FINITE ELEMENT ANALYSIS OF THE MAGNETIC FORCES ACTING ON AN ECCENTRIC ROTOR OF A HIGH-SPEED INDUCTION MOTOR

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ABSTRACT

The calculation of the radial forces acting on an eccentric rotor of a high-speed induction motor is based on a time-stepping, finite element analysis of the magnetic field of the motor. The field is assumed to be two-dimensional, and the field equation and the circuit equations of the windings are solved together as a system of equations. The forces are calculated from the air-gap field by a method based on the principle of virtual work. The method was verified by measuring the forces in a 34 kW, 36 000 rpm cage induction motor. The motor has magnetic bearings that could be used for measuring the forces they transmit. The non-symmetric flux distribution caused by the eccentricity induces circulating currents in the stator and rotor windings. These currents have a strong reducing effect on the radial total force.

INTRODUCTION

The development of power-electronic components has solved many of the problems associated with the power supply of high-speed a.c. motors, and convertors capable of producing up to 100 kVA within the frequency range 0–2.5 kHz are available. The construction of motors is now the limiting factor for many high-speed applications. When designing bearings for motors, one has to know the bearing loads. In addition to the weight of the rotor and the forces caused by the load machine, the electrical motor itself may generate electromagnetic forces that have to be supported by the bearings. This is the case, for instance, when the rotor is not concentric with the stator bore. The object of the present work is to estimate the magnetic forces caused by an eccentric rotor on the bearings of a high-speed motor. The analysis is restricted to rotors with static eccentricity, i.e. to cases in which the centre axis of the rotor remains stationary with respect to the stator.

When a motor has magnetic bearings, it is relatively easy to use the bearings for measuring the forces they transmit. This was done for a 34 kW, 36 000 rpm induction motor, and the forces obtained showed a clear disagreement with the values obtained from the force equations given by textbooks of electrical machines [1]. The forces measured were quite small, they were strongly dependent on the rotational speed, and they were not in the direction of the shortest air gap as the simple theory would predict.

A literature survey revealed that the reason for the disagreement was probably associated with the eddy currents induced in the rotor conductors. These currents try to equalise the flux distribution in which the rotor rotates, and thus, they reduce the force acting on the eccentric rotor. The survey also showed that such factors as the eddy currents, stator and rotor slotting, saturation and unipolar flux, which is caused by the eccentricity, are quite difficult to include in analytical force calculations. This gave the reason for using time-stepping, finite element analysis for calculating the forces.

Numerical methods have been rarely used for analysing eccentric rotors. DeBortoli et al. [2] used a time-stepping method for studying the equalising currents set up by an eccentric rotor in the parallel circuits of stator windings. They also presented some results of force calculation. On the other hand, analytical methods have been applied widely to problems associated with the eccentricity. Freise & Jordan [3] derived equations for the forces caused by the eccentricity. In these equations, they used damping factors for taking into account the force reduction caused by the equalising currents. They also noticed that these currents change the direction of the total force from the direction of the shortest air gap. In addition, they discussed the effect of eccentricity forces on the critical speeds of a rotor.

Meiler et al. [4] compared the forces measured for several asynchronous motors with the corresponding analytical results. The agreement was often quite bad. The authors longed for better methods to model the effects of equalising currents, stator and rotor slottings, leakage fluxes, and saturation. Some years later, Jaenicke & Jordan [5] presented equations for taking these factors approximately into account.
Belmans et al. [6] showed that an eccentric rotor in a two-pole machine may cause a significant unipolar flux closing through the shaft, bearings, end-caps and frame. The magnitude of the flux depends on the reluctance of this path. A unipolar flux increases the static pull and generates an additional force component varying at twice the supply frequency. The a.c. component may cause problems by exciting the natural vibrational modes of a rotor.

METHOD OF MEASUREMENT

Figure 1 presents a simplified schema of a radial magnetic bearing.

