DYNAMICS AND STABILIZATION OF MAGNETIC SUSPENSION USING TUNED LC CIRCUIT

Ju Jin
Kanagawa Academy of Science and Technology, Kawasaki City, Japan
Toshiro Higuchi
Department of Precision Machinery Engineering, University of Tokyo, Tokyo, Japan

ABSTRACT

To date, the dynamic model of suspension systems using tuned LC circuit has not been established. This paper develops a transfer function model for such systems by linearizing system equations near the equilibrium state. This model not only nicely explains the inherent dynamic instability, but also provides a theoretical basis for analyzing and synthesizing the dynamic behaviors. A new dynamic stabilization method is also proposed. This method applies damping to the movable "stator" rather than to the suspended object directly, and consequently, stable suspension without any mechanical contact is achieved. Experimental results are shown to confirm the theoretical analysis.

INTRODUCTION

As a simple and sensorless suspension method, suspension using a tuned LC circuit has attracted wide attention from scientists and engineers [1]-[3]. This technology uses the variation of inductance of the electromagnet, which is determined by the gap between the electromagnet and the suspended object, to modify the force-displacement characteristic of suspension systems. Fig. 1 shows a single degree of freedom version of a magnetic suspension system using tuned LC circuit. The electromagnet is the inductive part of the LC circuit. The LC circuit is designed in such a way that when the suspended object moves away from the electromagnet, inductance decreases. The LC circuit tends to become resonant, increasing coil current and hence attractive force, restores the suspended object to its original position. On the other hand, when the suspended object approaches to the electromagnet, inductance increases. The LC circuit goes away from the resonant state, coil current and thus attractive force decrease. The suspended object is pulled down by gravity. Fig. 2 shows its static force-displacement characteristic. It is obvious from Fig. 2 that if the suspended object moves within $\tilde{x} < \tilde{x}_{\max}$, attractive force acts as restoring force. If attractive force at a certain position, for example x_e ($x_e < \tilde{x}_{\max}$), is balanced against that of gravity, it is possible to obtain a stable equilibrium position for the suspended object.

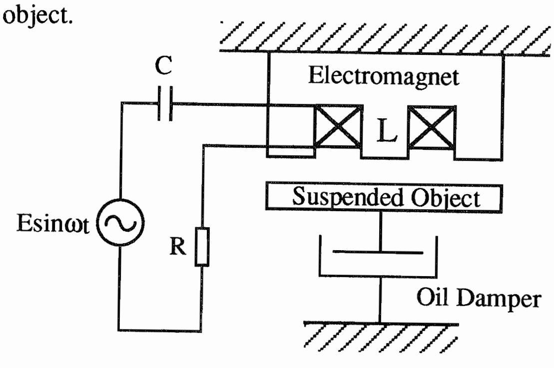

FIGURE 1 Tuned LC Circuit Suspension

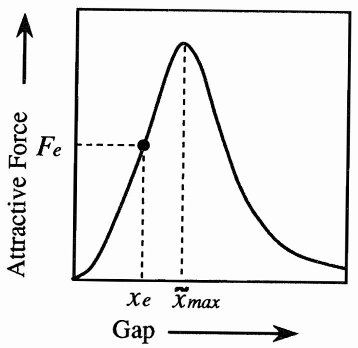

FIGURE 2 Force/Gap Characteristic

However, this system is dynamically unstable, the suspended object tends to vibrate divergently[2]-[3]. Additional damping must be applied to restrict the vibration. Generally an oil damper is used by submerging the suspended object in oil [1]-[2]. This damping method involves mechanical contact, so free suspension cannot be realized. Owing to this reason, magnetic suspension using tuned LC circuit has found its applications only in some specialized fields. Another reason which hampers the development of this suspension technology is the complexity of theoretical analysis. Generally, several coupled nonlinear equations are necessary to express completely such a suspension system. Practical analysis models have not been established so far. The main purposes of this paper is to eliminate these obstacles so as to promote the research and applications of this suspension technology.

