# Dynamic Bifurcations of the Active Magnetik Bearing–Rotor System

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Abstract:The Influence of nonlinear factors on the active magnetic bearing–rotor system (AMBRS) with closed loop controller can't be derived from the classical linear control theories. It was analysed for ther first time by the modern analytical methods used in nonlinear vibration. The Hopf bifureations of AMBRS at autonomous and no nautonomous cases were researched by the Normal Form theorem and the improved averaging method respectively. Results show that the non linear factor of magnetism is one of the important factors cauaing the self–excited vibration of AMBRS.

Key words: MBRS, Hopf bifuracation, Normal Formal Form theorem

## 1. INTRODUCTION

Magnetic suspension is a new and high–performance mechano– electronic technique which uses controllable magnetic force to levitate workpiecees(rotor,flatbed,etc.) without any contact. With no friction, no wear and high precision, low noise. It can be used even in many adverse circumstances such as high or super–low temperatures, vacuum, ra diation environment.All these special advantages have opened up vast vistas for its application. Researchers in many countries are working efficiently to take the lead in this competition that may bring about a revolution in our modern industry and the future society.

The successful application of magnetic suspension in the superspeed train and enormous profits in industries have greatly encouraged the research in this field. International Magnetic Bearing Symposiums were held once two years from 1988 in order to facilitate international interchange and coopearation. Meanwhile Magnetic Bearing Technique Conferences were held twice in America. Judging from the papers publi shed, we can see that research is focused on key parts of the magnetic bearing, namely, sensors and magnetic executors, controllers in particu lar. The digital controllers are becoming much more promising onse. This paper deals mainly with the five– axis Active Magnetic Bearing rotor system (AMB)(Fig.1). As its radial bearing is of the popular eight–polar construction, the sestem chosen is representative. The nonlinearity of the magnetic force can make the dynamic model more precise. With this in mind we have investigated the influence of the parameters on its nonlinear vibration under the control of the P. D. controller. The results are conducive to the designing of both analog and digital magne tic bearings.

## 2. THE DYNAMIC EQUATIONS OF THE SYSTEM

### 2.1 The formula of the magnetic force of AMB

The formula of the magnetic force of the axial magnetic bearing can be given as follows:

$$F = K_F(G_0 + C_{10}u + C_{01}i + C_{20}u^2 + C_{11}ui + C_{02}i^2 + C_{30}u^3 + C_{21}u^2i + C_{12}ui^2 + C_{03}i^3) \quad (1)$$

Where Kr is the dimensionless coefficient of the magnetic force , u is the dimensionless displacement, $u = \beta u^* / g_0$, i is the dimensionless control electric current, $i = i_{CX} / I_0$, $g_0$ is the interspace of the magnetic field when the axis of the rotor is stable.

Without considering the effects of the magneticflux–leakage and the magnetic–coupling, the radial magnetic force of the eight–polar AMB can be expressed by equation (1).

### 2.2 The dynamic equations of the five–axis–controlled rotor

Neglecting the dynamic disequilibrium, gyroscopic effect and radial coupling, we obtain the dynamic equation of the rotor which is suspended wholly by magnetic force as follows:

$$m\ddot{u} = F_u - G_u + F_e \qquad (2)$$

Where m:the mass of the rotor

u:the displacement component of the rotor at the suspended point. In equ. (2),it can represent X,Y or Z direction.

$F_u$: the suspending force of the AMB

$F_v$: the perturbation force caused by the unequal mass

$G_u$: the static load of the rotor in u direction

### 2.3 The equation of the field–excitation electrocircuit and the output voltage of the controller (Fig.2)

Assuming that the controller, sensor, amplifier and field– excitation electrocircuit have the same construction, we have the equation of the field–excitation electrocircuit:

$$V_{CX} = Ri_{CX} - \frac{d\psi_1}{dt} + \frac{d\psi_2}{dt} \qquad (3)$$

$$\begin{cases} \psi 1 = \dfrac{\mu_0 N^2 A}{2(g_0 + \beta u^*)}(I_0 - i_{CX}) \\ \psi 2 = \dfrac{\mu_0 N^2 A}{2(g_0 - \beta u^*)}(I_0 I_r + i_{CX}) \end{cases} \qquad (4)$$

