REGULATING THE MAGNETIC GYROSCOPE'S MOTION *

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INTRODUCTION

Basic components of a magnetic gyroscope are known [1] to be:

a) a magnetic field-inducing system or a system of magnetic sources (a magnetic suspender) possessing an axial or a spherical symmetry;

b) a ball-like dynamic symmetric pherite rotor; and

c) an automatic regulator, for varying source fields in a way as to provide for a stable noncontacting magnetic support to a rotor with a fixed centre of mass.

Potentially high precision of such gyroscopes is based upon slightness of departing rotor-affecting moments, though the same reason makes the rotor, upon its spin-up, perform angle motions; these motions being generally specified by initial conditions. For normal operation of a device, it is necessary to obtain a zero value for a nutation angle and a capability of regulated change of the kinetic moment orientation with respect to a device case. To satisfy these conditions, regulating moments, providing needed operating conditions for some finite time, have to be applied to a gyroscope rotor.

To have the regulating moments obtained, it is preferable to employ the available conservative moments being caused by nonideal manufacture of the rotor and to gain a required effect through regulating the suspender field or external sources under some specific law.

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STRUCTURE OF MOMENTS

In order to describe rotor's angular motions around a centre of mass, we use the basic equation in the gyroscopic science,

$$\frac{d}{dt}\vec{K} + [\,\vec{\omega} \times \vec{K}\,] = \vec{M}\,,$$

and also the expressions for projections of kinetic moment $\vec{K}$ onto an axis of ellipsoid of rotor inertia, $(\vec{K} \cdot \vec{x}_i) = I_i\,\omega_i$.

At small departing angles α and β of vector $\vec{K}$ from the suspender symmetry axis OZ_3 $(\vec{K} = K(\beta\vec{z}_1 - \alpha\vec{z}_2 + \vec{z}_3))$, vector's time evolutions are described [2] as

$$K\frac{d\alpha}{dt} = -M_2\,; \quad K\frac{d\beta}{dt} = M_1\,; \quad \frac{dK}{dt} = M_3\,, \quad (1)$$

where M_j stands for projections of moment of external actions onto direction of kinetic moment (M_3) and in the plane (M_1, M_2) perpendicular to this moment. If a position of rotor dynamic axis OX_3 with respect to the kinetic moment $\vec{K}$ is specified through the nutation angle ϑ and through the precession angle ψ, then for angle ϑ we obtain

$$K\frac{d\vartheta}{dt} = -[M_1\cos\psi + M_2\sin\psi] = M_\vartheta\,. \qquad (2)$$

Since the external moments are slight, the motion equations, upon their normalization, allow an averaging of right parts with respect to the precession angle ψ and rotor's selfrotation φ. Upon performing these operations and upon a transition to $r = \beta - j\alpha$ and $M_{\perp} = M_1 + jM_2$ we shall obtain the following:

$$Kr^{\bullet} = \langle M_{\perp} \rangle = m_r\,; \quad K\vartheta^{\bullet} = -\langle M_{\vartheta} \rangle = m_{\vartheta}\,;$$

$$K^{\bullet} = \langle M_3 \rangle = m_k\,, \tag{3}$$

Note that if $M_j = M_j(r, \vartheta, K, \psi, \varphi)$, then, upon averaging, we have $m_j = m_j(r, \vartheta, K)$. Let us denote M_j^0 and m_j^0 as $M_j^0 = M_j(0, 0, K, \psi, \varphi)$ and $m_j^0 = m_j(0, 0, K)$ and represent Eq. (3) as

$$x_i^{\bullet} = f_i(x_1, x_2, x_3)\,.$$

It is not difficult to see that finite times for changing $x_i^{(1)} \to x_i^{(2)}$ are possible only if, for any point on a 3-dimension trajectory described by these equations, f_i does not transfer to zero. In particular, damping of a rotor nutation needs the condition $m_{\vartheta}(0, 0, K) \neq 0$ to be satisfied. In general, it is possible to write

$$\begin{aligned} M_j &= M_{j0}(r, \vartheta, K) \\ &\quad + \sum_{pq} M_{jpq}(r, \vartheta, K) \cos(\Omega_{pq}\tau + \alpha^o_{jpq})\,, \end{aligned} \tag{4}$$

and, accordingly, we obtain

$$\begin{aligned} M_j^0 &= M_{j0}(0, 0, K) \\ &\quad + \sum_{pq} M_{jpq}(0, 0, K) \cos(\Omega_{pq}\tau + \alpha^o_{jpq})\,, \end{aligned} \tag{5}$$

where $\Omega_{pq} = p\psi^{\bullet} + q\varphi^{\bullet}$.