LIST OF SYMBOLS

The symbols used in this paper are as follows. The subscript "e" denotes values at the equilibrium state and the superscript " ~ " denotes total values.

E: Maximum value of source voltage (V)
ω: Source angular frequency (rad/s)
C: Capacitance of LC circuit (F)
R: Resistance of LC circuit (Ω)
L: Inductance of electromagnet (H)
x: Air gap (m)
x_1: Displacement of movable "stator" (m)
x_2: Displacement of suspended object (m)
F: Attractive force (N)
i: Coil current (A)
A: Coefficient of $\sin(\omega t)$ (A)
B: Coefficient of $\cos(\omega t)$ (A)
m_1: Mass of movable "stator" (kg)
m_2: Mass of suspended object (kg)
F_d: Disturbance (N)
K_1: Spring coefficient (N/m)
D_1: Damping on electromagnet (Ns/m)
D_2: Damping on suspended object (Ns/m)
t: Time (s)

SYSTEM DESCRIPTION

The LC circuit in Fig. 1 can be described by

$$\frac{d^2(\tilde{L}\tilde{i})}{dt^2} + R\frac{d\tilde{i}}{dt} + \frac{\tilde{i}}{C} = E\,\omega\cos(\omega t) \quad (1)$$

It has been found in real systems that (1) Coil current is a suppressed carrier amplitude modulation signal in the form

$$\frac{d^2(\tilde{L}\tilde{i})}{dt^2}\tilde{i}(t) = \tilde{A}\sin(\omega t) + \tilde{B}\cos(\omega t) \quad (2)$$

where $\tilde{A}$ and $\tilde{B}$ denote the baseband envelope modulating signal and ω is the carrier angular frequency. (2) $\tilde{A}$ and $\tilde{B}$ are functions of the gap,

$$\frac{d^2(\tilde{L}\tilde{i})}{dt^2}\tilde{A} = \tilde{A}(\tilde{x},t), \qquad \tilde{B} = \tilde{B}(\tilde{x},t)$$

Attractive force acting on the suspended object is determined by

$$\tilde{F} = -\frac{1}{2}\frac{d\tilde{L}}{d\tilde{x}}\tilde{i}^2$$

$$= -\frac{1}{2}\frac{d\tilde{L}}{d\tilde{x}}\left[\frac{\tilde{A}^2+\tilde{B}^2}{2}+\frac{\tilde{B}^2-\tilde{A}^2}{2}\cos(2\omega t)+\tilde{A}\tilde{B}\sin(2\omega t)\right] \quad (3)$$

Compared to the source frequency, the motion of the suspended object is relatively slow, so only the low frequency components of attractive force are important to the movement of the suspended object. Therefore the components with 2ω can be neglected. As a result, attractive force may be approximated by

$$\tilde{F} \approx -\frac{1}{4}\frac{d\tilde{L}}{d\tilde{x}}\left[\tilde{A}^2 + \tilde{B}^2\right] \quad (4)$$

In order to calculate attractive force, $\frac{d\tilde{L}}{d\tilde{x}}$ is needed. The functional relation between inductance $\tilde{L}$ and the gap $\tilde{x}$ can be determined experimentally. The following approximate formula is often used to express this relation.

$$\tilde{L}(\tilde{x}) = L_\infty + \frac{\kappa_1}{\tilde{x} + \kappa_2} \quad (5)$$

where κ_1 and κ_2 are positive constants and L_∞ denotes inductance when the gap is infinite. From equation (5), $\frac{d\tilde{L}}{d\tilde{x}}$ can be obtained.

The equation of motion of the suspended object is

$$m_2\ddot{\tilde{x}}_2 = F_d + m_2 g - \tilde{F} \quad (6)$$

These five equations, (1),(2),(4),(5),(6), together represent suspension systems using tuned LC circuit.

