MAGNETIC BEARINGS AND NON-STATIONARY DYNAMICS OF ROTORS

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ABSTRACT

This contribution is an attempt to spur interest in the application of neural networks in actively supported rotors.

After some introductory remarks non-stationary rotordynamics is reviewed in short, restricted to the Jeffcott Rotor (section 2). Section 3 than gives an overview of a simple neural network trained by the backpropagation procedure. The latter is then utilized in section 4 to avoid suspended stays of the non-stationary rotor. Section 5 outlines future improvements, realizations and other possible applications. A short summary concludes the contribution.

1 INTRODUCTION

Rotatory Machinery is supposed to accelerate and deaccelerate through critical speeds with respect to bending. The acceleration behavior of the rotor then depends on the applied drive torque. A description of this behavior is given by a known set of nonlinear differential equations for the Jeffcott Rotor (elastic shaft with centered disc), being the most simple rotor.

By using modal analysis, the partial differential equations valid for a rotor with a continuous mass distribution can be reduced to a set of non-linear ordinary differential equations. These equations, each of which applies to one eigenmode, have the same structure as those for the Jeffcott Rotor. For this we restrict considerations henceforth to the Jeffcott Rotor and refer to [1] for a more detailed discussion on mass distributed rotors.

Active Magnetic Bearings (AMB) supporting an accelerating rotor can be utilized to overcome critical behavior specifically so called suspended stays, which are due to a lack of drive torque. Since the occurrence of a suspended stay depends also on the qualities of the rotor system which are greatly determined by AMBs, these have the potential to eliminate stalling of rotors. Inhere we use a neural network approach to adapt the qualities of the rotor system in order to cope with suspended stays.

2 NON-STATIONARY ROTOR

The governing equations for the Jeffcott Rotor (Figure 1) can be found in most textbooks on Dynamics of Rotors and are normalized inhere for further reference. Equations (1) depict the complete set without any assumptions like a given acceleration or neglecting the non-linear coupling between the bending degrees of freedom and those of torsion. The latter leading to the well-known ordinary differential equations for the Jeffcott Rotor.

$$\begin{aligned} r_c'' + 2Dr_c' + r_c &= \exp(i\varphi) \\ \varphi'' - \tfrac{1}{2}\kappa^2(r_c\exp(i\varphi) + \bar{r}_c\exp(-i\varphi)) &= T \end{aligned}$$

$$r_c \in \mathcal{C}, i = \sqrt{-1} \quad (1)$$

, where

$$r_c(\tau) = \frac{z_c(\tau)}{\varepsilon} + i\frac{y_c(\tau)}{\varepsilon}$$

$$T(\tau) = \frac{M}{ck^2}$$

$$D = \frac{d}{2\sqrt{cm}}$$

$$\kappa = \frac{\varepsilon}{k}$$

$$\tau = \sqrt{\frac{c}{m}}t$$

r_c indicates the displacement of the disc's COM, φ is the angle turned through by the disc and T is the drive torque resulting from the applied torque M, all normalized according to the above. Parameters D and κ constitute of the stiffness c and structural damping d of the rotor system, mass m and radius of gyration k of the disc. The prime $'$ denotes the derivative with respect to the normalized time τ.

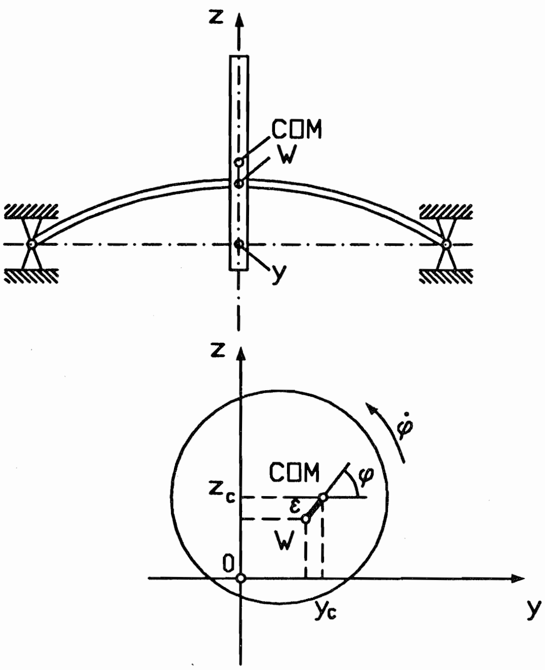

FIGURE 1 : Non-Stationary Jeffcott Rotor

These and further equations have been studied by several authors namely for the sake of solution methods and stability analysis. Although straightforward solving of eq. (1) is unknown, e.g. [1] and [2] describe a solution by successive approximation as well as calculating the minimum required torque without damping which will avoid a suspended stay. Acceleration behavior of the rotor is sensitive to parameter changes as can be recognized in Figure 2 which shows speed φ' vs. time τ. With $D = 0.165$ a suspended stay occurs while switching to $D = 0.17$ the rotor accelerates further. Figure 3 depicts the displacement $|r_c|$ vs. φ'. With the appropriate D displacement diminishes as one expects for a flexible rotor operating beyond the first bending critical speed.