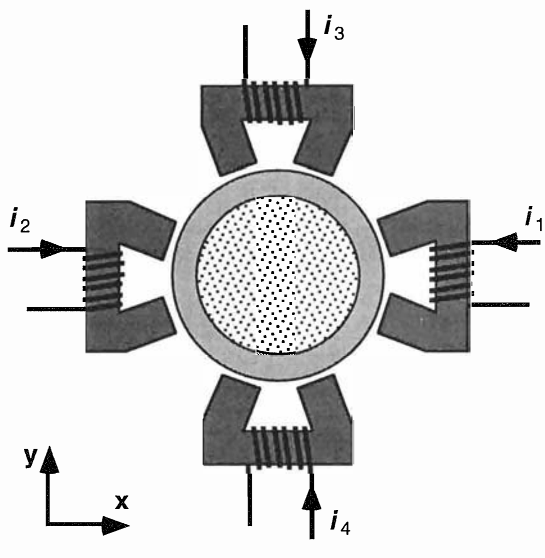

FIGURE 1. A simplified schema of a radial magnetic bearing.

The forces supported by the bearing can be determined by measuring the four currents i_1, ... i_4. In a linearised model, the forces in the two perpendicular directions are obtained from the equations

$$\begin{aligned} F_x &= k_1 i_1^2 - k_2 i_2^2 \\ F_y &= k_3 i_3^2 - k_4 i_4^2 \end{aligned} \qquad (1)$$

The equations have four unknown parameters (k_1, ..., k_4) that can be determined by applying four external forces on the rotor and measuring the values of the currents. In our case, the support of the motor was so constructed that is was easy to turn the motor from the horizontal position to the vertical one. Thus, it was natural to use the weight of the rotor for determining the k_i coefficients. The validity of Eqs. (1) was verified by applying an additional radial force on the shaft and computing the bearing forces both from Eqs. (1) and from the mechanical force balance for the rotor. An accuracy of ± 5 % was achieved in the force range of interest.
The first task in the measurement process was to find the central position for the rotor. This was done by supplying a d.c. current successively in the three phase windings and searching for such a rotor position in which the currents did not produce any measurable force in the bearings. We noticed soon that it was impossible to find such a rotor position in which the no-force condition would be satisfied exactly. The reason is probably a slightly non-circular stator bore, the radius of which might contain, for instance, a small third harmonic component. In the best 'central position' the forces caused by the d.c. stator currents were so small, when compared with the forces obtained for the eccentric rotor, that this position was considered acceptable. After this, the proper eccentricity was obtained by tuning the electronics of the magnetic bearings and measuring simultaneously the movement of the rotor by micrometers. For measuring the results of this paper the rotor was displaced by 0.11 mm, i.e. the eccentricity was 15 % of the mean air gap.

METHOD OF ANALYSIS

The calculation of the operating characteristics of the motor is based on the numerical solution of the magnetic vector potential in the core region of the machine. The magnetic field is assumed to be two-dimensional, and the two-dimensional field equation is discretized by the finite element method. The effects of end-region fields are taken into account approximately by constant end-winding impedances in the circuit equations of the windings. The finite element approximation of the vector potential is substituted in the circuit equations and they are solved together with the discretized field equations.
The time-dependence of the field is modelled by the Crank-Nicholson method. The rotor is rotated by changing the finite element mesh in the air gap. The nonlinear system of equations obtained at each time-step is solved by the Newton-Raphson method. The details of the method of analysis are given in Reference [7].
Several approximations have to be made in order to keep the amount of computation at a low enough level. The main simplification is the assumption of a two-dimensional magnetic field in the core region. The laminated iron core is treated as a non-conducting, magnetically nonlinear medium, and the non-linearity is modelled by a single valued magnetisation curve. Thus

the various loss components usually included under the title 'core losses' are neglected when solving the magnetic field.

Calculation of the forces. The method presented by Coulomb [8] was used for computing the electromagnetic forces. It is based on the virtual-work principle, and the forces are obtained as volume integrals computed in a cylindrical air layer surrounding the rotor. In the two-dimensional case the calculation reduces to a surface integration over the finite elements in the air gap. This method was chosen as it and other methods based on a similar integration have given good results when computing the torques of electrical machines [7], [9].