STATIC CHARACTERISTIC

First, let us consider the static characteristic of tuned LC circuit suspension systems. When the suspended object stays in the equilibrium position, inductance L_e is a constant, and equation (1) becomes an ordinary differential equation.

$$L_e \frac{d^2 i_e}{dt^2} + R\frac{di_e}{dt} + \frac{i_e}{C} = E\,\omega\cos(\omega t) \qquad (7)$$

The solution of equation (7) is

$$i_e = A_e \sin(\omega t) + B_e \cos(\omega t) \qquad (8)$$

where

$$A_e = \frac{ER}{\rho^2 + R^2},\ B_e = \frac{E\rho}{\rho^2 + R^2},\ \rho = \frac{1}{C\omega} - L_e\omega \qquad (9)$$

Substituting equation (9) into equation (4), attractive force at the equilibrium state can be obtained.

$$F_e = -\frac{1}{4}\frac{d\tilde{L}}{d\tilde{x}}\bigg|_{x_e}\left[A_e^2 + B_e^2\right] = -\frac{1}{4}\frac{d\tilde{L}}{d\tilde{x}}\bigg|_{x_e}\frac{E^2}{\rho^2+R^2} \qquad (10)$$

Equation (10) is the mathematical representation of the static force-displacement characteristic shown in Fig. 2.

DYNAMIC MODEL

Now let us proceed to studying the dynamic characteristic of the magnetic suspension system using tuned LC circuit shown in Fig. 1.

Current Equation

Substituting equation (2) into equation (1), comparing the coefficients of sin(ωt) and cos(ωt) separately, the next two equations can be obtained.

$$\frac{d^2(\tilde{L}\tilde{A})}{dt^2} + R\frac{d\tilde{A}}{dt} - 2\omega\frac{d(\tilde{L}\tilde{B})}{dt} + \tilde{\rho}\omega\tilde{A} - R\,\omega\tilde{B} = 0 \qquad (11)$$

$$\frac{d^2(\tilde{L}\tilde{B})}{dt^2} + R\frac{d\tilde{B}}{dt} + 2\omega\frac{d(\tilde{L}\tilde{A})}{dt} + \tilde{\rho}\omega\tilde{B} + R\,\omega\tilde{A} = E\,\omega \qquad (12)$$

where

$$\tilde{\rho} = \frac{1}{C\omega} - \tilde{L}\omega$$

Near the equilibrium state, system parameters can be approximated in the forms

$$\begin{cases} \tilde{x} = x_e + x & \tilde{L} = L_e + L \\ \tilde{A} = A_e + A & \tilde{B} = B_e + B \\ L = \frac{d\tilde{L}}{d\tilde{x}}\Big|_{x_e} x = K_x \cdot x & \frac{d\tilde{L}}{dt} = \frac{d\tilde{L}}{d\tilde{x}}\dot{x} \\ \frac{d\tilde{L}}{d\tilde{x}}\Big|_{\tilde{x}} = \frac{d\tilde{L}}{d\tilde{x}}\Big|_{x_e} + \frac{d^2\tilde{L}}{d\tilde{x}^2}\Big|_{x_e} x = K_x + K_{2x}x \end{cases} \qquad (13)$$

Inserting these formulae equations into (11) and (12), neglecting the nonlinear components of minute variation, we can get the functional relation between coil current and gap variation.

$$A(s) = \frac{\sum_{i=0}^{4} p_i s^i}{\sum_{i=0}^{4} \gamma_i s^i} X(s) \qquad B(s) = \frac{\sum_{i=0}^{4} \eta_i s^i}{\sum_{i=0}^{4} \gamma_i s^i} X(s) \qquad (14)$$

where

$$\begin{cases} p_4 = -L_e A_e K_x \\ p_3 = -R A_e K_x \\ p_2 = -2L_e\omega^2 A_e K_x - \frac{1}{C}A_e K_x + R\,\omega B_e K_x \\ p_1 = \frac{2}{C}\omega B_e K_x - R\,\omega^2 A_e K_x \\ p_0 = \frac{1}{C}\omega^2 A_e K_x - L_e\omega^4 A_e K_x + R\,\omega^3 B_e K_x \end{cases}$$

