IDENTIFICATION OF PHYSICAL PARAMETERS IN ACTIVE MAGNETIC BEARING SUSPENSION

J. Lottin and L. Saïdi
Laboratoire d'Automatique et de MicroInformatique Industrielle
LAMII - CESALP - Université de Savoie
41, avenue de la plaine BP. 806
74016 ANNECY cedex. FRANCE.
Phone : (33)-50-66-60-52- Fax : (33)-60-66-60-20
E-mail : Lottin @ esia.univ-savoie.fr

KEYWORDS
Identification, Active magnetic bearing, passive thrust, MIMO systems, non linear model.

ABSTRACT
This paper describes a simplified procedure that permits the identification of some parameters of the model of an active magnetic bearing suspension. It is shown that a direct identification of the model is not possible due to the non linear, unstable and multivariable features of the whole system. However, since some terms of the model can be determined in advance by means of simple measures on the roughly stabilized system, it is possible to converge towards a good estimation of the remaining parameters.

1-INTRODUCTION

A good control scheme requires a good knowledge of the model of the plant. Analysis of phenomena that involve in the process evolution provides with informations about the structure of the searched model. Then comes the most difficult step which consists in determining the value of some parameters. In the linear case, many identification procedures are available either for black box or physical model identification.

In the non linear case the problem is more complex, due to the eventual undiscernability of some parameters in one hand, or to the complexity of identification procedures with associated numerical analysis difficulties on the other hand.

The situation becomes severe when the plant is open loop unstable and multi input multi output. The problem which is considered in this paper concerns the identification of some parameters that appear in modelling the mechanical and electromagnetical parts of the magnetic suspension of a rotor shaft by means of active magnetic bearings.

The process under consideration is composed of an horizontal rotating shaft, suspended by means of two active magnetic bearings located in parallel planes orthogonal with rotation axis. The problem is to maintain the rotation axis in a specified position despite of various disturbances (working efforts or unbalance) by acting on electromagnets feeding. Among the six degrees of freedom of a body in three dimensional space, four of them are controlled by means of electromagnetic forces. The two others are rotation and translation along the main axis of the rotor. The former is controlled by angular velocity regulation, while the latter is passively controlled by annular permanent magnets located on the rotor and on the stator in such a way that equilibrium position is stable when considering displacement along the main axis. Of course, this device introduces destabilizing effects in radial directions, which must be compensated by control of active magnetic bearings.

We focus our attention on the identification of the non linear relation involved in force production by means of electromagnets, namely :

$$F(t)=\frac{D.i^2(t)}{\left(w_0-w(t)\right)^2}$$

where D is a constant parameter, w_0 is the equivalent magnetic circuit length, $w(t)$ is the rotor displacement, and $i(t)$ is the coil current.

The other parameter to identify is the stiffness of the destabilizing reaction introduced by the passive thrust in radial directions. Then it is desirable to consider the fluctuation of the negative stiffness with respect to the

relative angular position of the rotor and the stator, since magnetization is not homogeneous.
Some physical insights make possible the determination of acceptable values, allowing the synthesis of a stabilizing discrete control law.

2-MODELLING

2.1-Denotation

In this section, we derive the state equation of a magnetic bearing system. Plant and denotations are described in figure 1. Four electromagnets at both ends of the shaft sustain the rotor in the centered position without mechanical contact.

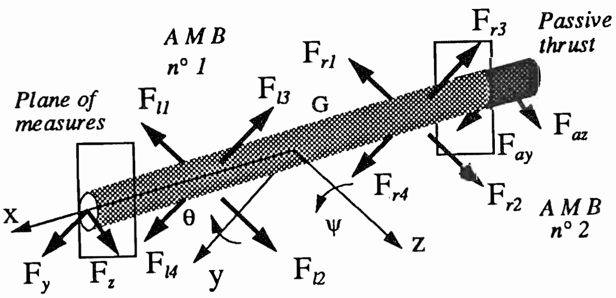

FIGURE 1 : Magnetic bearing structure

F_{ay}, F_{az} : components of thrust reaction

The modelling of the magnetic suspension suppose that the rotor is treated as a rigid body and its rotation is around its inertial axis. We represent the fixed frame by (O_0, X_0, Y_0, Z_0) and denote (G, X_2, Y_2, Z_2) the frame attached to the rotor. X_2 axis is obtained by rotating X_0 axis, first by ψ on the horizontal plane and next by θ on the vertical plane. We do the assumption that both of θ and ψ are small and we denote by p, q and r respectively the angular velocity of rolling, pitching and yawing (figure 2).

