

## EXPERIMENTAL CHARACTERISATION AND MATHEMATICAL MODELLING OF HIGH-TC SUPERCONDUCTORS FOR MAGNETIC BEARINGS

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### ABSTRACT

An experimental study of the levitation forces aimed to supply informations for the design of superconducting magnetic bearings was conducted on melt-textured YBCO materials. Such materials demonstrate sufficiently good levitation capabilities and complicated strongly non-linear force-position dependence. An analytical model for levitation force which is based on the Bean critical state concept and that takes into account the inhomogeneity of the materials is proposed in this paper. The comparison of experimental and numerical results shows that the proposed model can be used for quantitative prediction of the behaviour of magnetic levitation systems.

### INTRODUCTION

Probably the most promising near future practical application of high-Tc superconductors (HTS) is the construction of various levitation devices, such as passive superconducting magnetic bearings (SMB), flywheel energy storage systems, vibration dampers and others. The levitation force is the most important item of these device and can be considered as a result of induced current loops interacting with the magnetic field. Now it is well known that, due to flux pinning, the levitation force between magnet and HTS can be repulsive as well as attractive and depends on magnetic field history and time. The oriented YBCO samples obtained by MPMG or QMG technologies demonstrated very good  $J_c$  and flux pinning parameters [1], [2]. The interest in superconductive magnetic bearings (SMB) is mostly due to the predictable improvements in the characteristics of HTS in terms of critical current, pinning effect and critical temperature. In December 1993 some tests showed the

possibility of obtaining superconductive behaviour above 0 °C [3]; if the material involved will be obtained in practical forms, the future potential applications of SMB will change drastically.

The analytical model proposed in this paper predicts the levitation behaviour of magnets above a superconductor surface with remarkable accuracy in spite of the drastic assumptions involved and gives a possibility to obtain a quantitative description for the levitation force. This model may be regarded as a first approximation useful in the design of SMB [4].

### EXPERIMENTAL CHARACTERISATION

The experimental device used for the measurement of the levitation force is basically the same described by the authors in a previous work [5]. The HTS sample with its cooling system was secured to the table of a small milling machine which can move in three perpendicular directions using three graduate disks. The measurement of the position of the table is performed using an optical collimator with a precision of 25  $\mu\text{m}$  in each direction. The magnet was suspended to a load cell (DS EUROPE Mod. 535Q) bolted on the frame of the machine: the relative displacement between the magnet and the HTS specimen was then due to the displacement of the latter. Two different axially magnetised NdFeB magnets were used for all tests. The maximum field strength on their surface was respectively of 0.265 T and 0.4 T. The system was cooled by liquid nitrogen and the temperature was measured by a Cr-Au thermocouple; the flow of nitrogen was regulated in order to keep constant the temperature during the experiments. A Hall probe connected to a gaussmeter was used to measure the applied magnetic field on the HTS sample's surface.

Owing to the relaxation of the levitation force a data acquisition system consisting in a HP computer connected through HP-IB to the digital multimeter (HP 3478A, accuracy 1  $\mu$ V) was used. All the experiments were performed following the procedure here outlined:

- set the HTS tile on a support of hoxigen-free copper on the table of the machine;
- locate the thermocouple and the Hall probe on the surface of the HTS;
- put the magnet in the initial position with respect to the tile;
- cool the tile by liquid nitrogen until the temperature drops to 77 K;
- start of the program that times the measuring cycle and performs the data acquisition.

The measuring was performed every time using purposely written programs.

The experimental device allows to study the levitation force after zero field cooling as well as cooling within the magnetic field, the microcycles for static stiffness, the lateral force and the time behaviour of the levitation force. Small modifications allow the study of damping characteristics. HTS samples were prepared by CISE S.p.A. [6], using melt-texturing technique, in the form of pellets of 30 mm diameter and about 5 mm thickness. Magnetic measurement, using SQUID magnetometer, yielded a sufficiently high critical current in the range 1500-6000 A/cm<sup>2</sup>. It must be noted that all samples were highly inhomogeneous with contents from 20 to 50% of the non-superconducting 211 phase.

The SMB is made by an array of hexagonal tiles embedded in a block of hoxigen-free copper which is directly cooled by the liquid nitrogen; the experimental characterisation of the whole bearing is reported in [6]; here the work performed on single tiles is dealt with. A total of 10 specimens were used in the present tests; the specimens were labelled with a letter, designating the production batch, and a number (see Table 1).