$$\begin{cases} \eta_4 = -L_e B_e K_x \\ \eta_3 = -R B_e K_x \\ \eta_2 = -2L_e\omega^2 B_e K_x - \frac{1}{C}B_e K_x - R\,\omega A_e K_x \\ \eta_1 = -\frac{2}{C}\omega A_e K_x - R\,\omega^2 B_e K_x \\ \eta_0 = \frac{1}{C}\omega^2 B_e K_x - L_e\omega^4 B_e K_x - R\,\omega^3 A_e K_x \end{cases} \qquad (15)$$

$$\begin{cases} \gamma_0 = (\rho^2 + R^2)\omega^2 \\ \gamma_1 = \frac{2R}{C} + 2RL_e\omega^2 \\ \gamma_2 = R^2 + \frac{2L_e}{C} + 2L_e^2\omega^2 \\ \gamma_3 = 2L_e R \\ \gamma_4 = L_e^2 \end{cases} \qquad (16)$$

Force Equation

Next let us consider the attractive force near the equilibrium state. When the nonlinear items of minute variation are neglected, equation (4) can be simplified to

$$\tilde{F} \approx -\frac{1}{4}\frac{d\tilde{L}}{d\tilde{x}}\left[\tilde{A}^2 + \tilde{B}^2\right]$$

$$\approx F_e - \frac{K_x A_e}{2}A - \frac{K_x B_e}{2}B - \frac{A_e^2+B_e^2}{4}K_{2x}x \qquad (17)$$

Change of attractive force due to gap variation is

$$F = -\frac{K_x A_e}{2} A - \frac{K_x B_e}{2} B - \frac{A_e^2 + B_e^2}{4} K_{2x} x \qquad (18)$$

Substituting equation (14) for A and B respectively, gives its Laplace transformation

$$F(s) = \frac{\sum_{i=0}^{4} \beta_i s^i}{\sum_{i=0}^{4} \gamma_i s^i} X(s) \qquad (19)$$

where "s" denotes the Laplace transformation and

$$\begin{cases} \beta_4 = Q\,(2L_e K_x^2 - L_e^2 K_{2x}) \\ \beta_3 = 2Q\,(RK_x^2 - RL_e K_{2x}) \\ \beta_2 = Q\left[4L_e \omega^2 K_x^2 + \frac{2K_x^2}{C} - R^2 K_{2x} - 2L_e \omega \rho K_{2x}\right] \\ \beta_1 = 2Q\,(R\,\omega^2 K_x^2 - \frac{R}{C} K_{2x} - L_e R\,\omega^2 K_{2x}) \\ \beta_0 = Q\left[\frac{-2\omega^2 K_x^2}{C} + 2L_e \omega^4 K_x^2 - \rho^2 \omega^2 K_{2x} - R^2 \omega^2 K_{2x}\right] \end{cases} \qquad (20)$$

$$Q = \frac{A_e^2 + B_e^2}{4}$$

Transfer Function Model

The suspended object is governed by equation (6). At the equilibrium state, $F_e = m_2 g$, Equation (6) can be rewritten as

$$m_2 \ddot{x}_2 + F = F_d \qquad (21)$$

In traditional tuned LC circuit suspension systems, the electromagnet is fixed, therefore $\tilde{x}_2 = \tilde{x}$ and hence $x_2 = x$. Substituting equation (19) into (21), gives its Laplace transformation as

$$X_2(s) = \frac{\sum_{i=0}^{4} \gamma_i s^i}{\sum_{i=0}^{6} \alpha_i s^i} F_d(s) \qquad (22)$$

where

$$\begin{cases} \alpha_6 = m_2 \gamma_4 \qquad \alpha_5 = m_2 \gamma_3 \\ \alpha_4 = m_2 \gamma_2 + \beta_4 \qquad \alpha_3 = m_2 \gamma_1 + \beta_3 \\ \alpha_2 = m_2 \gamma_0 + \beta_2 \qquad \alpha_1 = \beta_1 \qquad \alpha_0 = \beta_0 \end{cases} \qquad (23)$$

Equation (22) is the transfer function model of tuned LC circuit suspension systems.