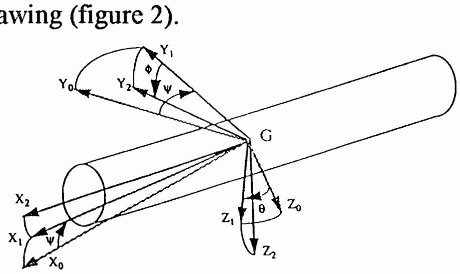

FIGURE 2 : Motion of the shaft

2.2-Equation of motion

The dynamical equations of the magnetic bearing system are given as follows :

$$\sum F_{ext} = m.\Gamma_G \qquad \delta_G = M_G \qquad (1), (2)$$

where :

m : mass of the rotor
F : resultant of external forces acting upon the body.
M_G : resultant of external moments about the mass center.
Γ_G : acceleration of the mass center.
δ_G : dynamical moment

Let the components of the vector OG be denoted by x_0, y_0, z_0. It shows the displacements of the gravity center in the fixed frame. From (1) we obtain the following equations of motion :

$$\ddot{y}_0 = \frac{F_{l4} - F_{l3} + F_{r4} - F_{r3} + F_y + P_y + F_{ay}}{m} \qquad (3)$$

$$\ddot{z}_0 = \frac{F_{l2} - F_{l1} + F_{r2} - F_{r1} + F_z + P_z + F_{az}}{m} \qquad (4)$$

P_y, P_z : components of the weight

The first term of the second dynamical equation gives :

$$\delta_G = \frac{d\sigma_0}{dt}\bigg|_{R0} = \begin{bmatrix} \dot{p}.J_x \\ \dot{q}.J_y + r.p.J_x - r.p.J_y \\ \dot{r}.J_y + q.p.J_y - q.p.J_x \end{bmatrix} \qquad (5)$$

where J_x is the moment of inertia around X axis and J_y is the moment of inertia around Y axis which is equal to the one around Z.
Considering the sole rotations θ and ψ, the second dynamical (2) equation gives :

$$\begin{bmatrix} \ddot{\theta} \\ \ddot{\psi} \end{bmatrix} = \frac{p.J_x}{J_y} \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} \dot{\theta} \\ \dot{\psi} \end{bmatrix} + \frac{1}{J_y} \begin{bmatrix} -F_z.l_b + (F_{l1} - F_{l2}).l_1 + (F_{r2} - F_{r1}).l_2 + F_{az}.l_a \\ F_y.l_b + (F_{l4} - F_{l3}).l_1 + (F_{r3} - F_{r4}).l_2 - F_{ay}.l_a \end{bmatrix}$$

The displacements of the rotor, introduced by the passive thrust in radial directions are
-in the plane (G, X_0, Z_0) : $\Delta_{rz} = z_0 + l_a.\theta$
-in the plane (G, X_0, Y_0) : $\Delta_{ry} = y_0 - l_a.\psi$
where l_a denotes the distance from G to the passive thrust.

The radial forces produced by the passive thrust in the corresponding directions are :
$F_{ay} = -K_r.\Delta_{ry} = -K_r.y_0 + K_r.l_a.\psi$
$F_{az} = -K_r.\Delta_{rz} = -K_r.z_0 - K_r.l_a.\theta$