RESULTS

Figure 2 shows a quadrant of the cross-section of the 34 kW motor used in the measurements, and Table 1 gives its main parameters. It is a solid-rotor induction motor equipped with an aluminium rotor cage. The rotor, which is unskewed, has circumferential grooves on its surface which are 1 mm wide and 2.5 mm deep. They are used for reducing eddy-current losses caused by stator slot harmonics on the solid rotor surface. The grooves are a potential error source for the two-dimensional analysis. Their effect was modelled by reducing the conductivity and permeability of the steel in the grooved surface layer. The stator winding has two parallel paths. According to the analysis of References [5] and [6], the eccentricity causes a second space-harmonic component in the air-gap flux. This component is able to induce circulating currents in the parallel circuits. Thus, there is the possibility for the equalising currents to exist both in the rotor cage and stator winding.

Figure 3 shows the radial forces measured and computed for the unloaded motor as functions of the stator current. The components for the force are given on two perpendicular axes, the x-axis being in the direction of the shortest air gap. The frequency of the sinusoidal supply voltage is used as the parameter. The force in the x-direction decreases significantly with the increasing supply frequency. This is caused by the equalising currents trying to symmetrise the flux distribution in the motor. At the lowest frequencies the flux variation is too slow to induce effective equalising currents, and the force in the x-direction is relatively strong. At low frequencies, the equalising currents affect the direction of the force and there is quite a strong force component in the y-direction, too.

The agreement between the measured and computed forces is, in general, relatively good. The variation in the measured y-components shows that at least the smallest values are quite close to the limit set by the accuracy of

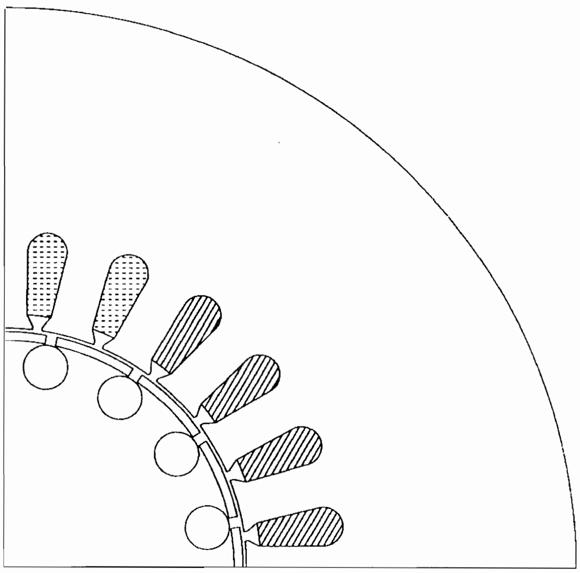

FIGURE 2. Cross-sectional geometry of the 36 000 rpm, 34 kW induction motor used for the measurements.

TABLE 1. Main parameters of the test motor.

Number of pole pairs	1
Number of phases	3
Number of parallel paths	2
Stator diameter [mm]	235
Rotor diameter [mm]	99
Air gap [mm]	0.75
Core length [mm]	90
Weight of the rotor [N]	260
Connection	star
Rated voltage [V]	380
Rated frequency [Hz]	600
Rated current [A]	72
Rated power [kW]	34

the measuring system. The measurements were restricted to the low supply frequencies at which the magnetic pull is the largest

The effect of the frequency can be seen clearly in Figure 4 presenting the components of the unbalanced magnetic pull as functions of the supply frequency. The curves have been computed for an unloaded motor, and the fundamental air-gap flux of the motor has been kept constant for all the frequencies. The equalising currents change both the amplitude and the direction of the total force. At the highest rotational speeds, the total force is less than 20 % of its value at the lowest speeds. The perpendicular force component has the maximum at about 2 Hz, and the maximum deviation of the force from the direction of the shortest air gap is somewhat over 40 degrees.