DYNAMIC INSTABILITY

From equation (23), we find the coefficient α_1 is

$$\alpha_1 = \beta_1 = \frac{A_e^2 + B_e^2}{2}\left[-R\,\omega^2 (L_e K_{2x} - K_x^2) - \frac{R}{C} K_{2x}\right]$$

Since inductance of the electromagnet is inversely proportional to the gap as indicated by equation (5), it is found that the following condition is always satisfied:

$$L_e K_{2x} - K_x^2 > 0 \qquad (24)$$

Combining formula (24) with α_1, it is easy to find that α_1 is always negative. This means that the system has at least one pole lying in the right half s-plane, so the system is unstable, i.e., tuned LC circuit suspension systems are inherently dynamically unstable.

INDIRECT DAMPING METHOD

In this section, we will discuss a new damping method, the indirect damping method. Fig. 3 illustrates the suspension systems using this method. Compared to the traditional suspension system shown in Fig. 1, the new system has two remakable features. (1) The "stator" , the electromagnet in this case, is movable, (2) Damping is applied to the movable "stator" instead of to the suspended object directly. Damping effect applied to the movable "stator" is transferred to the suspended object by attractive force, it suppresses any self-sustained oscillation of the suspended object, as a result, a stable suspension is achieved.

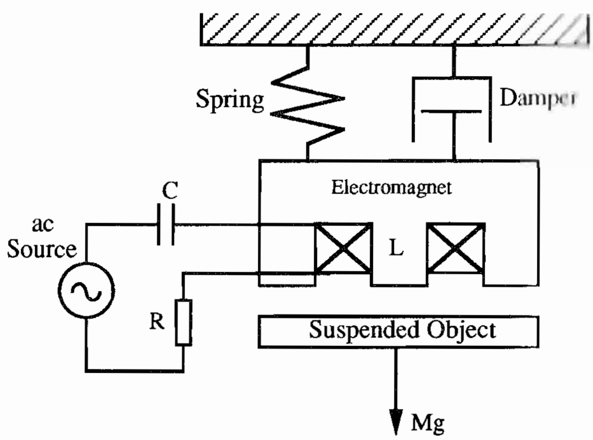

FIGURE 3 Indirect Damping Method

Transfer Function Model of New System

The suspension system shown in Fig. 3 can be described by

$$m_1\ddot{x}_1 = F - D_1\dot{x}_1 - K_1x_1 \quad (25)$$

$$m_2\ddot{x}_2 = F_d - F \quad (26)$$

where F is the change in attractive force due to $(x_2 - x_1)$.

According to equation (19) in section 5, attractive force in this case can be described in the form

$$F(s) = \frac{\sum_{i=0}^{4}\beta_i s^i}{\sum_{i=0}^{4}\gamma_i s^i}[X_2(s) - X_1(s)] \quad (27)$$

Rewriting equations (25) and (26) in the forms of Laplace transformation, and combining them with equation (27), yields

$$X_1(s) = \frac{\sum_{i=0}^{4}\beta_i s^i}{\sum_{i=0}^{8}\lambda_i s^i}F_d(s),\ X_2(s) = \frac{\sum_{i=0}^{6}\xi_i s^i}{\sum_{i=0}^{8}\lambda_i s^i}F_d(s) \quad (28)$$

where (β_i, $i = 0,1,2,3,4$) and (γ_i, $i = 0,1,2,3,4$) are the same as what defined in equations (16) and (20) in section 5, and the others are defined as follows.

$$\begin{cases}\lambda_8 = m_1m_2\gamma_4\\ \lambda_7 = m_1m_2\gamma_3 + m_2D_1\gamma_4\\ \lambda_6 = m_1m_2\gamma_2 + m_2D_1\gamma_3 + K_1m_2\gamma_4 + (m_1+m_2)\beta_4\\ \lambda_5 = m_1m_2\gamma_1 + m_2D_1\gamma_2 + K_1m_2\gamma_3 + (m_1+m_2)\beta_3 + D_1\beta_4\\ \lambda_4 = m_1m_2\gamma_0 + m_2D_1\gamma_1 + K_1m_2\gamma_2 + (m_1+m_2)\beta_2 + D_1\beta_3 + K_1\beta_4\\ \lambda_3 = m_2D_1\gamma_0 + K_1m_2\gamma_1 + (m_1+m_2)\beta_1 + D_1\beta_2 + K_1\beta_3\\ \lambda_2 = K_1m_2\gamma_0 + (m_1+m_2)\beta_0 + D_1\beta_1 + K_1\beta_2\\ \lambda_1 = D_1\beta_0 + K_1\beta_1\\ \lambda_0 = K_1\beta_0\end{cases}$$

