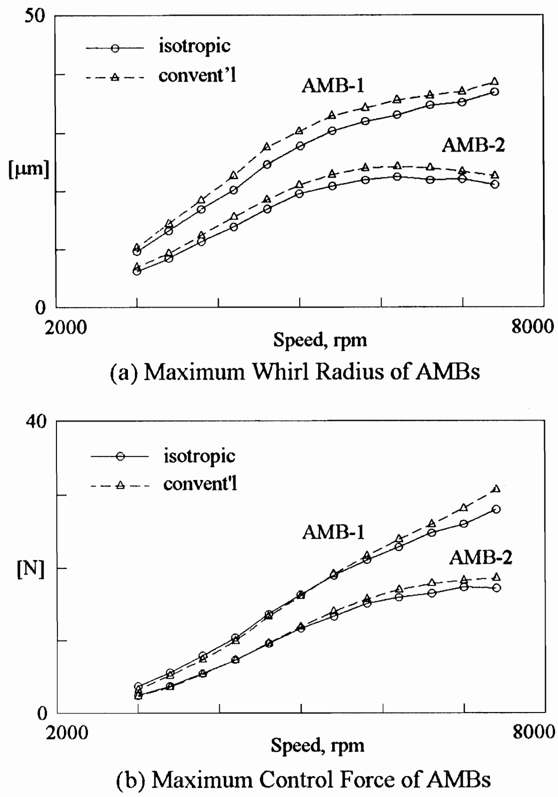

FIGURE 4 : Maximum Whirl Radii and Control Forces of AMBs for Unbalance.

u_1=61g·mm, u_2=37g·mm, as the rotational speed is varied. **FIGURE 5** is the plot for the backward and forward components of the unbalance responses in FIGURE 4(a). FIGURES 4 and 5 clearly indicate that the major whirl radius remains to be smaller for the isotropic optimal control than the conventional optimal control as the rotational speed varies. In addition, the maximum required control force for the isotropic optimal control also tends to be slightly less than that for the conventional optimal control except over the low rotational speed range. This is mainly due to the fact that, in the isotropic optimal control, the backward whirling component is eliminated by changing the phase and/or radius of backward control force component. The phase shift alone, unlike the change in radius, does not affect the maximum control force.

CONCLUSION

Isotropic control of anisotropic rotor bearing system in complex state space is proposed, which assigns isotropic eigenstructure to the controlled system. The isotropic optimal control of active magnetic bearing system is performed experimentally and the control performance is compared with the conventional optimal control method. It can be concluded that the isotropic optimal control method, which essentially eliminates the backward unbalance response component, is more efficient than the conventional optimal control in that it gives smaller major whirl radius and yet it often requires less control effort.

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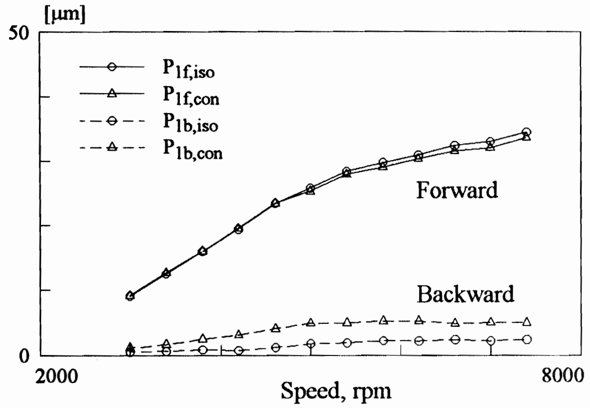
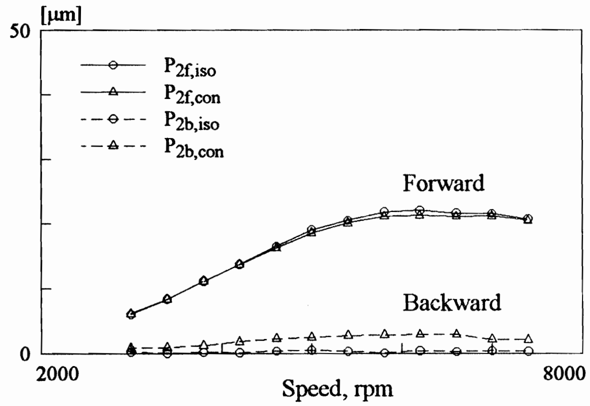


(a) Whirl Radius of AMB-1



(b) Whirl Radius of AMB-2

FIGURE 5 : Forward and Backward Whirl Radii of unbalance responses in FIGURE 4 (a)

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