The output voltage of the P.D. controller is

$$V_{CX} = \lambda K_p(u^* + Kd\frac{du^*}{dt}) \qquad (5)$$

Select the appropriate value for Io,Ir, then $KIG_0 = G_U$

### 2.4 The dynamic equation of the system

Having simplified equs.(1)–(5), we obtain the dynamic equation of the system

$$\dot{x} = Ax + f(\Omega\tau) + F(x) \qquad (6)$$

Where

$$x = (u\frac{du}{d\tau}\frac{di}{d\tau})^T = (x_1 x_2 x_3)^T$$

$$A = \begin{vmatrix} 0 & 1 & 0 \\ C_{10} & 0 & C_{01} \\ a_2 & a_3 & a_4 \end{vmatrix}$$

$$f(\Omega\tau) = \begin{vmatrix} 0 \\ \wedge sin(\Omega\tau) \\ 0 \end{vmatrix}$$

$$F(x) = \begin{vmatrix} 0 \\ (C_{20}x_1^2 + C_{11}x_1x_3 + C_{02}x_3^2 + C_{30}x_1^3 \\ + C_{21}x_1^2x_3 + C_{12}x_1x_3^2 + C_{03}x_3^3) \\ b_0x_1x_2 - a_2x_1^3 + 2x_1x_2x_3 \\ - a_4x_1^2x_2 + a_3x_1^2x_3 \end{vmatrix}$$

Take the accommodation coefficients of the control electrocircuit as the bifurcation parameters.

## 3. THE NONLINEAR VIBRATION OF THE SYSTEM

### 3.1 The autonomous case

The system can be regarded as autonomous when we investigate its axial activity or the radial activity while the center of the rotor is stable:

$$\dot{x} = Ax + F(x) \qquad (6.1)$$

when $\lambda$ is at the critical value, the parameters of the system satisfy

$$a_2 + a_3 a_4 = 0 \qquad (7)$$

The eigenvalues of matrix A are $\pm\, \omega_i a_4 (a_4 < 0)$, physical conditionis is naturally satisfied and $\omega$ satisfies $\omega^2 = -(c_{10} + a_3 a_4)$

If we denote the critical values of the parameters when $\lambda = \lambda_0$by signs in the preceding equations,and take perturbation $\lambda = \lambda_0 + \mu$, we have $A(\mu) = A_0 + A_1\mu$

where

$$A_0 = \begin{vmatrix} 0 & 1 & 0 \\ C_{10} & 0 & C_{01} \\ -a_3 a_4 & a_3 & a_4 \end{vmatrix}$$

$$A_1 = \begin{vmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ -k_p a_4 & -k_d a_4 & 0 \end{vmatrix}$$

Substituting transition x=T * T / 0 y into equ.(6.1),we have:

$$\dot{y} = J_0 y + J_1 \mu y + T_0^{-1} F(T_0 y) \qquad (8)$$

where

$$C = \frac{a_4(a_4 k_p - k_D \omega^2)}{2det(T_0)},$$

$$d = -\frac{a_4 \omega(k_p + a_4 k_D)}{2det(T_0)},$$

$$e = \frac{a_4 \omega(k_p + a_4 k_D)}{2det(T_0)},$$

$$J_0 = \begin{vmatrix} 0 & -\omega & 0 \\ \omega & 0 & 0 \\ 0 & 0 & a_4 \end{vmatrix}$$

$$J_1 = \begin{vmatrix} d & -c & 0 \\ c & d & 0 \\ 0 & 0 & e \end{vmatrix}$$

$$T_0 = \begin{vmatrix} 1 & 0 & 1 \\ 0 & -\omega & a_4 \\ -\frac{c_{10} + \omega^2}{c_{01}} & 0 & \frac{a_4^2 - c_{10}}{c_{01}} \end{vmatrix}$$