$$\begin{cases}\xi_6 = m_1\gamma_4\\ \xi_5 = m_1\gamma_3 + D_1\gamma_4\\ \xi_4 = m_1\gamma_2 + D_1\gamma_3 + K_1\gamma_4 + \beta 4\\ \xi_3 = m_1\gamma_1 + D_1\gamma_2 + K_1\gamma_3 + \beta 3\\ \xi_2 = m_1\gamma_0 + D_1\gamma_1 + K_1\gamma_2 + \beta 2\\ \xi_1 = D_1\gamma_0 + K_1\gamma_1 + \beta_1\\ \xi_0 = K_1\gamma_0 + \beta_0\end{cases} \quad (29)$$

Stability Condition

As mentioned in section 6, β_1 is usually negative. However, it is noted that β_0 is always positive in real systems. Hence from the definitions of λ_1 and λ_0, if K_1 and D_1 are chosen as

$$K_1 > 0, \qquad D_1 > -K_1\frac{\beta_1}{\beta_0} \quad (30)$$

it is possible to make λ_1 positive. This means that it is possible to make the system shown in Fig. 3 stable by using the indirect damping method.

EXPERIMENTAL RESULT

To confirm the analysis discussed in previous sections, we carried out an experimental verification using the device shown in Fig. 4. The suspended object is a cantilever, it moves in the vertical plane. The electromagnet for suspension is fixed to one side of a parallel spring. As the damping material, silicon Gel is used. The system parameters are listed in Table 1.

TABLE 1 System Parameters

E	61.0 (V)	R	15.8(Ω)
C	9.54(μF)	L_e	25.1 (mH)
K_x	-11.2 (H/m)	K_{2x}	1.35×10^4 (H/m^2)
x_{1e}	0.5×10^{-3} (m)	x_{2e}	1.25×10^{-3} (m)
m_1	1.6 (kg)	m_2	1.5 (kg)
I	0.22 (kg·m^2)	f	400.0 (Hz)
l	0.236 (m)		

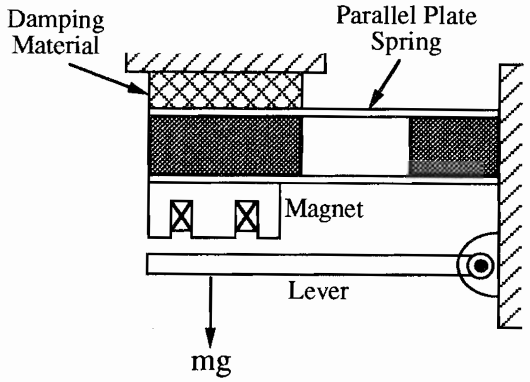

FIGURE 4 Experimental System

The system response to a pulse disturbance force applied directly to the suspended object is illustrated in Fig. 5. It is obvious from that stable suspension was achieved.