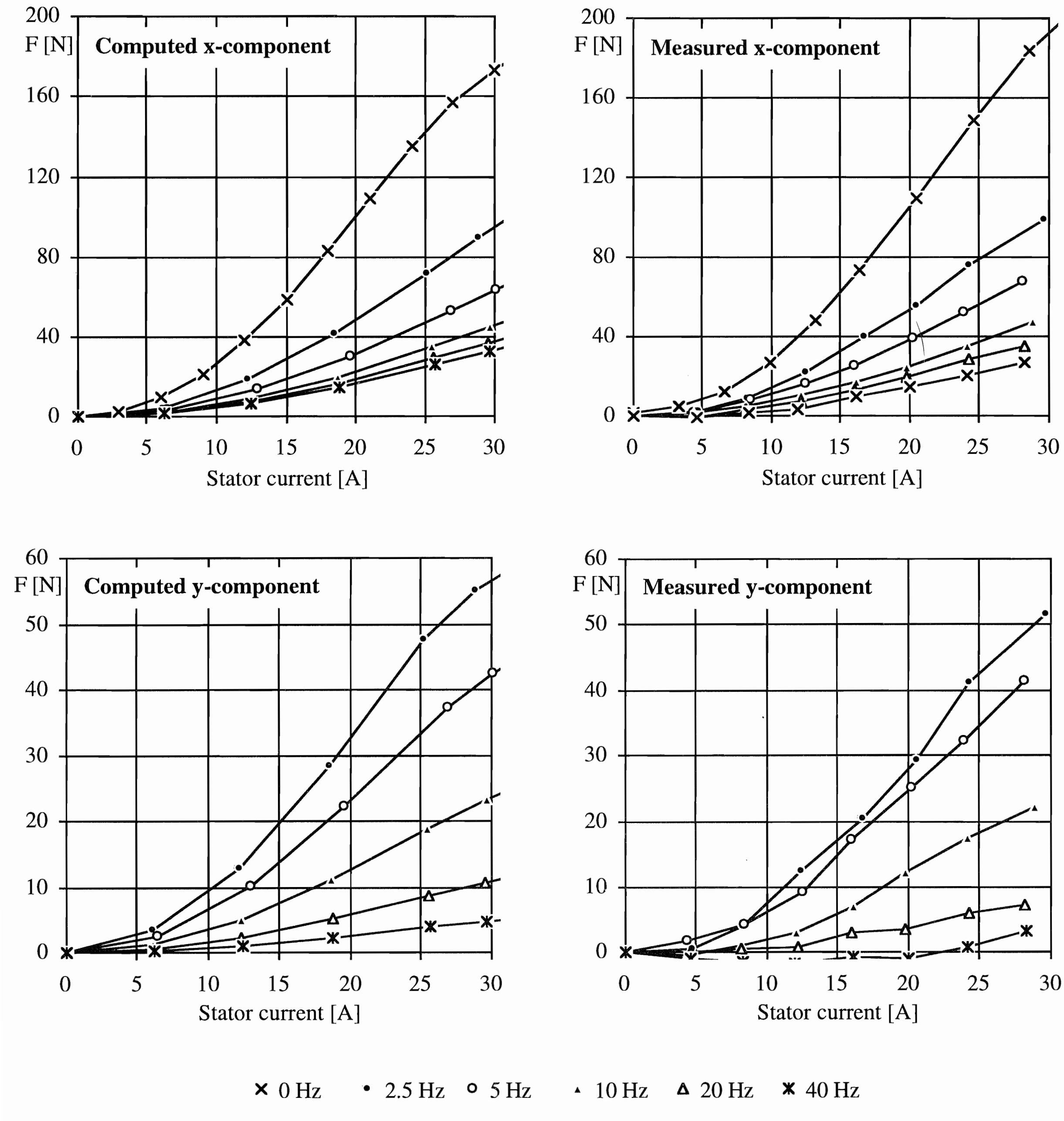

FIGURE 3. The force components computed (on the left) and measured (on the right) for the unloaded motor with the rotor displaced by 15 % of the mean air gap. The *x*-axis coincides with the direction of the shortest air gap. The forces are shown as functions of the stator current, and the supply frequency has been used as the parameter. The rated no-load current of the motor is 28 A and, for reference, the weight of the rotor is 260 N.

It was checked by numerical simulations that the reduction of the force was mainly caused by the currents induced in the rotor. When simulating a no-load operation, the equalising rotor currents can be suppressed by reducing the conductivities of the rotor materials. The equalising currents in the stator can be eliminated by connecting the parallel paths in series. If both these methods are used, one obtains the undamped value for the unbalanced magnetic pull. In the simulations for the no-load operation at 600 Hz, the equalising stator currents could reduce the force by 42 % from the undamped value, the rotor currents alone caused a 81 % reduction, and a 82 % reduction was obtained by the common action of both the currents.