Using the Center Mainfold Theorem, we obtain the center mainfold of the system(8):

$$y_3 = H_{20} y_1^2 + H_{11} y_1 y_2 + H_{02} y_2^2 \qquad (9)$$

Substituting equ.(9) into the first two of equ.(8) , we obtain the following equations of the center mainfold:

$$\begin{cases} \dot{y}_1 = d\mu y_1 - (\omega + c\mu) y_2 + a_{20} y_1^2 + a_{11} y_1 y_2 \\ \quad + a_{02} y_2^2 + a_{30} y_1^3 + a_{21} y_1^2 y_2 + a_{12} y_1 y_2^2 + a_{03} y_2^3 \\ \dot{y}_2 = (\omega + c\mu) y_1 + d\mu y^2 + b_{20} y_1^2 + b_{11} y_1 y_2 \\ \quad + b_{02} y_2^2 + b_{30} y_1^3 + b_{21} y_1^2 y_2 + b_{12} y_1 y_2^2 + b_{03} y_2^3 \end{cases} \qquad (10)$$

The pioncare Birkhopf Normal Form of the system is

$$\begin{cases} \dot{r} = r(d\mu + ar^2) + h.o.t \\ \dot{\theta} = \omega + c\mu + br^2 + h.o.t \end{cases} \qquad (11)$$

It can be easily seen that when du / a>0 , there will be the limit cycle in the system;when a>0 the limit cycle is unstable, otherwise it is stable, and there will be self–excited vibration in the system.

### 3.2 The nonautonomous case

If we take the unequal mass of the rotor into consideration, the radial motion of the rotor is nonautonomous. Then let us study its resonance and disresonance, respectively.

a) Disresonance

Rewrite the equations of the system as $(u = \varepsilon\eta)$

$$\dot{x} = A_0 x + f(\Omega\tau) + \varepsilon[A_1 \eta x + F(x)] \qquad (6.2)$$

From the othogonality of the fundamental solutions of a equation and its adjoint equations, we can derive that G(t) satisfies normalizing condition $G^*(\tau)G(\tau) = E$ ( E is 3 × 3 identity matrix )

We also know that $x^*(\tau) = \frac{\wedge}{\omega^2 - \Omega^2}(sin\Omega\tau \Omega cos\Omega\tau a_3 sin\Omega\tau)^T$ is the particular solution of equ. $X = A_0 X + j(\Omega\tau)$ which is the lineari zing equation of equ.(6.2). Then take transition

$$X = G(\tau)b + X^* \qquad (12)$$

Substituting it into equ.(6.2), we obtain

$$b = \varepsilon a^*(\tau)[A\eta x + F(G(\tau)b + x^*)]$$
$$= \varepsilon H(b, \omega\tau, \Omega\tau) \qquad (13)$$

$$Let \begin{cases} b_1 = rcos\theta \\ b_2 = rsin\theta \\ b_3 = B_3 e^{-a_4\tau} \end{cases} \qquad (14)$$

$$then \begin{cases} \dot{r} = \varepsilon[H_1 cos\theta + H_2 sin\theta] \\ \dot{\theta} = \frac{\varepsilon}{r}[-H_1 sin\theta + H_2 cos\theta] \\ \dot{B}_3 = a_4 B_3 + \varepsilon\tilde{H}_3 \end{cases}$$

Assuming the series solution of this equation is

$$B_3 = B_{32} + B_{33} + \cdots \qquad (15)$$

Where $B^{3j}$(j=2,3,…) is the j–th homogeneous multinomal of rcosωτ, rsinωτ, cosΩτ, sinΩτ

Substituting it back into the first two of the preceding equations, we get the dimension–reduced standard equations of equations(6.2):

$$\begin{cases}\dot r = \varepsilon\varphi = \varepsilon[\varphi_0 + \varepsilon\varphi_1 + \cdots] \\ \dot\theta = \varepsilon\varphi^* = \varepsilon[\varphi_0^* + \varepsilon\varphi_1^* + \cdots]\end{cases} \quad (16)$$