From it, it follows that if a regulating moment is generated by employing the component M_{jpq}, then the interaction parameters (e.g., rotational frequences of the field or its amplitudes, directions, etc) should be modulated with the frequency Ω_{pq}.

APPLICATION OF MAGNETIC GYROSCOPES

Now we will demonstrate how regulating moments are obtained via interaction between a rotor and a suspender field or an external field, when these interactions are stipulated by rotor's surface that is different from that of an ideal sphere.

If the surface is axially symmetric, then energy of conservative interaction of aspheric rotor with axially symmetric magnetic field can be represented through a sum of Legendre polynomials [3]:

$$W(\Theta) = \sum_n A_n P_n(\cos\Theta = \vec{s}_o \cdot \vec{h}_o)\,, \tag{6}$$

where $\vec{h}_o$ and $\vec{s}_o$ are unit vectors of symmetry axes of the field and of the rotor surface. From here, for a departing moment we may write

$$\begin{aligned} \vec{M} &= \frac{\vec{s}_o \times \vec{h}_o}{\sin\Theta} \frac{dW}{d\Theta} \\ &= -(\vec{s}_o \times \vec{h}_o) \sum_n A_n P_n{}'(\cos\Theta)\,. \end{aligned} \tag{7}$$

Here, harmonic amplitudes A_n are determined both by amplitude of the n-th harmonic in the rotor surface shape and also by a field whose variance will cause a variance of the departing moment.

Let an angle between $\vec{s}_o$ and $\vec{x}_3$ be denoted by χ ($\cos\chi = (\vec{s}_o \cdot \vec{x}_3)$); and orienting $\vec{h}_o$ within the coordinate system OZ_k, associated with the suspender, will be designated by spherical angles δ and ϕ ; and then we shall have:

$$\vec{h}_o = \vec{h}_o[\sin\delta\cos\phi\,, \sin\delta\sin\phi\,, \cos\delta\,]_{OZ}\,.$$

If in (7), in a rotor surface shape only a second harmonic is regarded (e.g., it is stipulated by rotor's ellipsoidality), then for $r = \vartheta = 0$ it is not difficult to derive the below expressions for the associated components of the moment :

$$\begin{aligned} M_r^0 = -jA_2\{ &[P_2(\cos\delta)e^{j\alpha_{11}} \\ &-\frac{1}{2}\sin^2\delta e^{-j(\alpha_{11}-2\phi)}]\sin 2\chi - [P_2(\cos\chi)e^{j\phi} \\ &-\frac{1}{2}\sin^2\chi e^{j(2\alpha_{11}-\phi)}]\sin 2\delta\}; \end{aligned}$$

$$\begin{aligned} M_3^0 = 2A_2 &\sin\chi\sin\delta[\cos\chi\cos\delta\sin(\alpha_{11} - \phi) \\ &+ \frac{1}{2}\sin\chi\sin\delta\sin 2(\alpha_{11} - \phi)\,]\,; \end{aligned}$$

$$\begin{aligned} M_{\vartheta}^0 = -\,A_2[&P_2(\cos\delta)\sin\alpha_{01} \\ &+ \frac{1}{2}\sin^2\delta\sin(\alpha_{21} - 2\phi)]\sin 2\chi \\ &+ [P_2(\cos\chi)\sin(\alpha_{10} - \phi) \\ &+ \frac{1}{2}\sin^2\chi\sin(\alpha_{12} - \phi)]\sin 2\delta\,. \end{aligned} \tag{8}$$

Let us consider some specific cases significant from the point of view of their applications.