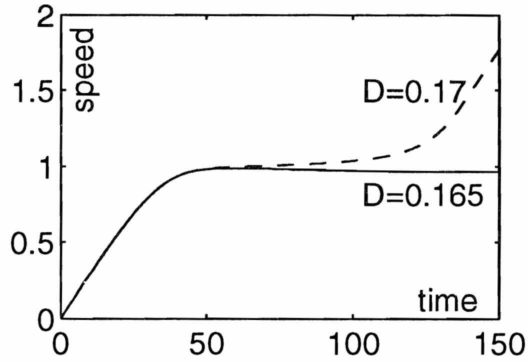

FIGURE 2 : φ' vs. τ with $T = 0.03$, $\kappa = 0.1414$

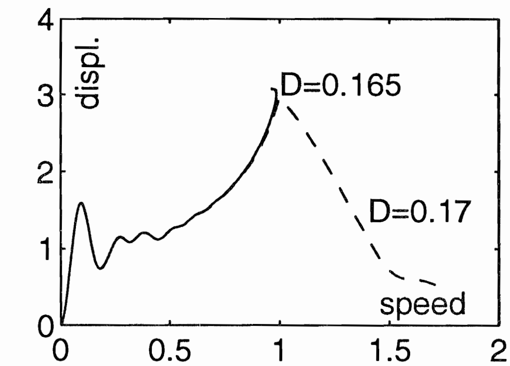

FIGURE 3 : $|r_c|$ vs. φ' with $T = 0.03$, $\kappa = 0.1414$

3 NEURAL COMPUTATION

The neural network approach stems from the field of biology respectively neurosciences and was originally conceived as a model for brain tissue. Nevertheless the results from this can be applied to general dynamical systems as shown by [3].

We like to show one of the many possible applications of neural networks in the field of dynamics of rotors specifically when active supports are utilized. For this reason a brief outline of topics from neural networks used in the next section is stated below. For further reference e.g. [4] can be used.

Neural networks estimate functions from sample data. The basic problem definition for a neural network reformulated from [5] is this:

> Store a set of patterns, consisting of input and output values, in such a way that when presented with a new pattern the network responds by producing whichever one of the stored patterns most closely resembles the new pattern

The structure of a neural network is depicted by Figure 4.

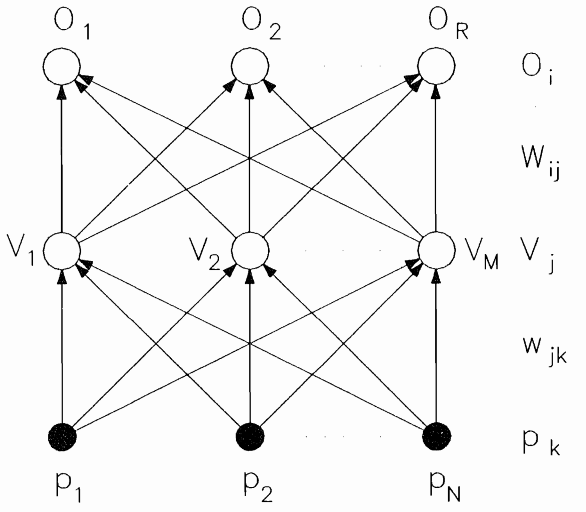

FIGURE 4 : Structure of a network

When a pattern μ is applied at the input, that is p_k^μ, the hidden unit receives a net input[1]

$$h_j^\mu = \sum_k w_{jk} p_k^\mu \text{ and produces } V_j^\mu = g_1(h_j^\mu).$$

$$\text{Output unit } i \text{ thus receives } h_i^\mu = \sum_j W_{ij} V_j^\mu.$$

The final output is then

$$O_i^\mu = g_2(h_i^\mu) = g_2(\sum_j W_{ij} g_1(\sum_k w_{jk} p_k^\mu)). \quad (2)$$