The equations which indicate respectively the motion of the body along and around the rotating axis are independent of others. Consequently, we not consider them in the subsequent discussion. We can rewrite the previous equations in the following matrix form :

$$\begin{bmatrix} \dot{y}_0 \\ \dot{z}_0 \\ \dot{\theta} \\ \dot{\psi} \\ \ddot{y}_0 \\ \ddot{z}_0 \\ \ddot{\theta} \\ \ddot{\psi} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ \frac{-K_r}{m} & 0 & 0 & \frac{K_r.l_a}{m} & 0 & 0 & 0 & 0 \\ 0 & \frac{-K_r}{m} & \frac{-K_r.l_a}{m} & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{-K_r.l_a}{J_y} & \frac{-K_r.l^2{}_a}{J_y} & 0 & 0 & 0 & 0 & \frac{-p.J_x}{J_y} \\ \frac{K_r.l_a}{J_y} & 0 & 0 & \frac{-K_r.l^2{}_a}{J_y} & 0 & 0 & \frac{p.J_x}{J_y} & 0 \end{bmatrix} . \begin{bmatrix} y_0 \\ z_0 \\ \theta \\ \psi \\ \dot{y}_0 \\ \dot{z}_0 \\ \dot{\theta} \\ \dot{\psi} \end{bmatrix} + \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{m} & \frac{1}{m} \\ \frac{1}{m} & \frac{1}{m} & 0 & 0 \\ \frac{-l_1}{J_y} & \frac{l_2}{J_y} & 0 & 0 \\ 0 & 0 & \frac{l_1}{J_y} & \frac{-l_2}{J_y} \end{bmatrix} . \begin{bmatrix} F_{l2} - F_{l1} + \left(\frac{\sqrt{2}}{2} . \frac{l_2}{l_1 + l_2} .m.g \right) \\ F_{r2} - F_{r1} + \left(\frac{\sqrt{2}}{2} . \frac{l_1}{l_1 + l_2} .m.g \right) \\ F_{l4} - F_{l3} + \left(\frac{\sqrt{2}}{2} . \frac{l_2}{l_1 + l_2} .m.g \right) \\ F_{r4} - F_{r3} + \left(\frac{\sqrt{2}}{2} . \frac{l_1}{l_1 + l_2} .m.g \right) \end{bmatrix}$$

2.3-Output equations

Rotor displacements caracterized by y_0 and z_0, θ and ψ are deduced from measures m_i provided by four sensors. These determine gap deviations along orthogonal directions in two planes parallel to magnetic bearings. Linear relations linking measures to state variables are :

$$\begin{bmatrix} m_1 \\ m_2 \\ m_3 \\ m_4 \end{bmatrix} = \begin{bmatrix} 0 & 1 & -d_1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & d_2 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & d_1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & -d_2 & 0 & 0 & 0 & 0 \end{bmatrix} . \begin{bmatrix} y_0 \\ z_0 \\ \theta \\ \psi \\ \dot{y}_0 \\ \dot{z}_0 \\ \dot{\theta} \\ \dot{\psi} \end{bmatrix} \qquad (8)$$

2.4-Electromagnetical modelling

It concerns the production of forces by means of electromagnets. A well known model is given by :

$$F(t) = \frac{D.i^2(t)}{\left(w_0 - w(t)\right)^2}$$

where D is a constant resuming the characteristics of the electromagnet, $i(t)$ is the current in the coil, w_0 is the nominal equivalent magnetic circuit length (including nominal air gap and iron length weighted by relative permeabilities) and $w(t)$ is air gap deviation which is directly related to state variables of the mechanical model.

It should be noticed that $F(t)$ is always positive (attractive force). It is why two electromagnets are necessary to produce any desired force along one direction (referred as $F_{l_i} - F_{l_{i+1}}$ or $F_{r_i} - F_{r_{i+1}}$ in the mechanical model)

2.5-Electrical model

This last part deals with coil feeding. We assume here that the control variable is the voltage at the ends of the coils. In that case, each coil is modelled through the following equation :

$$u(t) = R.i(t) + L.\frac{di(t)}{dt} + i(t).\frac{dL}{dt} \qquad (9)$$

since L is likely to vary according to air gap evolution. Other control of energy sources could be used, like pulse width modulation for instance. Since it has no influence on the identification procedure, these aspects are not developed here.

3-SIMPLIFIED CONTROL SCHEME

3.1-Model simplification

In this section we consider the case where the shaft is not rotating. Consequently, there is no gyroscopic effects, and the system can be described by two models describing the displacements of the shaft into two orthogonal planes which intersect along the revolution axis of the shaft. Since the gravity force has to be compensated by means of electromagnetic forces, it is enough to consider the upper electromagnets, and the linearized model for this equilibrium.