Where

$$\varphi_0 = \frac{ra_4c_{01}}{2(a_4^2+\omega^2)}\{(K_P + a_4K_D)\eta + (a_3 - 0.5a_5)(r^2 + \frac{a_3\wedge^2}{\omega^2+\Omega^2})\}$$

$$\varphi_0^* = 0.5\{\frac{1}{\omega}(c_{30} + a_3c_{21} + a_3^2c_{12})(0.75r^2 - \frac{1.5\wedge^2}{\omega^2-\Omega^2} + \frac{c_{01}a_4^2k_p\eta}{(a_4^2+\omega^2)\omega} - \frac{c_{01}\omega}{a_4^2+\omega^2}[a_4k_D\eta + (a_3 - 0.5a_5)(r^2 + \frac{\omega\wedge^2}{\omega^2-\Omega^2})]\}$$

Using the averaging method, we can easily know that the linear approximate stationary solution satisfies

$$\frac{0.5y_0c_{01}a_4}{a_4^2+\omega^2}[(k_p + a_4k_D)\eta + \frac{\wedge^2a_3(a_3-0.5a_5)}{\omega^2-\Omega^2} + (a_3 - 0.5a_5)y_0^2] = 0 \quad (17)$$

This is the Hopf bifurcation equation of this system. Obviously, when

$$\frac{(k_p + a_4k_D)\eta}{a_3 - 0.5a_5} + \frac{a_3\wedge^2}{\omega^2-\Omega^2} < 0,$$

the limit cycle exists, and when $a_3 - 0.5a_5$ 0, the limit cycle is steady and self–excited vibration appears in the system. Otherwise the cycle is unsteady.

$(\omega - \Omega = \varepsilon\delta, \delta < 0(1))$

b) Main resonance

Rewrite the equation of the system as

$$\dot x = A_0x + \varepsilon[A_1\eta x + f(\Omega\tau) + F(x)] \quad (6.3)$$

Take transition $G = a(\tau)$ b, and the equations(6.2b) can be transformed into

$$\dot b = \varepsilon G^*(\tau)[A_1\eta G(\tau)b + f(\Omega\tau) + F(G(\tau)b)] = \varepsilon H \quad (18)$$

Then substituting equ.(13), we have

$$\begin{cases}\dot r = \varepsilon(H_1\cos\theta + H_2\sin\theta) \\ \dot\theta = -(\omega-\Omega) + \frac{\varepsilon}{r}(-H_1\sin\theta + H_2\cos\theta) \\ \dot B_3 = a_4b_3 + \varepsilon B(r,\theta,\Omega\tau)\end{cases} \quad (19)$$

Substituting it back into equ.(19), we have the dimentionreduced standard equations

$$\begin{cases} r = \varepsilon\varphi = \varepsilon[\varphi_0 + \varepsilon\varphi_1 + \cdots] \\ \theta = \varepsilon\varphi^* = -(\omega-\Omega) + \varepsilon[\varphi_0^* + \varepsilon\varphi_1^* + \cdots]\end{cases} \quad (20)$$

Then using the averaging method, we obtain the response equations of the resonant system:

$$\begin{cases}\frac{ya_4c_{01}}{a_4^2+\omega^2}[(K_P + a_4k_D)\eta + y^2(a_3 - 0.5a_5)] + \frac{\wedge\cos y}{\omega} = 0 \\ \Omega - \omega + \frac{\wedge\sin y}{2\omega y} + 0.5\{-\frac{a_4c_{31}\eta}{a_4^2+\omega^2}(\omega k_D - \frac{a_4k_p}{\omega}) + y^2[-\frac{c_{01}\omega}{a_4^2+\omega^2}(a_3 - 0.5a_5) + 0.75\omega(c_{30} + a_3c_{31} + a_3^2c_{12})]\} = 0\end{cases}$$

(21)

## 4 CONCLUSIONS (Fig.1–Fig.4)

(1) The nonlinearity of the magnetic force is also one of the important causes that stimulate the self–excited vibration in the AMB.

(2) The rules of conditional stability: In the case of nonresonance, the system can work normally if any one of these two conditions is satisfied:

a) The amplitude of the unstable limit cycle is greater than g0.

b) The amplitude of the stable limit cycle is smaller than go.

(3) The leapperformance will appear in the system in the condition of resonance. But the resonance peak can be eliminated by changing proportionally the parameters of the system.

(4) The differentiate parameters of the control circuit have great effect on the function of the system.