From Figure 4 one can draw the conclusion that the equalising currents are so efficient in reducing the unbalanced magnetic pull that the eccentricity does not cause any problems at the rated rotational speeds of high-speed motors. The situation is not, however, quite so simple. Figure 5 shows the effect of loading on the total force. When computing these results, a six-step voltage with one 120° pulse per half cycle was used for modelling the convertor supply. The RMS value of the fundamental component of the voltage was 380 V.

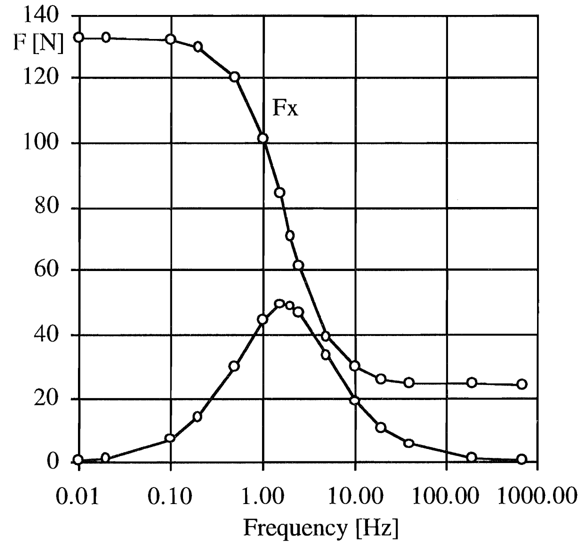

FIGURE 4. Components of the unbalanced magnetic pull as functions of the supply frequency. The forces have been computed for the unloaded motor with the rotor displaced in the *x*-direction by 15 % of the air gap. The motor was supplied by such sinusoidal voltages that the fundamental air-gap flux was constant at all the frequencies.

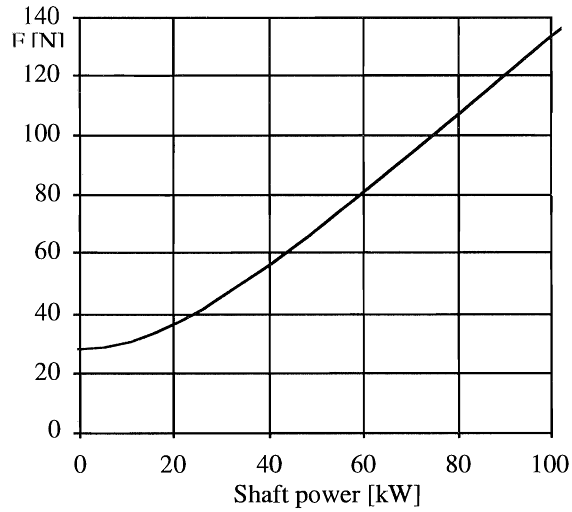

FIGURE 5. The effect of loading on the total force computed for the test motor when supplied by the rated voltage. The force is given as a function of the shaft power of the motor.

The magnetic pull increases with the load. In Reference [4] this behaviour is explained as the effect of the increased leakage flux in a loaded machine. The largest shaft power included in the figure exceeds the rated power (34 kW) considerably. At the rated power, the total force still remains much smaller than the values obtained for the unloaded motor at low supply frequencies. High-speed motors are usually supplied by frequency convertors. By preventing the over-loading of the motor, the convertor helps to keep the radial forces under control.

Only the static components of the radial forces have been discussed so far. Figure 6 shows the time variation of the total force acting on the loaded rotor. In the upper figure the motor is supplied by a sinusoidal voltage and in the lower one by the six-step voltage, the fundamental harmonics of the two voltages being equal. The forces contain a relatively large a.c. component varying at twice the supply frequency. In Reference [6] this kind of variation was shown to activate the natural vibrational modes of a rotor. The effect of the non-sinusoidal supply can be clearly seen in the wave form of the lower figure although the average value for the force is almost independent of the supply mode.