1. As an object for regulation, a field of the axially symmetric suspender itself is chosen: $\vec{h} = \vec{z}_3$ (when $\delta = 0$) . In this case, we have

$$M_r^0 = -jA_2 e^{j\alpha_{11}} \sin 2\chi \ ; \quad M_3^0 = 0 \ ;$$

$$M_\vartheta^0 = -A_2 \sin \alpha_{01} \sin 2\chi \ . \tag{9}$$

In the above expressions, the condition $m_r^0 \neq 0$ is satisfied provided the field magnitude or amplitude of moment A_2 is modulated with frequency $\omega = \alpha_{11}{}^\bullet = \psi^\bullet + \varphi^\bullet$:

$$A_2 = a_2[1 + 2\varepsilon_{11} \cos(\alpha_{11} + \phi_{11})] \, . \tag{10}$$

Here, the value of nutation angle ϑ does not vary ($m_\vartheta^0 = 0$). Monotonous variance of this angle can be effected through modulating a suspender field amplitude by the frequency $\omega = \alpha_{01}{}^\bullet = \varphi^\bullet$:

$$A_2 = a_2[1 + 2\varepsilon_{01} \cos(\alpha_{01} + \phi_{01})] \ . \tag{11}$$

Note that in this case no change in kinetic moment orientation occurs.

To concurrently affect both the orientation of $\vec{K}$ and the nutation angle ϑ is possible through modulating coil currents in a way that

$$A_2 = a_2[1 + 2\varepsilon_{01} \cos(\alpha_{01} + \phi_{01}) + 2\varepsilon_{11} \cos(\alpha_{11} + \phi_{11})] \ . \tag{12}$$

2. It happens very often that an axis of rotor surface symmetry coincides with rotor's dynamic axis ($\vec{s}_0 = \vec{x}_3$ or $\chi = 0$) . Here we obtain

$$M_r{}^0 = jA_2 e^{j\phi} \sin 2\delta \ ; \quad M_3{}^0 = 0 \ ;$$

$$M_\vartheta{}^0 = -A_2 \sin(\alpha_{10} - \phi) \sin 2\delta \ . \tag{13}$$

First it is worthy of noting that regulating a gyroscope is possible if only $\vec{h}_0$ is not coincident with the suspender symmetry axis OZ_3, i.e., for $\delta \neq 0$. By specifying $\delta = \delta^*$, it becomes possible to force vector K to depart along some specified direction. To damp a nutation, a field can be regulated in the following ways:

a) by modulating field's magnitude with frequency $\alpha_{10}{}^\bullet = \psi^\bullet$:

$$A_2 = a_2[1 + 2\varepsilon_{10} \cos(\alpha_{10} - \phi + \phi_{10})] \ ,$$

when

$$m_\vartheta^0 = \varepsilon_{10} a_2 \sin \phi_{10} \sin 2\delta \ ; \tag{14}$$

b) by rotating the field at the same frequency around suspender symmetry axis OZ_3, when $\phi = \alpha_{10} + \phi_{10}$, and

$$m_\vartheta^0 = a_2 \sin \phi_{10} \sin 2\delta \ ;$$

and

c) by rotating the field when $\delta = \frac{1}{2}[\alpha_{10} + \phi_{10}]$. In this case, we have

$$m_\vartheta^0 = -\frac{1}{2} a_2 \cos \phi \cos \phi_{10} \ .$$

It is easy to see that for the case *a*) the regulating moment will be drastically less ($\sim \varepsilon$) than that for the other two.

Let us observe in more details the case when gyroscope motion is regulated by algorithm (12). Here the averaged motion equations will be of the form

$$K r^\bullet = -ja_2[\varepsilon_{11} e^{-j\phi_{11}} \sin 2\chi + 2rP_2(\cos\chi)]$$
$$K\vartheta^\bullet = -\varepsilon_{01} a_2 \sin \phi_{01} \sin 2\chi$$
$$K^\bullet = \frac{1}{2}\varepsilon_{11} a_2 [r e^{j\phi_{11}} + \overline{r} e^{-j\phi_{11}}] \sin 2\chi \ , \tag{15}$$

where $\overline{r} = \beta + j\alpha$.