A first group of tests consisting in 12 measurements of the levitation force as function of the distance, 13 tests for the axial stiffness and 5 tests for the radial stiffness was performed on the tiles. During each test run a large number of experimental readings were taken, both with increasing and decreasing distance between the magnet and the superconductor, as shown in figure 1. The results are summarised in table 1. All tests have been performed in quasi-static conditions.

The levitation force becomes attractive when the distance magnet-tile increases, owing to the trapped magnetic flux; as a consequence the second hysteretic loop differs from the initial one. The subsequent loops however are almost identical and a stabilisation of the hysteretic loops is observed. Experimental results performed on the same tile show an increase of the

force with increasing field strength. This increase is particularly noticeable for what the stiffness is concerned, as seen on the test performed on specimens R2, T1 and T3.

TABLE 1: Results of the tests on the tiles

Levitation force				
Tile	F <sub>Z</sub> (max) [N]	F <sub>Z</sub> (min) [N]	Cooling	Magnet [mm*mm] [mT]
R1	2.4	-0.86	ZFC <sup>1</sup>	15*15 <sup>2</sup> 400 <sup>3</sup>
R2	1.66	-0.68	ZFC	28*10 265
S1	2.15	-1.18	ZFC	28*10 265
S2	1.30	-0.62	ZFC	28*10 265
S3	1.27	-0.73	ZFC	28*10 265
S4	1.09	-0.53	ZFC	28*10 265
T1	2.44	-0.96	ZFC	28*10 265
T1	3.48	-1.34	ZFC	15*15 400
T3	3.05	-1.32	ZFC	28*10 265
T3	3.47	-1.37	ZFC	15*15 400
U1	5.69	-1.45	ZFC	28*10 265
U2	5.09	-1.82	ZFC	28*10 265

Axial microcycle - axial stiffness				
Tile	K <sub>Z</sub> <sup>4</sup> [N/m]	Cooling	Magnet [mm*mm] [mT]	z <sub>c</sub> <sup>5</sup> [mm]
R1	1964	FC 6 <sup>6</sup>	15*15 400	2
R2	1656	FC 6	28*10 265	2
R2	1963	FC 6	15*15 400	2
S1	2655	FC 4	28*10 265	2
S2	1607	FC 4	28*10 265	2
S3	1943	FC 4	28*10 265	2
S4	1614	FC 4	28*10 265	2
T1	2832	FC 4	28*10 265	2
T1	2913	FC 4	15*15 400	2
T3	2932	FC 4	28*10 265	2
T3	3044	FC 4	15*15 400	2
U1	3164	FC 4	28*10 265	2
U2	3178	FC 4	28*10 265	2

Radial microcycle - radial stiffness				
Tile	K <sub>X</sub> <sup>7</sup> [N/m]	Cooling	Magnet [mm*mm] [mT]	z <sub>c</sub> [mm]
T1	1025	FC 2	28*10 265	2
T1	1616	FC 2	15*15 400	2
T3	760	FC 2	28*10 265	2
T3	1250	FC 2	15*15 400	2
U1	1030	FC 2	28*10 265	2

<sup>1</sup> Zero Field Cooling.

<sup>2</sup>diameter\*thickness of tile.

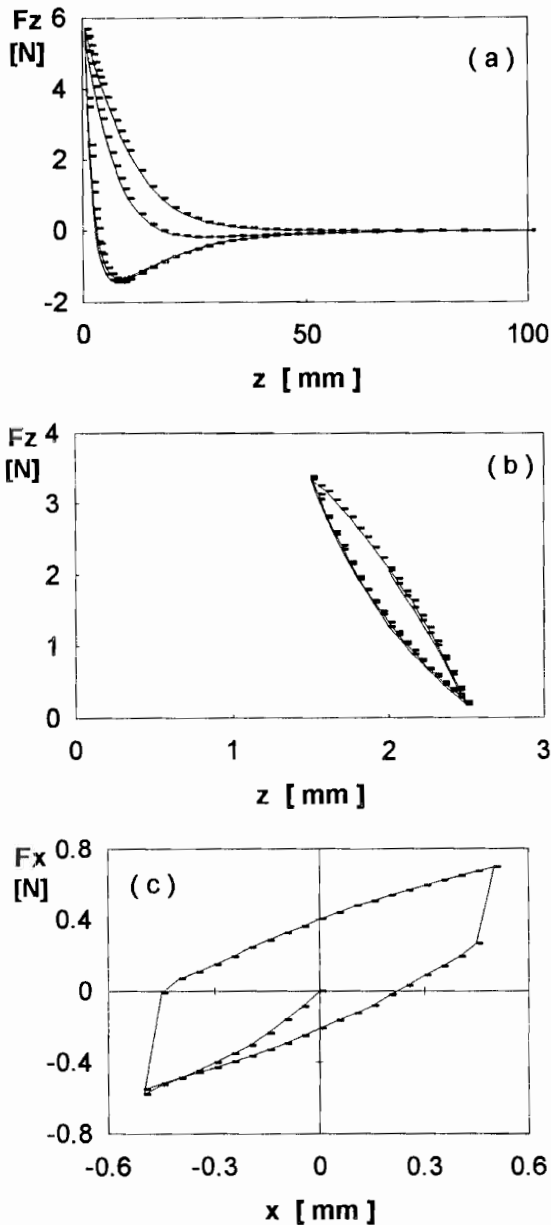
<sup>3</sup>maximum strength of magnetic field.