This model has two inputs and two outputs. The passive thrust can be considered as simple springiness with negative stiffness. Notations are given in figure 3.

The dynamical equations of the new system are given as follows in a matrix form :

$$\begin{bmatrix} \dot{x} \\ \ddot{x} \\ \dot{\theta} \\ \ddot{\theta} \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ \frac{-K_r}{m} & 0 & \frac{-K_r.l_a}{m} & 0 \\ 0 & 0 & 0 & 1 \\ \frac{-K_r.l_a}{J_y} & 0 & \frac{-K_r.l_a^2}{J_y} & 0 \end{bmatrix} . \begin{bmatrix} x \\ \dot{x} \\ \theta \\ \dot{\theta} \end{bmatrix} + \begin{bmatrix} 0 & 0 & 0 \\ \frac{1}{m} & \frac{1}{m} & \frac{-1}{m} \\ 0 & 0 & 0 \\ \frac{l_1}{m} & \frac{-l_2}{m} & 0 \end{bmatrix} . \begin{bmatrix} F_1 \\ F_2 \\ P_y \end{bmatrix}$$

$$\begin{bmatrix} m_1 \\ m_2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & d_1 & 0 \\ 1 & 0 & -d_2 & 0 \end{bmatrix} . \begin{bmatrix} x \\ \dot{x} \\ \theta \\ \dot{\theta} \end{bmatrix} \qquad (10), (11)$$

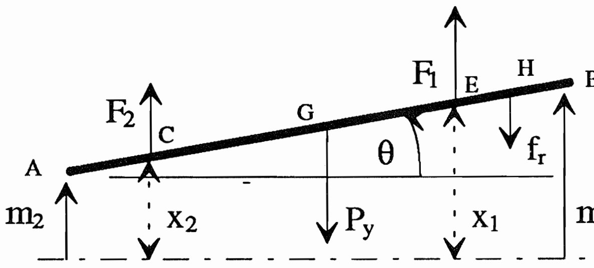

FIGURE 3 : Simplified mechanical model

where :

$$GC = l_1,\ GE = l_2,\ GH = l_a$$
$$GA = d_1,\ GB = d_2$$

3.2-Controllers synthesis

The aim of this section is the presentation of a simplified control structure synthesis, in order to stabilise the position of the shaft and then to be able to get measures that will be exploited in the identification procedure.

The model that is considered here is two inputs, two outputs, four state variables as shown in section 3.1. If one is not familiar with multivariable control schemes, it is possible to derive two simple controllers supposed independent, each one acting in a bearing plane : its role is to determine the force to be produced, according to the evolution of the position of the shaft. To derive one controller, we make the assumption that the other bearing performs well, that is that the shaft is able to rotate around the center of the second bearing : we then face a problem of order two. The behaviour of the electromagnet can be linearized since weight leads to an equilibrium that doesn't coincide with singular positions of the nonlinear model :

Let us denote : $i(t) = i_0 + \Delta\ i(t)$

$\Delta\ i(t) << i_0$ and $w(t) << w_0$, then :

$$F(t) = \frac{D.i^2(t)}{(w_0 - w(t))^2} \cong D.\frac{i_0^2}{w_0^2}\left(1 + 2.\frac{\Delta\ i}{i_0} + 2.\frac{w}{w_0}\right) \qquad (12)$$

Then we obtain two independent linear models of order two for the displacements m_1 *et* m_2 :

$$\ddot{m}_1 = \alpha_1.\Delta\ i_1 + \beta_1.m_1 \qquad (13)$$
$$\ddot{m}_2 = \alpha_2.\Delta\ i_2 + \beta_2.m_2 \qquad (14)$$

where : α_k, β_k resume dependence upon characteristics of electromagnets $(D,\ w_0,\ i_0)$ and mechanical model $(d_1,\ d_2,\ l_1,\ l_2,\ l_a,\ J_y,\ m,\ K_r)$.