Neglecting the variance of K or supposing it compensated, from the first equation in (15), the vector $\vec{K}$ is seen to precess with the frequency

$$\Omega_r = \frac{2a_2 P_2(\cos\chi)}{K}$$

around the equilibrium state

$$r^* = -\varepsilon_{11} \frac{\sin 2\chi}{2P_2(\cos\chi)} e^{-j\phi_{11}} \ . \tag{16}$$

In order to shift $\vec{K}$ from the position r_1 to the position r_2 during some finite time, the depth ε_{11} and the modulation phase ϕ_{11} are chosen from the condition

$$| \, r_1 - r^* \, | = | \, r_2 - r^* \, | = R \ .$$

Then, the vector $\vec{K}$ will travel from the point r_1 to the point r_2 along the arc of radius R for the time

$$T_{12} = \frac{1}{\Omega_r} \arcsin \frac{| \, r_2 - r_1 \, |}{2R} \ . \tag{17}$$

Here, the nutation angle will decrease from its initial value ϑ_0 to 0 for the time

$$T(\vartheta_0) = K\vartheta_0 [\varepsilon a_{01} \sin \phi_{01} \sin 2\chi]^{-1} \ . \tag{18}$$

This time becomes minimal for the modulation phase

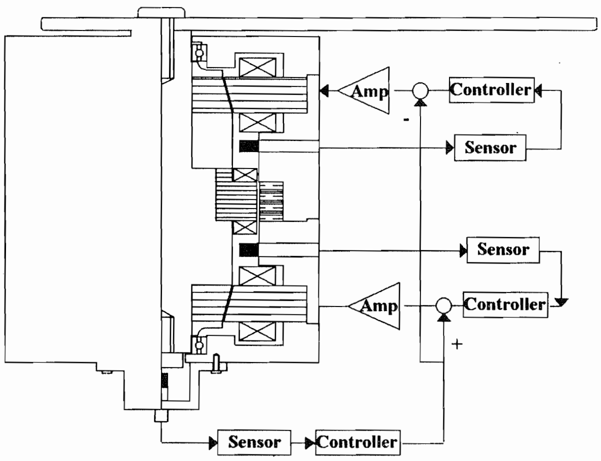

FIGURE1:Magnetically Suspended Robot Joint System

$$g_{7,8} = g_o + x\sin\beta \pm (z + l_2\theta_y)\cos\beta , \qquad (2)$$

where l_1, l_2 are the distances of the upper and lower bearings from the mass center, respectively, and β is the inclined angle of magnet core. As a function of the gap and the current, the change in magnetic force can be written as

$$F_1 = \frac{\alpha_g \mu_o A N^2 (I_{o1} + i_1)^2}{4\left(g_o - cx\sin\beta - c(y + l_1\theta_z)\cos\beta\right)^2} \qquad (3)$$

$$\alpha_g = \frac{\sin\theta_2 - \sin\theta_1}{\theta_2 - \theta_1}, \; c = \frac{\cos\theta_1 + \cos\theta_2}{2}$$

where I_{o1} is the bias current in the upper bearing, g_o is the steady state air gap, i_1 is the control current, α_g is the force factor and c is the shape factor. Assuming that the changes in current and displacement of rotor are relatively small compared with the bias current and the nominal air gap, the magnetic force in Eq.(3) can be linearized, using Taylor series expansion, as

$$F_1 \approx \frac{\alpha_g \mu_o A N^2}{4g_o^2}\left[1 + 2\frac{i_1}{I_{o1}}\right]\left[1 + 2c\frac{x\sin\beta + (y + l_1\theta_z)\cos\beta}{g_o}\right]$$

$$F_{1,2} = F_{o_1} + K_{i_1} i_1 + K_{q_1} x\sin\beta \pm K_{q_1}(y + l_1\theta_z)\cos\beta \qquad (4)$$

$$F_{3,4} = F_{o_1} + K_{i_1} i_3 + K_{q_1} x\sin\beta \pm K_{q_1}(z - l_1\theta_y)\cos\beta$$

$$F_{5,6} = F_{o_2} + K_{i_2} i_5 - K_{q_2} x\sin\beta \pm K_{q_2}(y - l_2\theta_z)\cos\beta$$