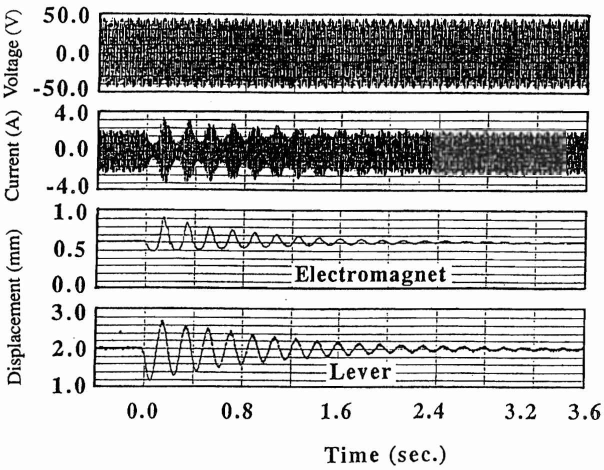

FIGURE 5 System Response to Pulse Disturbance

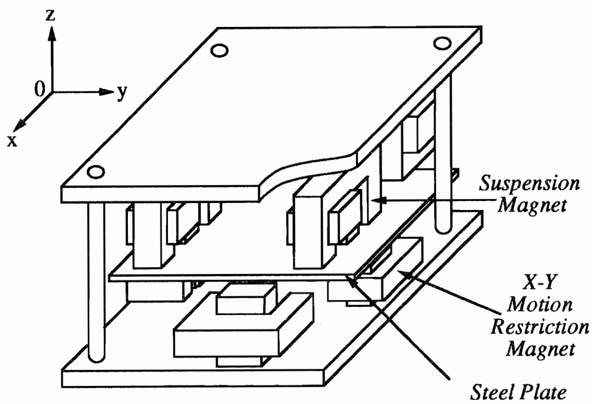

FIGURE 6 5-DOF Suspension System

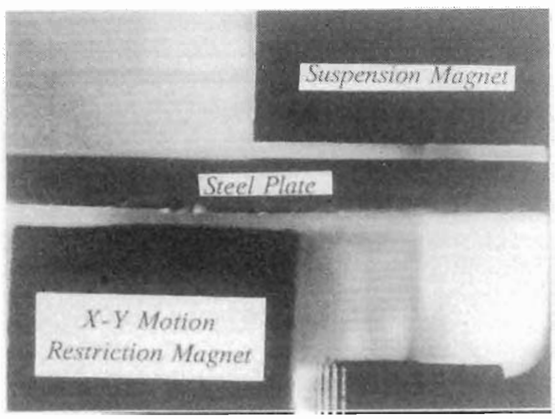

FIGURE 7 Steel Plate in Suspension State

In addition, we have carried out a 5 degrees-of freedom suspension experiment using a system shown in Fig. 6. A steel plate with thickness 3 mm and area 300 cm^2 was suspended successfully. Fig. 7 is the photograph shows the suspension state.

CONCLUSION

In this paper, we have developed a transfer function model for magnetic suspension systems using tuned LC circuit and proposed a new dynamic damping method. Their validity are verified by experimental results. In addition, since the electrostatic suspension using tuned LC circuit is very similar to the tuned LC circuit magnetic suspension, the theoretical analysis and the proposed "Indirect Damping Method" mentioned above can also be used to analyze and stabilize electrostatic suspension systems using tuned LC circuit. We believe that the model and the new damping method discussed in this paper will play an important role in the research and applications of this suspension technology.

REFERENCES

[1] B. Z. Kaplan, "Analysis of a method for magnetic levitation", *Proceedings of IEE,* Vol. 114, No. 11, pp. 1801-1804, November 1967.

[2] J. W. Henn, "Linear perturbation models for a.c magnetic suspension systems: experimental and theoretical results", *Proceedings of IEE*, Vol. 127, Pt. D., No. 2, pp. 64-74, march, 1980.

[3] J. Jin, "Magnetic suspension using tuned LC circuit", Doctoral Thesis, University of Tokyo, 1992.

[4] J. Jin and T. Higuchi, "Modeling and stabilization for magnetic suspension system using tuned LCR circuit", *Proceedings of the IEEE International Conference on System Engineering,* pp. 411-415, September 1992.

[5] J. Jin and T. Higuchi, "Dynamics and stability of magnetic suspension systems using tuned LCR circuit", *Proceedings of the 1st International Conference on Motion and Control,* pp. 633-638, September 1992.

[6] J. Jin and T. Higuchi, "A new approach to sensorless magnetic suspension system using tuned LCR circuit: theoretical analysis", *Proceedings of SICE'91,* pp. 1001-1004, July 1991.