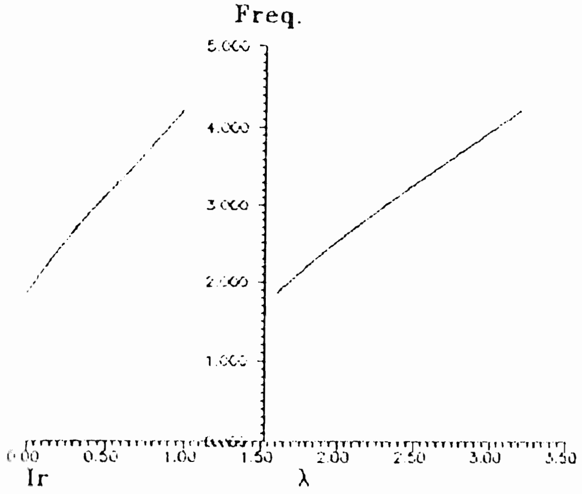


Fig.1 Ir's and λ's effects on Freq

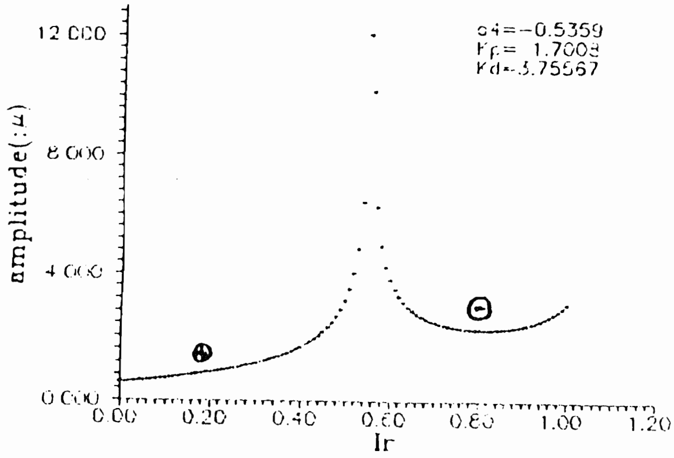


Fig.2 Ir's effects on amplicude

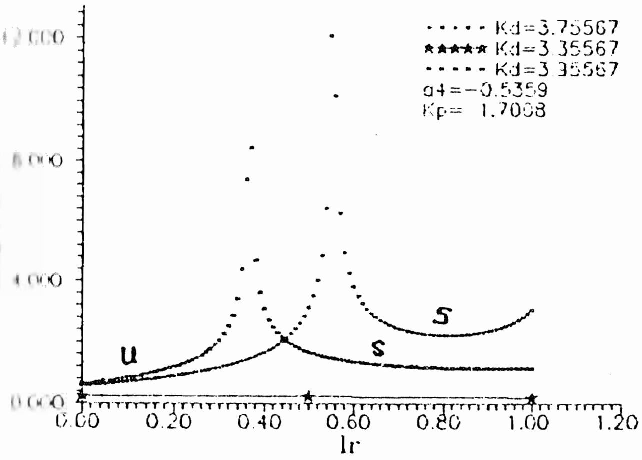


Fig.3 Ir's effects on amplitude

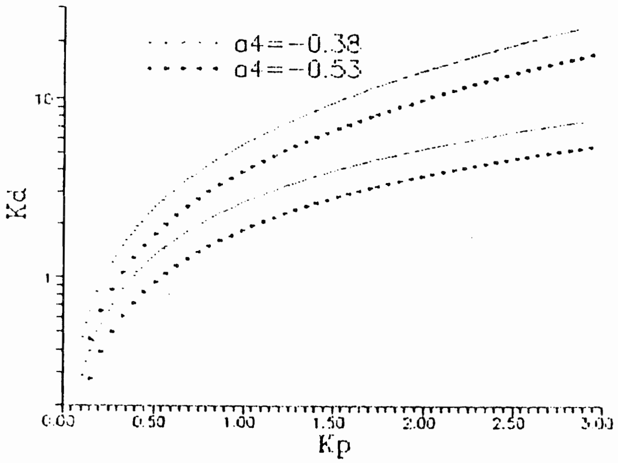


Fig.4 Kp's effects on $K_D$

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