$$F_{7,8} = F_{o_2} + K_{i_1} i_7 - K_{q_2} x\sin\beta \pm K_{q_2}(z + l_2\theta_y)\cos\beta$$

where

$$F_{o_1} = \frac{\alpha_g \mu_o A N^2 I_{o_1}^2}{2g_o^2}, \; F_{o_2} = \frac{\alpha_g \mu_o A N^2 I_{o_2}^2}{2g_o^2}$$

$$K_{q_1} = \frac{c\alpha_g \mu_o A N^2 I_{o_1}^2}{2g_o^3}, \; K_{q_2} = \frac{c\alpha_g \mu_o A N^2 I_{o_2}^2}{2g_o^3}$$

$$K_{i_1} = \frac{\alpha_g \mu_o A N^2 I_{o_1}}{2g_o^2}, \; K_{i_2} = \frac{\alpha_g \mu_o A N^2 I_{o_2}}{2g_o^2}$$

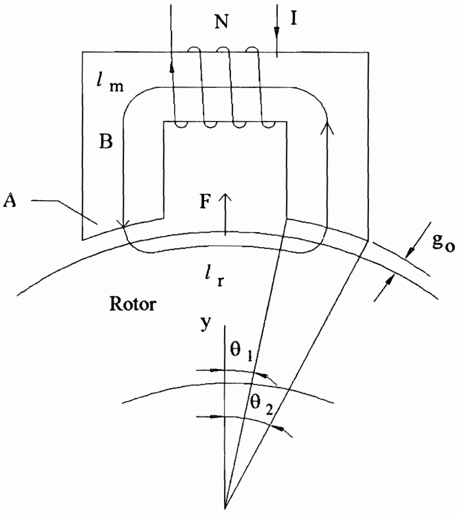

FIGURE 2 : Configuration of Magnet

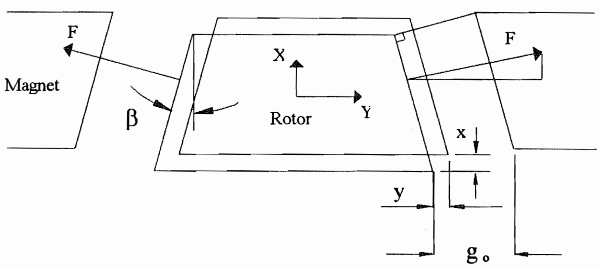

FIGURE 3 : Inclined Magnet Geometry

and I_{o_2} is the bias current in the lower bearing, i_1, i_2, i_3, . . . ,i_8 are the control currents, F_{o_j}, j=1,2,, are the steady state magnetic forces, K_{i_j}, j=1,2, are the magnetic force sensitivity for current and K_{q_j}, j=1,2, are the magnetic force sensitivity for displacement.

Equations of Motion

Assuming that the rotor is rigid, as shown in **FIGURE** 4, we can write the equations of motion in the mass center coordinate(x, y, z, θ_y, θ_z) as,

$$m\ddot{x} = \sum_{i=1}^{4} F_i \sin\beta - \sum_{i=5}^{8} F_i \sin\beta - mg$$

$$m\ddot{y} = (F_1 - F_2 + F_5 - F_6)\cos\beta$$

$$m\ddot{z} = (F_3 - F_4 + F_7 - F_8)\cos\beta \qquad (5)$$

$$I_d\ddot{\theta}_y + \Omega I_p\dot{\theta}_z = (F_4 - F_3)l_1\cos\beta + (F_7 - F_8)l_2\cos\beta + (F_3 - F_4 - F_7 + F_8)R_m\sin\beta$$

$$I_d\ddot{\theta}_z - \Omega I_p\dot{\theta}_y = (F_1 - F_2)l_1\cos\beta + (F_6 - F_5)l_2\cos\beta + (F_2 - F_1 - F_6 + F_5)R_m\sin\beta$$

where R_m is the effective radius of magnet core, I_d is the diametrical moment of inertia, I_p is the polar moment of inertia and Ω is the rotational speed. Since the control is performed in the bearing coordinate(x, y_1, y_2, z_1, z_2), the equations of motion must be transformed