O_i^μ in general deviates from the desired output pattern or target T_i^μ. By adjusting weights w_{jk} and W_{ij} O_i^μ can be shifted to T_i^μ, which can be understood as learning. Updating the weights is done by the so called backpropagation algorithm, which relies on gradient descent rules to minimize the sum-squared error[2]

$$E\,[w_{jk}, W_{ij}] = \frac{1}{2}\sum_{\mu,i}(T_i^\mu - O_i^\mu)^2.$$

The backpropagation rule gives[3]

$$\Delta W_{ij} = -\eta \frac{\partial E}{\partial W_{ij}} = \eta \sum_\mu \delta_i^\mu V_j^\mu \quad (3)$$

$$\text{where } \delta_i^\mu = g_2'(h_i^\mu)[T_i^\mu - O_i^\mu]$$

and

$$\Delta w_{jk} = -\eta \frac{\partial E}{\partial w_{jk}} = \eta \sum_\mu \delta_j^\mu p_k^\mu \quad (4)$$

$$\text{where } \delta_j^\mu = g_1'(h_j^\mu) \sum_i W_{ij} \delta_i^\mu.$$

Once increments ΔW_{ij} and Δw_{jk} are effective, the rule is repeatedly applied until an error goal or a maximum number of cycles has been reached. η serves as a tuning factor to improve convergence and is referred to as learning rate. The activation function g_1 and g_2 must be differentiable and should saturate at both sides. Thus, so called sigmoid type functions like $tanh(.)$ are usually applied.

One of the drawbacks of the backpropagation is its slowness due to the gradient descent. More sophisticated algorithms than gradient descent exist but these induce more complexity. A simpler way for improving effectiveness is the addition of a momentum, that is part of the previous increments are added:

$$\Delta W_{ij}(t+1) = -\eta \frac{\partial E}{\partial W_{ij}} + \alpha \Delta W_{ij}(t) \quad (5)$$

$$\Delta w_{jk}(t+1) = -\eta \frac{\partial E}{\partial w_{jk}} + \alpha \Delta w_{jk}(t) \quad (6)$$

α is the momentum parameter in the range of $(0, 1)$.

4 AVOIDING A SUSPENDED STAY

In section 2 it was shown that a rotor system is sensitive to parameter changes in regard to occurrences of suspended stays. By investigation of the non-stationary equations of the Jeffcott-Rotor, eq. (1), it is obvious that parameter D depends both on the

[1] $g(.)$ denotes a function, typically sigmoid

[2] summed over μ and i

[3] $'$ denotes derivative with respect to W_{ij} and w_{jk}

system stiffness (c) and damping (d). Thus, if a rotor is actively supported by Magnetic Bearings switching c and d respectively D can be used to overcome a suspended stay. Moreover, the ratio of the eccentricity, ε, to the radius of gyration of the disc, κ, greatly influences the acceleration.

In regard to the above we would like to find the smallest D which ensures continuous acceleration. This is equal in meaning to evaluate the function $D_{\text{min}} = f(T, \kappa)$ resulting from the constraint of $\min(\varphi'') = 0^+$. Unfortunately f is analytical intractable. For this reason we use a neural network approach according to section 3 to estimate D_{min}.

The basic approach is then, first to sample a sufficiently large number of values from $f(T, \kappa)$. These samples are obtained by repeatedly simulating equation (1) and increasing D while holding on to T and κ until φ'' becomes just larger than zero during the whole simulation. In this way data triples (D_{min}, T, κ) are determined. With these patterns a network structure according to Figure 4 is trained, where we just have the two inputs T and κ and the single output D_{min}. Thus, the structure in Figure 4 is restricted to $N = 2$ and $R = 1$.

4.1 ESTIMATION WITH CONSTANT κ

In the simplest case κ can be assumed constant, which reduces the network to just one input and output ($N = R = 1$). Figure 5 shows the result of estimating D_{min} with $\kappa = 0.1414$. A network with 10 ($M = 10$) tan-sigmoid neurons with activation function

$$g_1 = \tanh(h_j) + b_1$$

and one linear output neuron

$$g_2 = h_i + b_2$$

is used. Here biases b_1 and b_2 are added, which are also updated with the rules given in equations (3) and (4).

Eleven training sets (D_{min}, T) lead to the output sets depicted in Figure 5. Training takes 59 seconds on a INTEL 486/66-Processor with a final sum-squared error of $E\left[w_{jk}, W_{ij}\right] = 0.0011$. The learning rate η is set to 1.