It is then easy to derive a continuous controller of the form : $C(s) = \frac{a_0 + a_1.s}{1 + b_1.s}$ that stabilises the simplified problem. This controller is then discretized according to the chosen sampling period using an approximation like Tustin method for instance. Simulation experiments, considering the whole system in the plane, with the two simplified controllers, permit to select the appropriate dynamics which provide with a good robustness with respect to badly known parameters.

4-IDENTIFICATION PROCEDURE

Fortunately for this kind of process, it is possible to determine in advance with good accuracy some of the physical parameters appearing in the model : it is the case for the mass m and inertia J_y, various distances l_1, l_2, d_1, d_2, l_a. However, other parameters remain difficult to measure without experiment in situ : it is the case for the stiffness of the passive thrust, with involves permanent magnet with eventually anisotropic effects, and for characteristics of electromagnetic behaviour of electromagnets. For instance the nominal gap w_0 depends on the length of the magnetic circuit in the air, which is easily measured, but also in the iron, through the stator and the rotor. Even if these two portions of circuit are weighted by relative permeability, it can be shown that their contributions become comparable, since length of air gap is much smaller than length in iron.

The objective is to determine a fairly accurate value of these equivalent nominal gaps in order to synthesize more performing control laws, that don't assume linearization around a working point.

To sum up, considering the dynamical model presented in section 3, four parameters have to be identified. Due to the nonlinear feature of this model, it worries to envisage a direct identification. It is why we attempt to evaluate this parameters through static measures.

Since the system is stabilized by means of simplified controllers, it is possible to consider several equilibrium points, either by modifying the reference position feeding controllers, or by adding external constant signal to control variables.
Once the equilibrium is obtained, since the shaft is almost horizontal, the forces that are produced by electromagnets have to compensate the effects of the weight. So each equilibrium is characterized by two equations :

$$\delta . D_1 + \varepsilon . D_2 + \gamma . K_r = P_y \quad (15)$$
$$\mu . D_1 + \nu . D_2 + \lambda . K_r = 0 \quad (16)$$

where : $\delta, \varepsilon, \gamma$ are related to measures (i, m_1, m_2) by means of known parameters (l_a, d_1, d_2) and estimated gaps,
and : $\mu = l_1 . \delta$, $\nu = l_2 . \varepsilon$, $\lambda = l_a . \gamma$

We use the following procedure to determine the parameters :
-Choose reasonable values for nominal gaps (these can be chosen close to the nominal air gap for instance)
-Solve the identification problem with respect to D_1, D_2 *and* K_r. Since, it involves linear relations with respect to parameters, it is easy to search a solution in the least square sense : then compute the value of the criterion.
-Improve the quality of the result by searching the best value of nominal gap. Since it is a two dimensional problem with a fairly good initial solution, it is easy to converge towards the optimal solution without using sophisticated gradient procedures.

5-EXPERIMENTAL RESULTS

The procedure has been tested on a pilot process of the laboratory. Once the simplified controllers have been determined and implemented in a computer, several campaigns of measures have been carried out. It is desired to get information in the largest domain of position of the shaft. However, controllers that are used have been derived by means of several assumptions : the simplicity of calculations is paid by poor performances when one goes away the nominal position, and by the difficulties to master the value of the equilibrium that is effectively reached, because of the multivariable feature of the whole system. Figure 4 shows the set of positions effectively measured : abscissa : m_1 ; ordinate m_2 (the unit is volt).

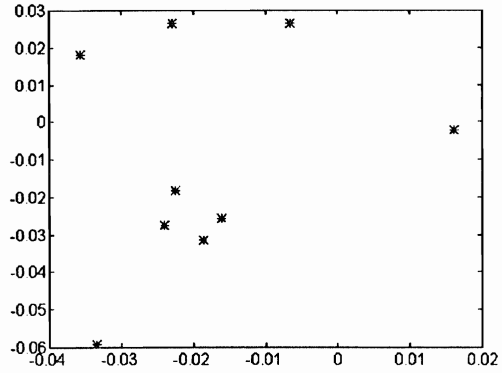

FIGURE 4 : Measures domain

Once measures are got, one has to examine the pertinence of these values. For example, on figure 5 which shows dependence of position upon control variable, it can be seen that one point is "outside" the good area, although this area is difficult to determine in advance : the wrong points are removed from the set before going further into the identification procedure.