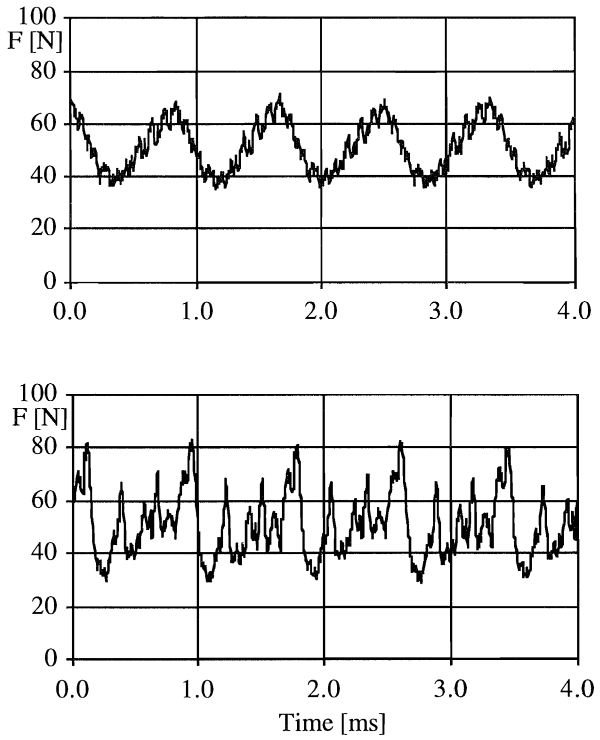

FIGURE 6. The time variation of the total radial force at the rated operation point of the motor. The upper figure is associated with the sinusoidal supply, the lower one with the inverter supply.

Figure 7 presents a comparison between the forces obtained by analytical equations [6] and by the numerical method. The results are given as functions of the fundamental air-gap flux density. The numerical results have been computed for the frequencies 0.01 and 600 Hz. The lower frequency corresponds, in practice, to the d.c. case but the forces were integrated over one period of the supply frequency in order to eliminate the effects of possible force fluctuations caused, for instance, by the slotting. The analytical results agree well with the numerical d.c. results as long as the core is non-saturated. The analytical model neglects the saturation and leads to a quadratic dependence of the flux density. The strong action of the equalising currents at higher frequencies is also missing from the analytical results.

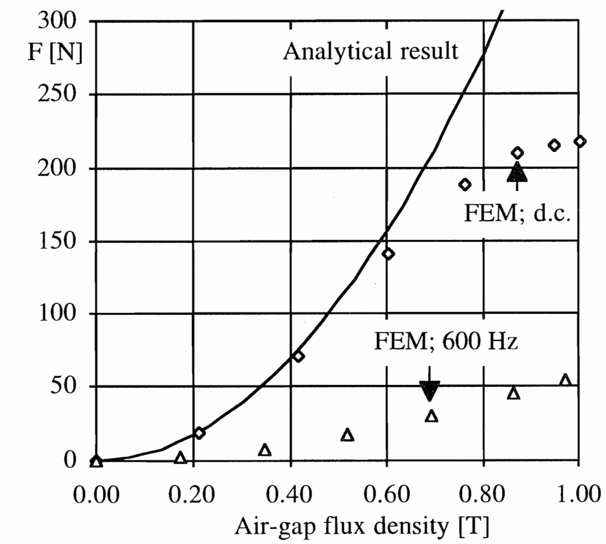

FIGURE 7. Comparison of the analytical result with the numerical ones. The radial force acting on the eccentric rotor is presented as a function of the air-gap flux density. The analytical equation is derived in Reference [6] and it has been applied by assuming that there is no unipolar flux. The numerical results have been computed for the unloaded motor supplied by 0.01 Hz sinusoidal voltages (d.c. case) and 600 Hz six-step voltages.

CONCLUSIONS

A time-stepping, finite element method has been used for computing the forces acting on an eccentric rotor of a high-speed induction motor. The forces were measured at the no-load operation, and the agreement between the measured and computed results is good. The method of analysis is versatile including the effects of equalising currents, slotting and saturation. The unipolar flux that may be caused by the eccentricity could not be taken into account in the two-dimensional analysis.

According to the measured and computed results, the conventional equations used for calculating the forces give too large values in the normal operation range of high-speed motors. In the special case studied, the total force was reduced by about 80 % because of the equalising currents flowing mainly in the rotor conductors. The largest radial forces are to be expected at very low frequencies when starting a motor.

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