Clearly, for this rather simple problem even the small utilized network can succeed in reasonable time. This will not be the case if the network grows, which is mandatory to estimate more complex data structures.

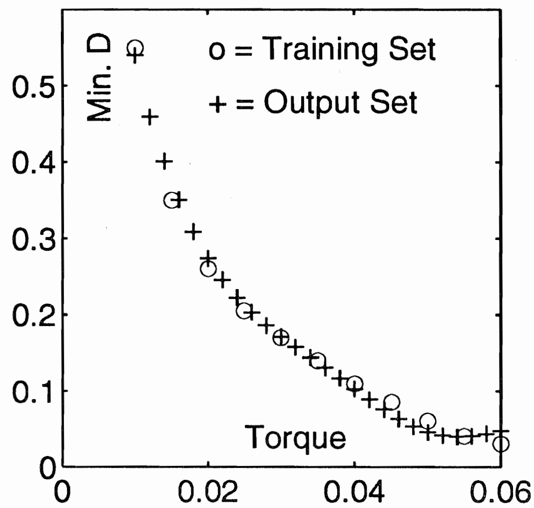

FIGURE 5 : D_{min} vs. T for $\kappa = 0.1414$

4.2 ESTIMATION OF $D_{\text{min}} = f(T, \kappa)$

Estimating $D_{\text{min}} = f(T, \kappa)$ is an extension to the previous section. Now the network has two inputs ($N = 2$). The same network structure as in section 4.1 is applied, but due to the increased number of inputs 50 ($M = 50$) neurons are used. Activation functions g_1 and g_2 are maintained according to the previous section. Training takes 28 minutes on a INTEL 486/66 Processor with the result of estimating the 63 training sets as depicted in Figure 6. Also, the evolution of the sum-squared error vs. the training cycles is shown in Figure 7.

From the latter Figure it can be deduced that training could be stopped much earlier since the error did not decrease substantially after 3000 cycles. After 5000 cycles the error is 0.11. During training the learning rate η is adapted according to the progress of the error after each training cycle. This accelerates learning.

The number of neurons is chosen from a trade-off. Although with increasing number of neurons better network performance can be achieved training time prolongs. On the other side limiting the number of neurons prevents the network from reaching satisfactory error goals.

5 PERSPECTIVES

The previous sections showed the possible application of the neural network approach in regard to adapt system qualities. Although the examples given can be interpreted as mere curve fitting, the advantage of the neural approach becomes obvious when it is applied to a real system. As can be seen from equation (3) the weight changes ΔW_{ij} are calculated solely from V_j^μ and δ_i^μ That means once δ_i^μ is known each ΔW_{ij} can be calculated independently. For Δw_{jk} in equation (4) the same is true. This gives access to parallel computation. For this reason we will implement neural networks on a parallel computer which will control a magnetically supported flexible rotor. The basic outline of the test-bed currently under construction is shown in Figure 8. The utilized net structure is not optimal in that it was not specifically designed for the problem. So we like to design networks which can help to solve problems in regard to actively supported rotors. One of it to improve methods in active balancing of flexible rotors, whereby e.g. [6] developed a method for active balancing which could benefit from a neural approach.

6 SUMMARY

From the example of non-stationary rotor behavior it was shown that neural networks can solve problems in the field of control for rotordynamics. Although specific net design might be preferred, even an ordinary net structure can be applied to avoid a suspended stay. Future work will implement networks on a parallel computer, such that the advantage of neural networks can be exploited.