<sup>4</sup>axial stiffness.

<sup>5</sup>displacement offset for microloops.

<sup>6</sup>Field Cooling with 6 mm magnet-tile displacement

<sup>7</sup>radial stiffness.



**FIGURE 1:** Results of the three tests performed on the superconducting tiles. a) levitation force vs. distance between magnet and superconductor; b) levitation force during an axial microcycle in a configuration aimed to measure the axial stiffness; c) lateral force vs. radial displacement.

**MATHEMATICAL MODEL**

The levitation force between a superconductor and a permanent magnet can be considered as a result of induced current loops interacting with the magnetic field. Using the Bean concept [7] of the critical state, the Lorentz equation for the force between magnet and superconductor can be written as

$$F = \mu_0 \int_V \mathbf{J}_c \times \mathbf{H} dv \tag{1}$$

where  $\mu_0$  is the vacuum permeability,  $\mathbf{J}_c$  the critical current density and  $\mathbf{H}$  the magnetic field. After some transformations equation (1) can be rewritten in the form

$$F = \mu_0 \int_V (\mathbf{M} \cdot \nabla) \mathbf{H} dv \tag{2}$$

where  $\mathbf{M}$  is the induced magnetisation and the integration is performed over all the region where the induced superconducting currents exist.

As follows from equations (1) and (2), the interaction between magnet and superconductor has a nonlocal character and so a very complicated boundary value magnetostatic problem must be solved for the calculation of the levitation force, even for very simple magnet-superconductor geometries.

This formulation is clearly insufficient for engineering applications. However, in practical design of superconductive magnetic bearings all what is usually needed is the dependence of the interaction forces on the position.

As a first assumption, the magnet can be regarded as a simple "average" magnetic field  $\mathbf{H} = \mathbf{H}(z)$ , which depends only on the distance  $z$  between the magnet and the superconductor. Equation (2) can be substituted by the following model

$$F = C \cdot M \cdot V \cdot \frac{dH}{dz} \tag{3}$$

where  $M$  is the induced magnetisation of the high-Tc superconductors,  $V$  is the volume of superconductor and  $C$  is a shape factor which accounts for the influence of the field's details, the sample geometry, the presence and distribution of 211 phase and other geometrical parameters. Clearly  $M$  is a functional of magnetic field history.

This functional may be derived in according to Bean theory applied to a simple slab of HTS with thickness  $d$ . The magnetic field distribution inside the superconductor can be computed by using the simple Bean equation

$$J_c = \frac{dH}{dz} \tag{4}$$

Although seemingly unrealistic, this model nevertheless predicts with relative accuracy the experimental data.

Actually it is based on the following simple and quite realistic assumptions:

- in the case of constant geometry the actual magnetic field may be reduced to a simple "average" magnetic field  $H(z)$ ;
- the superconductor can be characterised (beside its volume  $V$  and 123-phase content) by a specific superconducting characteristic, namely the critical current  $J_c$ , and by one "effective" geometric parameter, its thickness  $d$ , or, similarly, by the critical magnetic field penetration intensity  $H_p = J_c d$ ;
- all other geometrical parameters may be incorporated into a constant shape factor  $C$ .