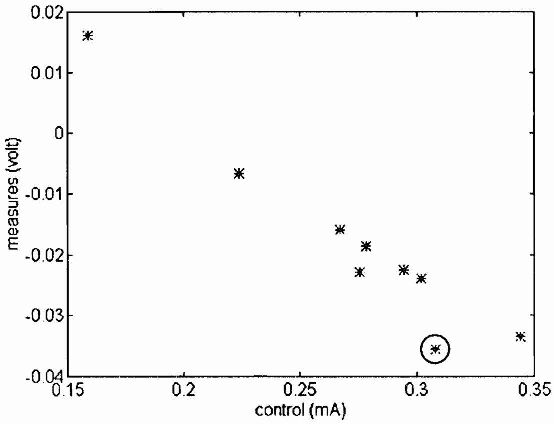

FIGURE 5 : Pertinence of measures

Figure 6 shows the evolution of the mean square error with respect to nominal gaps by successive approximations. The optimal solution is easily found ; mean square error is the ordinate and abscissa is the nominal gap of first electromagnet. Different curves correspond to different values of nominal gap of second electromagnet : decreasing values of w_{02} from left to right.

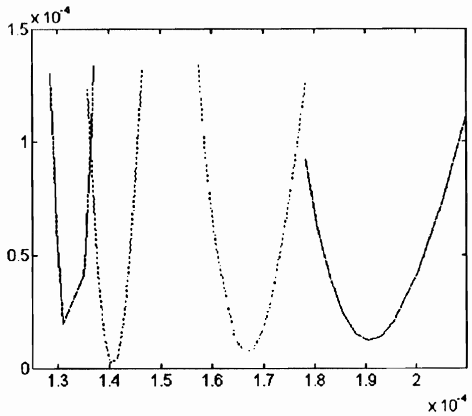

FIGURE 6 : Square error evolution with respect of nominal gaps

Several remarks can be made :

-by construction, there is symmetry between orthogonal planes : so the same values should be found when dealing with the second system : this is true for gap, but for electromagnets (D_1 and D_2) and stiffness, there could be some deviations, due to adjustments of digital and analog functions involved in measurements, and to anisotropy of passive thrust.

-this procedure allows the identification of the behaviour of one of both electromagnets involved in one direction in a bearing. Let us precise that there is no polarization current, so only one electromagnet is working at one time along one direction. The effect of weight allows the identification of the upper electromagnets. When it is not possible to make 180° rotation of the stator, thus exchanging the roles of upper and lower electromagnets, the following procedure should be used to identify lower electromagnets :

-feed lower coils with known currents and make the measures as in the previous procedure. Repeat for various values of these currents.

-since the behaviour of the upper electromagnets is known, it is possible to deduce the forces created by known electromagnets.

-then apply the same procedure to identify the lower electromagnets.

6-CONCLUSION

Direct identification of non linear features involved in a dynamical model is a very difficult problem. Furthermore an unstable system requires a stabilizing control scheme before an identification procedure is carried out.

This paper gives a procedure that allows the identification of several parameters appearing in the model of the shaft which levitates by means of electromagnetic forces. The parameters under consideration are those that cannot be determined by physical considerations. It requires the synthesis of robust simple controllers that stabilise the system despite of the bad knowledge of some parameters of the model. Then identification is carried out considering some particular situations that are easily analyzed. Determination of parameters is based on a good knowledge of their influence on the whole behaviour.

BIBLIOGRAPHY

1 Ljung L., "System identification : Theory for the user". Prentice-Hall, Inc., Englewood Cliffs, New jersey. 1987.

2 Mouille P., "Commande non linéaire de paliers magnétiques" Thèse de doctorat 3ème cycle, Université de Savoie, 1994.

3 Ljung L., Söderström T., "Theory and Practice of Recursive Identification". MIT press., Cambridge, Massachusetts. 1983.

4 Genta G., Delprete C., Tonoli A., Rava E., Mazzocchetti L., "Analytical and experimental investigation of a magnetic radial passive damper". Proc of the Third International Symposium on Magnetic Bearings. pp 255-264. July 1992. Virginia.