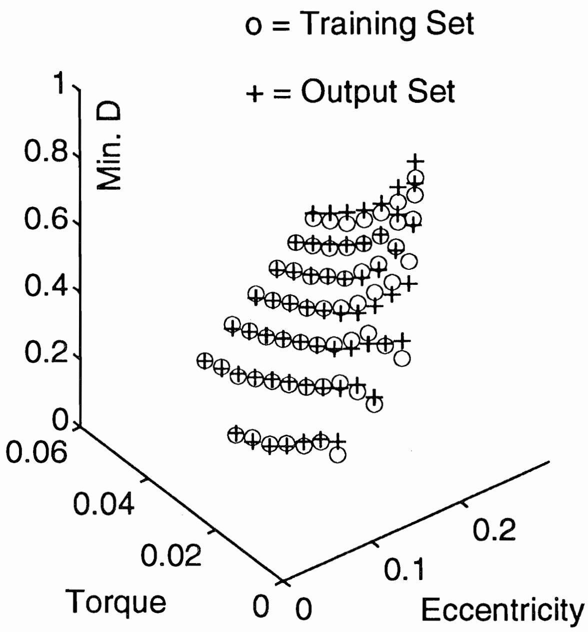

FIGURE 6 : D_{min} vs. T and κ

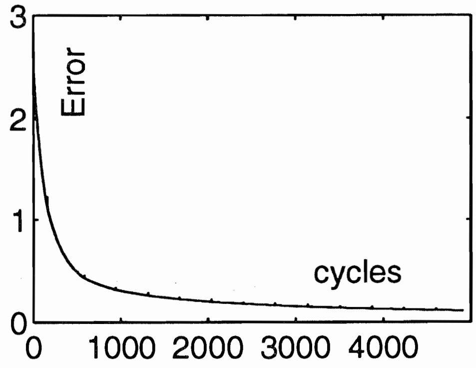

FIGURE 7 : Sum-squared error vs. learning cycles

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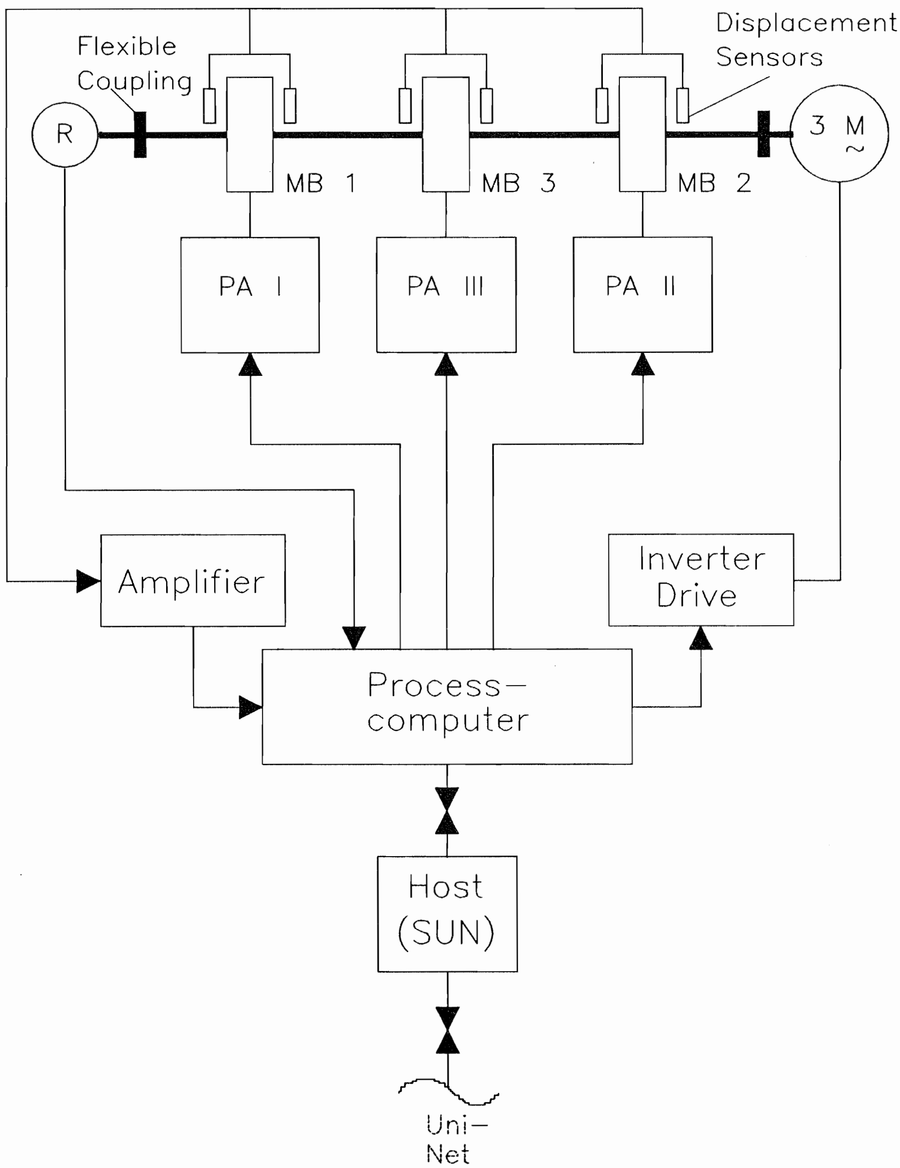

FIGURE 8 : Test-Bed Structure with Parallel Computer