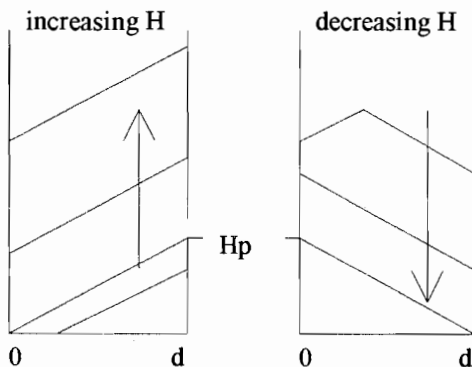
Under these hypotheses the internal magnetic field obtained from the model is described by linear functions within the interval  $[0, d]$ . The slope of these functions can be only  $J_c$  or  $-J_c$  respectively for the cases of increasing or decreasing  $H$  (fig. 2).

This situation can describe the hysteretic behaviour for both the macro-cycles (levitation force tests) and micro-cycles (stiffness tests).

When, in zero field cooling, the distance between magnet and HTS decreases for the first time, the linear function sloping upwards moves up until reaching the maximum value of applied magnetic field (fig. 2).

When moving the magnet toward the HTS a decreasing linear function describes the internal magnetic field inside the tile.

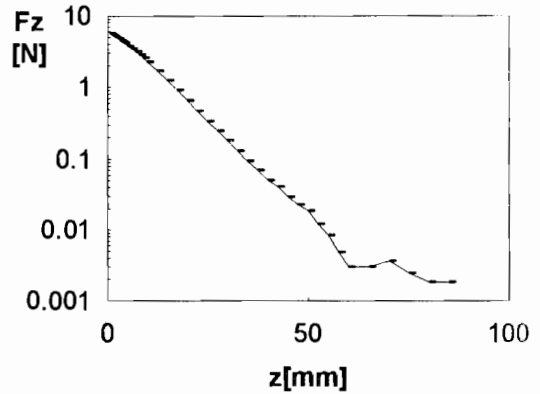
When the applied magnetic field becomes negligible, an induced internal magnetic field remains in the superconductor and the following loops will be different from the first one. This behaviour is always observed when measuring the levitation force (fig. 1a and fig. 4).



**FIGURE 2:** Example of behaviour of the magnetic field inside the HTS in the cases of increasing or decreasing applied magnetic field.

In all calculations the magnetic field was approximated as  $H = H_0 \exp(-\alpha z)$ , where  $H_0$  is the field on the surface of the magnet. This assumption was confirmed by measurements of magnetic field on the tiles using a Hall probe.

In this case it is easy to show that for the initial motion of the magnet towards the superconductor the levitation force obeys to the exponential law  $\ln(F) \approx z$  as it is experimentally observed (fig. 3).



**FIGURE 3:** Experimental data of the initial motion of the magnet towards the superconductor.

The values of the levitation force so computed were then compared with experimental results obtained from the HTS samples (fig. 4).

It is interesting to note that the initial levitation force follows the exponential relation  $F = F_0 \exp(-\gamma z)$  rather than the power law  $F \approx z^{-4}$  predicted on the basis of diamagnetic model.

The experimental results obtained in many tests gave values which are well below those obtained from the present model.

This is due to the assumption about the homogeneous structure of the HTS, while the superconductor samples used in the present experiments were highly inhomogeneous. Moreover, in MPMG processes a textured material is obtained and not all superconductor grains have the prescribed orientation.

To incorporate the inhomogeneity of the materials into the present model, it is possible to assume that the superconductor under consideration is a mixture of "good" and "bad" phases with critical currents  $J_{c1}$  and  $J_{c2}$  (where  $J_{c1} > J_{c2}$  and  $H_{p1} > H_{p2}$ ) and that  $\lambda$  is the percentage of the "good" phase.

The critical current density  $J_{c1}$  is assumed to correspond to the critical current parallel to plane a-b, and the  $J_{c2}$  to the current along c direction (where directions a, b and c are the directions of the orthorhombic cell of YBCO).

The levitation force for such "composite" material may be computed as the sum of the levitation forces due to each phase.

The use of an inhomogeneous model can greatly improve the correspondence between experimental and numerical results (figures 4 and 5).

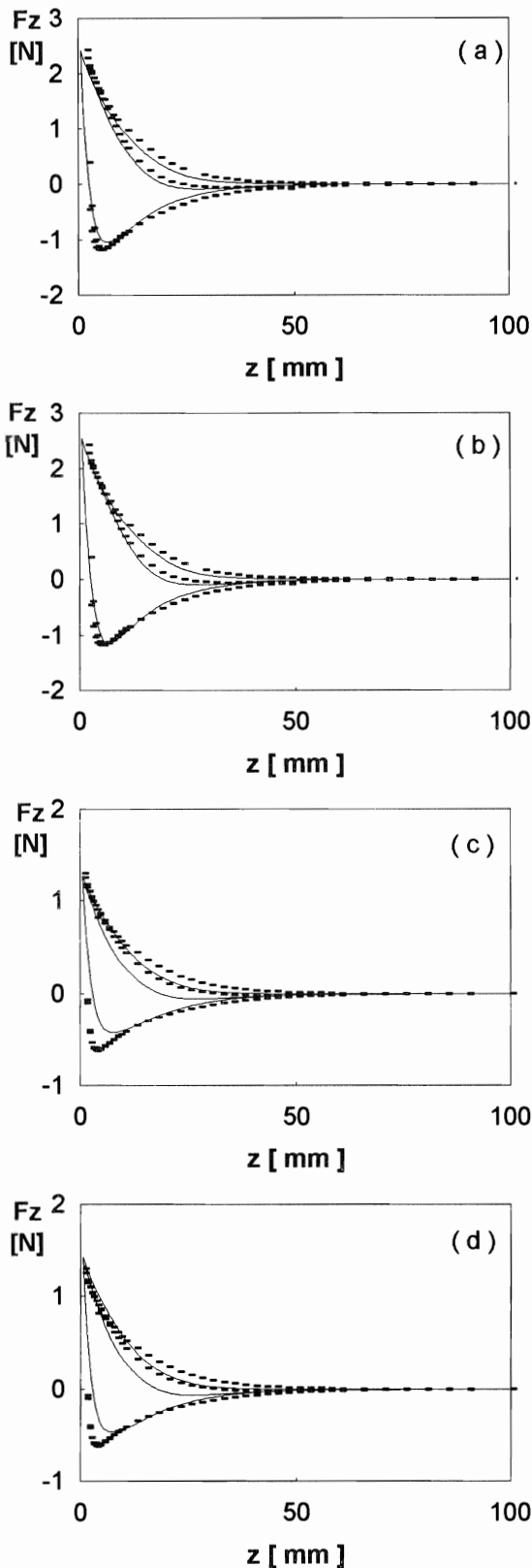


FIGURE 4: Experimental data and results from the inhomogeneous model. a), c) tiles S1 S2; b), c) tiles S1 and S2 with different values of  $H_0$ ,  $d$ ,  $\alpha$ ,  $J_c$ .

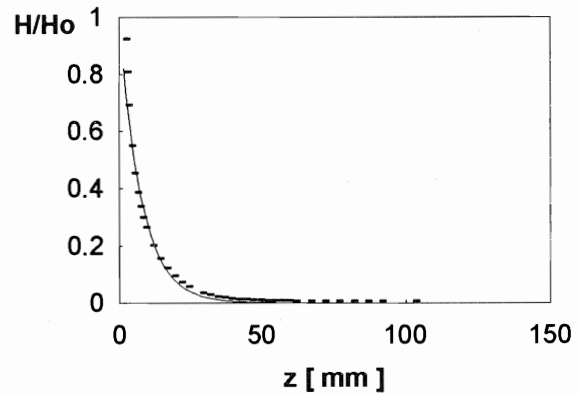


FIGURE 5 Comparison between experimental and theoretical results. Points: magnetic field measurements; line: theoretical results.

## CONCLUSIONS

The experimental results of measurements of levitation force and of axial and radial stiffness performed on superconductors tiles of YBCO, produced with melt textured technique, have been summarised.

The results highlight the non-linear behaviour of the levitation force which, due to the flux pinning, may be repulsive as well as attractive. The levitation force and, in a higher degree, the value of the stiffness increase with the applied magnetic field.

The numerical values here obtained substantiate the possibility of using the superconductors tested for the construction of magnetic bearings as well as other suspension systems which are intrinsically stable.

In order to perform a first approximated simulation of the performance of the HTS, an analytical model is here proposed. It can predict with sufficient precision the levitation behaviour of magnets above superconductors and gives a possibility to obtain quantitative description for the levitation force, in spite of the drastic assumptions involved.

This model takes into account the inhomogeneity of actual materials and may be regarded as a first approximation useful for the simulation of the behaviour of SMB.

A further way to improve the present model is to take into account the dependence of the critical current density from the magnetic field.

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